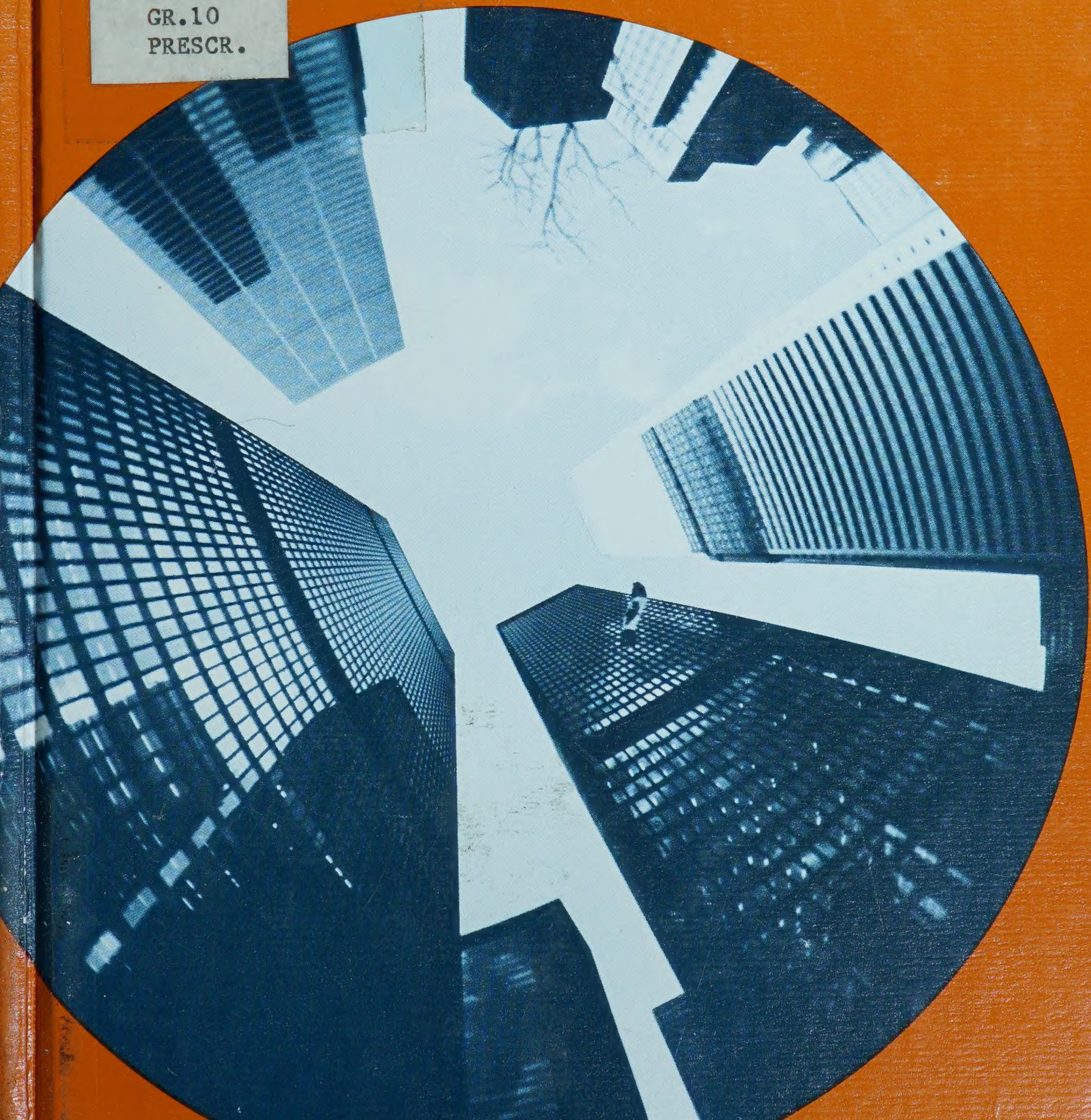
Book 2
Second Edition

Baxter
Newton
Del Grande

mathematics for a modern world

MATH
GR.10
PRESCR.



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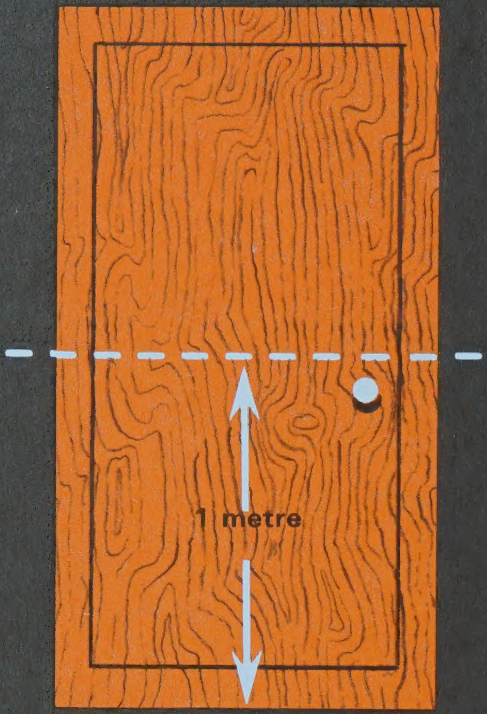
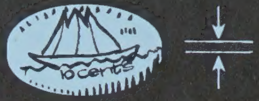
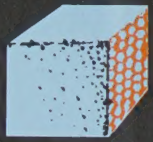


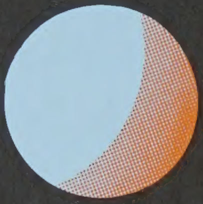
1 metre (1 m) is about
half the height of a door



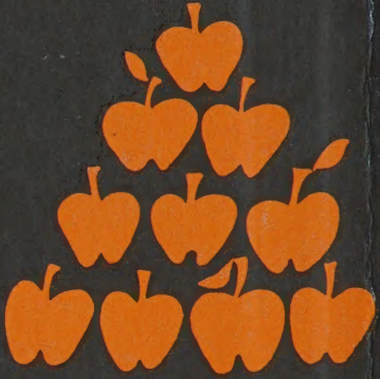
1 millimetre (1 mm) is
about the thickness of
a dime



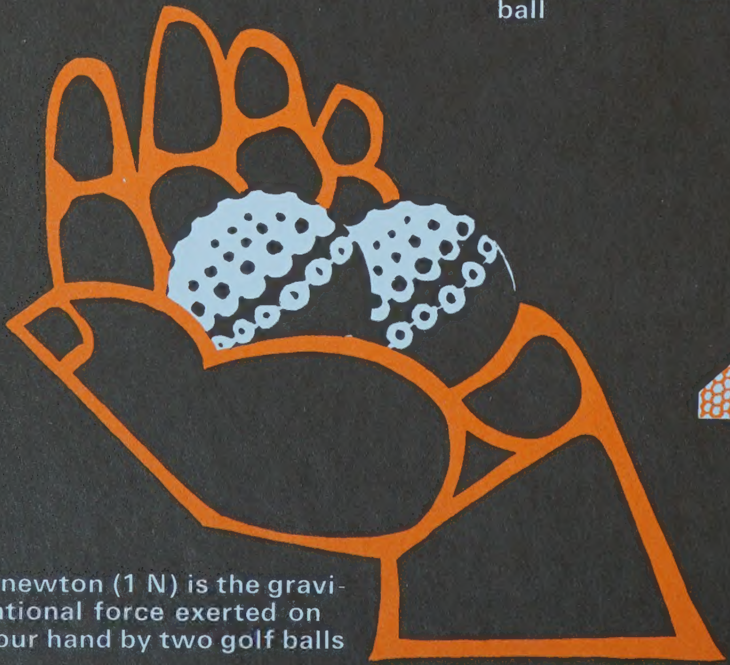
1 cubic centimetre
(1 cm^3) is smaller
than a sugar cube



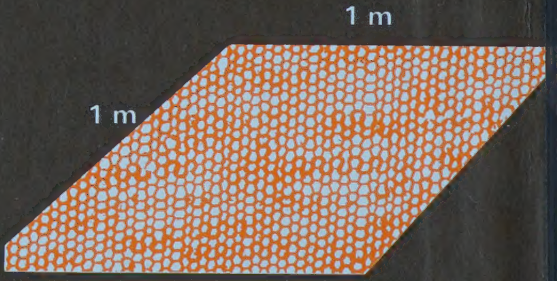
1 gram (1 g) is about
the mass of a ping-pong
ball



1 kilogram (1 kg) is
about the mass of te
medium-sized apple

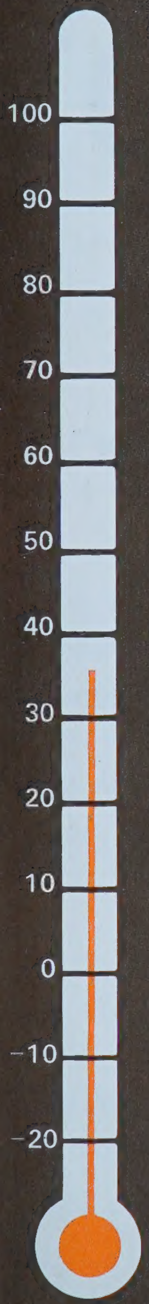


1 newton (1 N) is the gravitational force exerted on
your hand by two golf balls



1 pascal (1 Pa) is the pressure
produced on a square metre
by 1 N (the two golf balls,
ground up)

metric units



THE CELSIUS
THERMOMETER

water boils at 100°C

bath water

normal body
temperature 37°C

a fine warm day

cool weather

water freezes at 0°C

cold

BICYCLE
mass about 10 kg
racing speed about 40 km/h

AUTOMOBILE
mass about 1950 kg
(or nearly 2 t)
highway speed about 100 km/h
gas tank holds about 90 ℓ
tire pressure about 200 kPa

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Mathematics for a Modern World

Book 2, Second Edition

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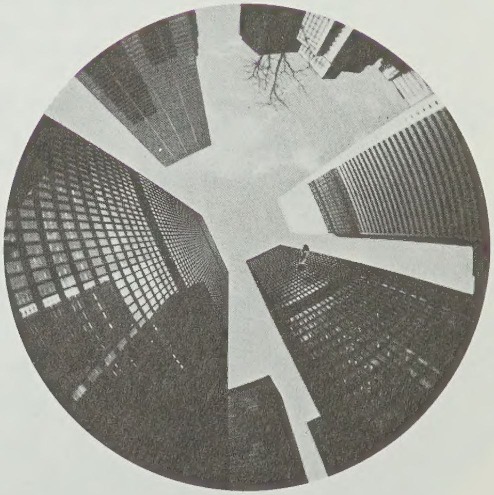

Published by Gage Educational Publishing Limited, Toronto

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Special thanks go to D'Arcy Glionna and Birgitte Nielsen, for their photographs of the students, and to the students for their versatility and good humor.

ISBN 0-7715-3593-7

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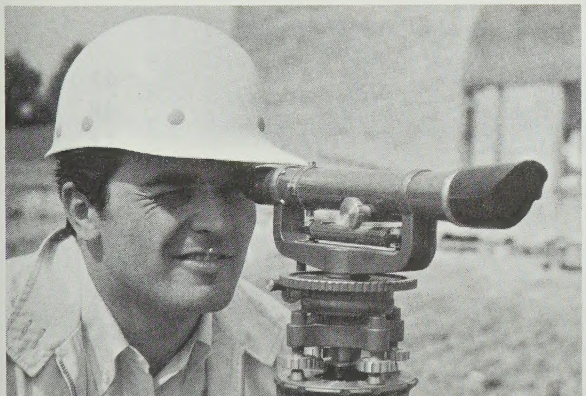
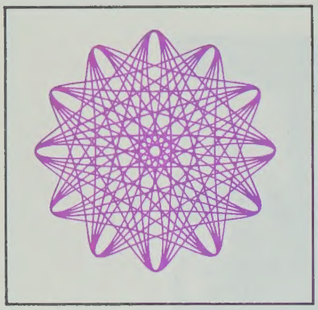
To the student:

Math — a stepping stone
to the future.
Don't slip!

The authors

TEXAS INSTRUMENTS





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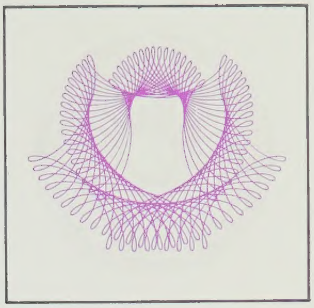

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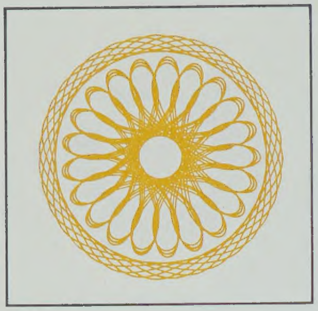

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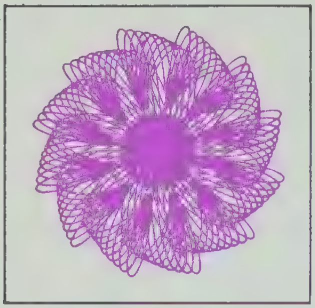

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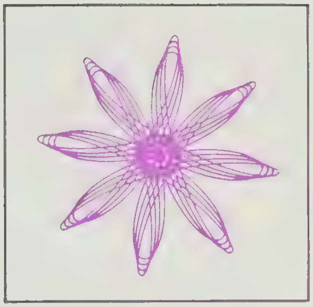

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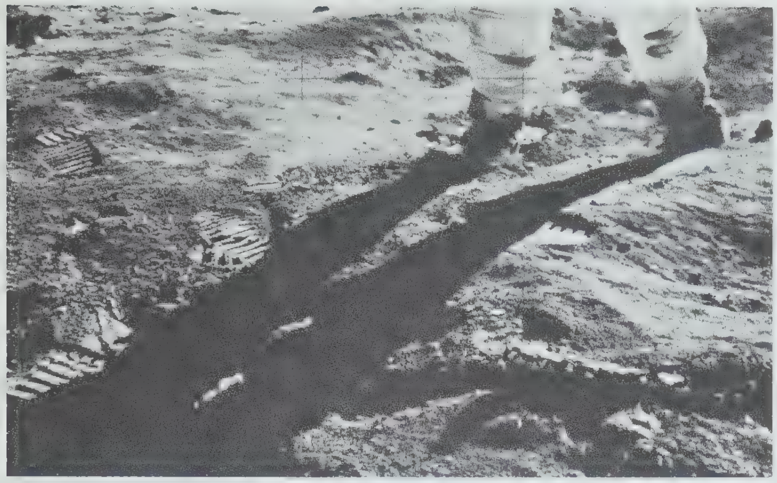


Unit 1
Computation

On Sunday, July 20, 1969, at 16 17 43 Eastern Daylight Time, Neil Armstrong and Edwin Aldrin touched down on the surface of the moon in their lunar module *Eagle*.

This first step on the stairway to the stars was made possible by that miracle of modern technology, the *computer*. The computer is really not very smart, but it is able to perform thousands (or even millions) of simple calculations in *seconds*. Without this partner in computation, we would still be bound to earth.

One of the astronauts' prime objectives was to set up experiments and measuring devices to give scientists on earth a more complete picture of our closest neighbour in space. Although the accuracy of measurement in these experiments staggers the imagination, remember that the people responsible for designing and executing them were once concerned with topics like those you are about to study.

EASEP
Early Apollo
Scientific Experiments
Package.

1. Solar wind experiment to trap electrified gases from the sun on a sheet of aluminum foil.

2. Lunar seismometer to detect tremors and quakes caused by meteors striking the moon, and to determine lunar "tides".

3. Supermirror to reflect laser beams from earth in order to determine the distance from earth to the moon with an error of less than 15 cm!

+

I'm a calculating whiz!

−

What's the difference?

1(e) 235 (m) 472 $\frac{19}{40}$
(g) $8\frac{1}{3}$ (o) 1.603
(i) $1\frac{7}{12}$ (q) 23.30
(k) $23\frac{11}{24}$

2(e) 257 (o) 73.27
(g) 768 (q) 8.915
(i) $5\frac{5}{28}$ (s) 788.21
(k) $64\frac{9}{20}$ (u) 0.0064
(m) $\frac{11}{30}$

Get-Set Exercise — Review of Arithmetic

1. Add.

(*a*) $\begin{array}{r}426\\319\\718\\\underline{265}\end{array}$

(*b*) $\begin{array}{r}7431\\6925\\7008\\\underline{1096}\end{array}$

(*c*) $\begin{array}{r}21\,563\\94\,812\\3\,594\\\underline{1\,226}\end{array}$

(*d*) $\begin{array}{r}2\,957\\34\,162\\28\,995\\\underline{317}\end{array}$

(*e*) $73 + 49 + 62 + 51$

(*f*) $2143 + 7625$

(*g*) $\begin{array}{r}4\frac{1}{2}\\\underline{3\frac{5}{6}}\end{array}$

(*h*) $\begin{array}{r}2\frac{1}{8}\\\underline{19\frac{1}{4}}\end{array}$

(*i*) $\begin{array}{r}\frac{5}{6}\\\underline{\frac{3}{4}}\end{array}$

(*j*) $\begin{array}{r}11\frac{2}{5}\\\underline{5\frac{3}{10}}\end{array}$

(*k*) $\begin{array}{r}2\frac{1}{4}\\13\frac{5}{6}\\\underline{7\frac{3}{8}}\end{array}$

(*l*) $4\frac{2}{7} + 5\frac{11}{14}$

(*m*) $216\frac{3}{5} + 92\frac{1}{4} + 163\frac{5}{8}$

(*n*) $\begin{array}{r}61.52\\7.03\\\underline{121.562}\end{array}$

(*o*) $\begin{array}{r}0.003\\0.502\\\underline{1.098}\end{array}$

(*p*) $\begin{array}{r}257.9\\6.053\\\underline{19.178}\end{array}$

(*q*) $17.32 + 5.98$

(*r*) $615.08 + 93.772 + 64.13$

2. Subtract.

(*a*) $\begin{array}{r}95\\\underline{42}\end{array}$

(*b*) $\begin{array}{r}6754\\\underline{2101}\end{array}$

(*c*) $\begin{array}{r}49\,376\\\underline{21\,553}\end{array}$

(*d*) $\begin{array}{r}1\,537\,961\\\underline{205\,834}\end{array}$

(*e*) $473 - 216$

(*f*) $42\,591 - 6304$

(*g*) $911 - 143$

(*h*) $\begin{array}{r}8\frac{5}{6}\\\underline{5\frac{1}{3}}\end{array}$

(*i*) $\begin{array}{r}24\frac{3}{7}\\\underline{19\frac{1}{4}}\end{array}$

(*j*) $\begin{array}{r}\frac{9}{10}\\\underline{\frac{3}{5}}\end{array}$

(*k*) $\begin{array}{r}143\frac{1}{4}\\\underline{78\frac{4}{5}}\end{array}$

(*l*) $15\frac{3}{5} - 12\frac{1}{2}$

(*m*) $2\frac{1}{6} - 1\frac{4}{5}$

(*n*) $193\frac{5}{8} - 116\frac{3}{4}$

(*o*) $\begin{array}{r}97.83\\\underline{24.56}\end{array}$

(*p*) $\begin{array}{r}317.9\\\underline{42.68}\end{array}$

(*q*) $\begin{array}{r}9.573\\\underline{0.658}\end{array}$

(*r*) $\begin{array}{r}2156.3\\\underline{425}\end{array}$

(*s*) $814.79 - 26.58$

(*t*) $9.351 - 8.56$

(*u*) $0.015 - 0.0086$

3. Multiply.

(*a*) $\begin{array}{r} 42 \\ 9 \\ \hline \end{array}$

(*b*) $\begin{array}{r} 316 \\ 41 \\ \hline \end{array}$

(*c*) $\begin{array}{r} 215 \\ 384 \\ \hline \end{array}$

(*d*) $\begin{array}{r} 2563 \\ 17 \\ \hline \end{array}$

(*e*) $\begin{array}{r} 42\,153 \\ 108 \\ \hline \end{array}$

(*f*) 16×84

(*g*) $32 \times 11 \times 15$

(*h*) $4 \times 72 \times 9$

(*i*) $326 \times 4271 \times 18$

(*j*) $\frac{2}{3} \times \frac{5}{7}$

(*k*) $4\frac{1}{2} \times 3$

(*l*) $2\frac{1}{5} \times 11\frac{1}{4}$

(*m*) $\frac{2}{3} \times \frac{5}{6} \times \frac{9}{20}$

(*n*) $\begin{array}{r} 42 \\ 1.9 \\ \hline \end{array}$

(*o*) $\begin{array}{r} 63.2 \\ 7.5 \\ \hline \end{array}$

(*p*) $\begin{array}{r} 9.183 \\ 0.94 \\ \hline \end{array}$

(*q*) $\begin{array}{r} 513.75 \\ 26.14 \\ \hline \end{array}$

(*r*) 7.9×1.2

(*s*) 0.93×16

(*t*) $32.5 \times 11.4 \times 0.653$

4. Divide.

(*a*) $41\overline{)4346}$

(*b*) $62\overline{)36\,518}$

(*c*) $11.7\overline{)2702.7}$

(*d*) $\dfrac{2112}{22}$

(*e*) $\dfrac{5.928}{10.4}$

(*f*) $\dfrac{35.22}{6}$

(*g*) $\dfrac{1196.98}{97}$

(*h*) $73\overline{)62.05}$

(*i*) $0.09\overline{)0.504}$

(*j*) $243.1\overline{)115.9587}$

(*k*) $6 \div \frac{3}{4}$ *

(*l*) $5\frac{1}{2} \div 2\frac{1}{8}$

(*m*) $\dfrac{3\frac{1}{4}}{4\frac{1}{3}}$

(*n*) $\frac{5}{6} \div 3$

(*o*) $3\frac{3}{4} \div 1\frac{1}{2}$

(*p*) $\dfrac{5}{\frac{3}{4}}$

(*q*) $1\frac{1}{3} \div 2\frac{1}{5}$

5. Evaluate.

(*a*) $43 + 95 - 65 + 100 - 29$

(*b*) $6511 - 2342 + 117$

(*c*) $147.3 - 19.56 + 24.31$

(*d*) $642.5 + 11.73 - 0.009$

(*e*) $\dfrac{5\frac{1}{2} + 3\frac{1}{4}}{\frac{5}{6}}$

(*f*) $\dfrac{7\frac{1}{8} - 2\frac{3}{4} + \frac{1}{2}}{1\frac{1}{3} + \frac{5}{6}}$

(*g*) $8.32 \times (16.1 + 11.03)$

(*h*) $(43.62 + 5.099) \div (11.48 - 1.18)$

(*i*) $\dfrac{0.03 \times 32\,922.2}{8.42}$

(*j*) $\dfrac{81 \times 6.7 \times 11.5}{9.2 \times 2.5}$

×

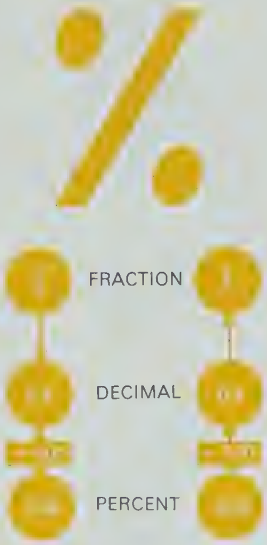
$$\begin{array}{rl} 2.3 & \rightarrow 1 \text{ decimal} \\ 5.7 & \rightarrow 1 \text{ decimal} \\ \hline 161 & \\ 115 & \\ \hline 13.11 & \rightarrow 2 \text{ decimals} \end{array}$$

÷

Remember!
To divide by a fraction, multiply by its reciprocal!

$$* \quad 6 \div \underbrace{\tfrac{3}{4} = 6 \times \tfrac{4}{3}}_{\text{Reciprocals}}$$

3(a)	378	(k)	$13\frac{1}{2}$
(c)	82 560	(m)	$\frac{1}{4}$
(e)	4 552 524	(o)	474.00
(g)	5280	(r)	9.48
(i)	25 062 228	(s)	14.88
4(a)	106	(k)	8
(c)	231	(m)	$\frac{3}{4}$
(e)	0.57	(o)	$2\frac{1}{2}$
(g)	12.34	(q)	$\frac{20}{33}$
(i)	5.6		
5(a)	154	(g)	225.7216
(c)	152.05	(i)	117.3
(e)	$10\frac{1}{2}$		



6. Change to percents.
(*a*) 0.57 (*c*) 0.04 (*e*) 5 (*g*) $\frac{5}{8}$ (*i*) $\frac{2}{5}$
(*b*) 0.63 (*d*) 1.73 (*f*) $\frac{3}{4}$ (*h*) $1\frac{1}{4}$ (*j*) $\frac{5}{6}$

7. Change to decimal numbers, then to common fractions in lowest terms.
(*a*) 25% (*b*) $33\frac{1}{3}\%$ (*c*) 37.5% (*d*) 112.5% (*e*) 260%

8. Hilda Himark's report card reads as follows.

Math	92%	History	59%
Science	73%	Geography	63%
Home Ec.	70%	English	67%
Phys. Ed.	68%	Drafting	71%

Calculate Hilda's average mark, correct to 1 decimal place.

9. In preparation for the opening of school, Freddie Frugal buys the following items at the corner variety store.

11 pencils	at 8¢ each
5 erasers	at 4¢ each
3 rulers	at 10¢ each
2 packages of notepaper	at 29¢ each
1 mathematical drawing set	at $1.19
1 binder	at $1.49
3 felt pens	at 49¢ each
8 ballpoint pens	at 19¢ each

If Freddie must pay an additional 6% sales tax on each item, how much must he pay the check-out clerk?

10. During a football series between the B.C. Lions and the Edmonton Eskimos, gate attendant Cathy Countemall recorded attendance figures 28 492, 31 714, 27 723, and 30 019. Calculate
(*a*) the total attendance at all 4 games, and
(*b*) the average attendance per game.

6(a) 57% (g) 62.5%
(c) 4% (i) 40%
(e) 500%
7(a) 0.25, $\frac{1}{4}$ (e) 2.6, $2\frac{3}{5}$
(c) 0.375, $\frac{3}{8}$
8. 70.4%
9. $8.09
10(a) 117 948 (b) 29 487

1.1 Accuracy

A Whole numbers are those that result from counting. They are *exact*. Measurements, however, can only be *approximated* by a number because measurements are limited to the precision of the measuring instrument used.

Approximations to numbers also occur when exact numbers are *rounded off*. Although the paid attendance at a wrestling match might have been 1623, for example, the sportswriter may report that "about 1600 people" were present.

1. Classify the number in each of the following statements as *exact* or *approximate*.
   - (*a*) There are 32 students in the class.
   - (*b*) About 400 people attended the dance.
   - (*c*) The length of the stick is 12 cm.
   - (*d*) The car consumes 14 ℓ of gasoline in 100 km.
   - (*e*) We sent out 114 Christmas cards last year.
2. Round off 1729.4 to the nearest
   - (*a*) hundred
   - (*b*) ten
   - (*c*) whole number

B The accuracy of a measurement depends upon the number of *significant digits* involved.

Example 1

State the number of significant digits in each number.

(*a*) 18

(*b*) 924.3

(*c*) 0.420

(*d*) 1.2004

(*e*) 0.000 63

(*f*) 217.0

Solution

(*a*) 2 significant digits

(*b*) 4 significant digits

(*c*) 3 significant digits

(*d*) 5 significant digits

(*e*) 2 significant digits

(*f*) 4 significant digits

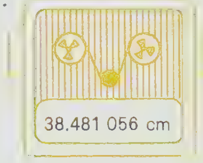
In part (*f*), the zero to the right of the decimal point indicates that the number has been rounded off to one decimal. (For instance, 217.03 = 217.0, correct to one decimal.)

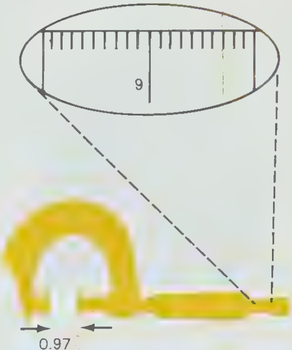
How long is it?

1. I say about 40 cm.

2. It is 38.5cm long.

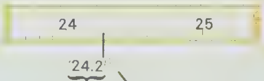
3.





Significant

Not significant.



We see it! ∴ Significant

Can't see it! ∴ Not significant.

If the next digit is
a) 4 or less, drop it,
b) 5 or greater,
round up.

Example 2

Round off 7.123 852 3 to
(*a*) 3 significant digits (*b*) 5 significant digits

Solution

(*a*) 7.123 852 3
Because this 4th digit is less than 5, we drop it.
∴ 7.123 852 3 = 7.12, to 3 significant digits.

(*b*) 7.123 852 3
Because this 6th digit is 5 or greater, we round up.
∴ 7.123 852 3 = 7.1239, to 5 significant digits.

Exercise 1.1

Discussion

1. Which of the following express approximate values, and which express exact values?
   (*a*) 13 footballs
   (*b*) 500 g mass
   (*c*) 11.4 km
   (*d*) 63 000 population
   (*e*) 95 kg
   (*f*) 472 pages
   (*g*) 12 seeds in an orange
   (*h*) 184 cm tall

2. Round off 7246.59 to
   (*a*) the nearest 1000,
   (*b*) the nearest 10,
   (*c*) the nearest unit,
   (*d*) 5 significant digits,
   (*e*) 3 significant digits.

Written Solutions

3. State the number of significant digits in each number.
   (*a*) 1763.1 (*c*) 6 (*e*) 11.956 (*g*) 2.004
   (*b*) 24.005 (*d*) 0.943 (*f*) 0.000 431 (*h*) 0.830

4. A length is measured and stated as 4.2 cm, to 2 significant digits. If the measuring instrument gave values to 2 decimal places, which of the following numbers could *not* have represented the measured length?
   4.14, 4.15, 4.16, 4.20, 4.22, 4.23, 4.24, 4.25, 4.27

3(a) 5 (e) 5
(c) 1 (g) 4

5. If the unit of measurement for each of the following is metres, state the smallest and largest possible values for each measurement, to 2 decimal places.
   (*a*) 17 m (*b*) 5 m (*c*) 11 m (*d*) 193 m

In other words, these have been rounded to the nearest metre.

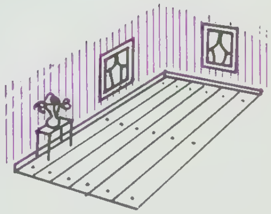
6. Each of the measured lengths that follow has been rounded off. For each length, state the smallest possible value and the largest possible value.
   (*a*) 7.2 cm (*b*) 63 cm (*c*) 0.12 m (*d*) 0.008 km

5(a) 16.50 m, 17.49 m
(c) 10.50 m, 11.49 m

6(a) 7.15 cm, 7.24 cm

1.2 Estimating Answers

It is often necessary to obtain an approximate answer to a problem, either to decide if the answer is reasonable or to locate the decimal point in the answer.



The floor measures 4.1 m by 9.8 m, so I guess I need about 40 m² of carpet.

1. Which product is easiest to determine? Why?
   (*a*) 22.1 × 6.2 (*c*) 22 × 6
   (*b*) 22.1 × 6 (*d*) 20 × 6

2. Which quotient is easiest to determine? Why?
   (*a*) $\frac{41.8}{11.3}$ (*c*) $\frac{40}{11}$
   (*b*) $\frac{41}{11}$ (*d*) $\frac{40}{10}$

Example 1

Find an approximate value.

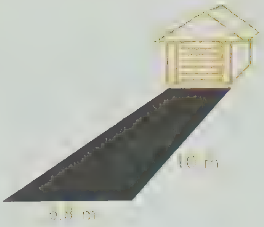
(*a*) 29.3 × 18.9 (*b*) $\frac{17.2 \times 72}{3.95}$

Solution

(*a*)
$$29.3 \times 18.9 \quad 30 \times 20$$
$$= 600$$
$$\therefore 29.3 \times 18.9 \doteq 600$$

(*b*)
$$\frac{17.2 \times 72}{3.95} \doteq \frac{\overset{4}{\cancel{16}} \times 70}{\cancel{4}}$$
$$= 280$$
$$\therefore \frac{17.2 \times 72}{3.95} \doteq 280$$

≐ means "is approximately equal to."



$5.8 \doteq 6$
$2.98 \doteq 3$

m^2
means "square metres"

Example 2

Harry Homeowner wants to blacktop his driveway. If the price of blacktop is \$2.98/m^2, estimate his cost.

Solution

$$\begin{aligned} \text{Approximate area} &= 10\text{ m} \times 6\text{ m} \\ &= 60\text{ m}^2 \\ \text{Approximate cost} &= \$60\ \cancel{\text{m}^2} \times \frac{3}{\cancel{\text{m}^2}} \\ &= \$180 \end{aligned}$$

∴ The estimated cost is \$180.

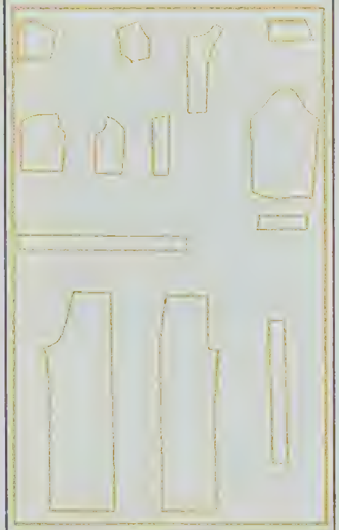
Example 3

The pattern for Sandra Sewitall's pantsuit requires 3.75 m of material. However, Sandra increases this amount by 50% to allow for matching plaids, and she adds a further 10% for waste. How many metres of material should she buy?

Solution

$$\begin{array}{ll} \text{Original material requirement} & \doteq 4\text{ m} \\ \text{Pattern matching allowance} & \doteq \underline{2\text{ m}} \\ & \quad 6\text{ m} \\ 10\%\text{ waste allowance} & \quad \underline{0.6\text{ m}} \\ \text{Total material required} & \quad 6.6\text{ m} \end{array}$$

∴ Sandra should buy 6.6 m of material.



$\frac{1}{2} \times 3.75\text{ m} \doteq \frac{1}{2} \times 4\text{ m}$
$= 2\text{ m}$
and $\frac{1}{10} \times 6\text{ m} = 0.6\text{ m}$

Exercise 1.2

Discussion

1. Three answers are given for each problem. In each case determine the correct answer.

(*a*)	19.1 × 6.2	118.4 or 11.84 or 1184
(*b*)	11.4 × 2.6	296 or 2.96 or 29.6
(*c*)	3.82 × 76	29.0 or 290 or 2.90
(*d*)	0.98 × 123.2	12.1 or 1.21 or 121
(*e*)	65.2 ÷ 11.9	5.48 or 548 or 54.8
(*f*)	403 ÷ 7.9	5.10 or 51.0 or 510

2. State an approximate answer.
   - (*a*) 4.2×3.1
   - (*b*) 62×5
   - (*c*) 11.2×19
   - (*d*) 704×5.2
   - (*e*) $63 \div 8$
   - (*f*) $924 \div 3.1$
   - (*g*) $83 \div 0.98$
   - (*h*) $\dfrac{29.2 \times 3.1}{14.7}$

Written Solutions

3. By estimation determine the correct answer.
   - (*a*) 419.2×37.5 — 157 or 1570 or 15 700
   - (*b*) $2941 \div 76.3$ — 38.5 or 385 or 3.85
   - (*c*) $48.5 \times 117 \div 4.91$ — 116 or 1160 or 11.6
   - (*d*) $\dfrac{2081 \times 4.28}{0.97}$ — 9180 or 918 or 9.18

4. Find an approximate answer.
   - (*a*) $0.32 \times 118 \times 9.5$
   - (*b*) $\dfrac{4216}{21.7 \times 8}$
   - (*c*) $319 \div 57.3 \div 2.04$
   - (*d*) $\dfrac{92.4 \times 613}{27.6 \times 1.18}$

5. Locate the decimal point correctly in each answer.
   - (*a*) $\dfrac{372 \times 4765}{0.982} = 1805$
   - (*b*) $\dfrac{692 \times 0.478}{2.95 \times 0.006} = 187$
   - (*c*) $\dfrac{0.007 \times 926}{0.59 \times 12.3} = 894$
   - (*d*) $\dfrac{795.4}{0.0023 \times 4973} = 695$
   - (*e*) $\dfrac{(72.5)^2}{619.5} = 849$
   - (*f*) $\dfrac{0.275 \times 0.498}{16 \times (0.035)^2} = 699$

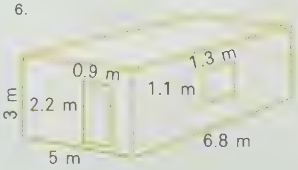
6. Patti Painter wants to paint the 4 walls of her room. If 1 ℓ of paint covers 10 m² and costs \$2.99, find Patti's approximate cost of painting the room.

7. Carl Catermore must order meat to serve 190 people. Allowing 0.25 kg of meat per person, and using a price of \$3.11/kg, estimate the cost of the meat.

8. A person wishes to sod the backyard of a property. The area to be sodded is rectangular in shape, measuring 15 m by 25 m, and contains a circular flower bed 6 m in diameter. If sod costs 60¢/m², find the cost of sodding the yard.

3(a) 15 700 (c) 1160
5(a) 1 805 000 (e) 8.49
(c) 0.894
6. approx. \$24
7. approx. \$160
8. about \$210

6.



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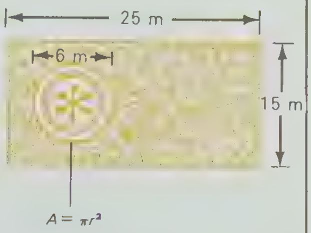
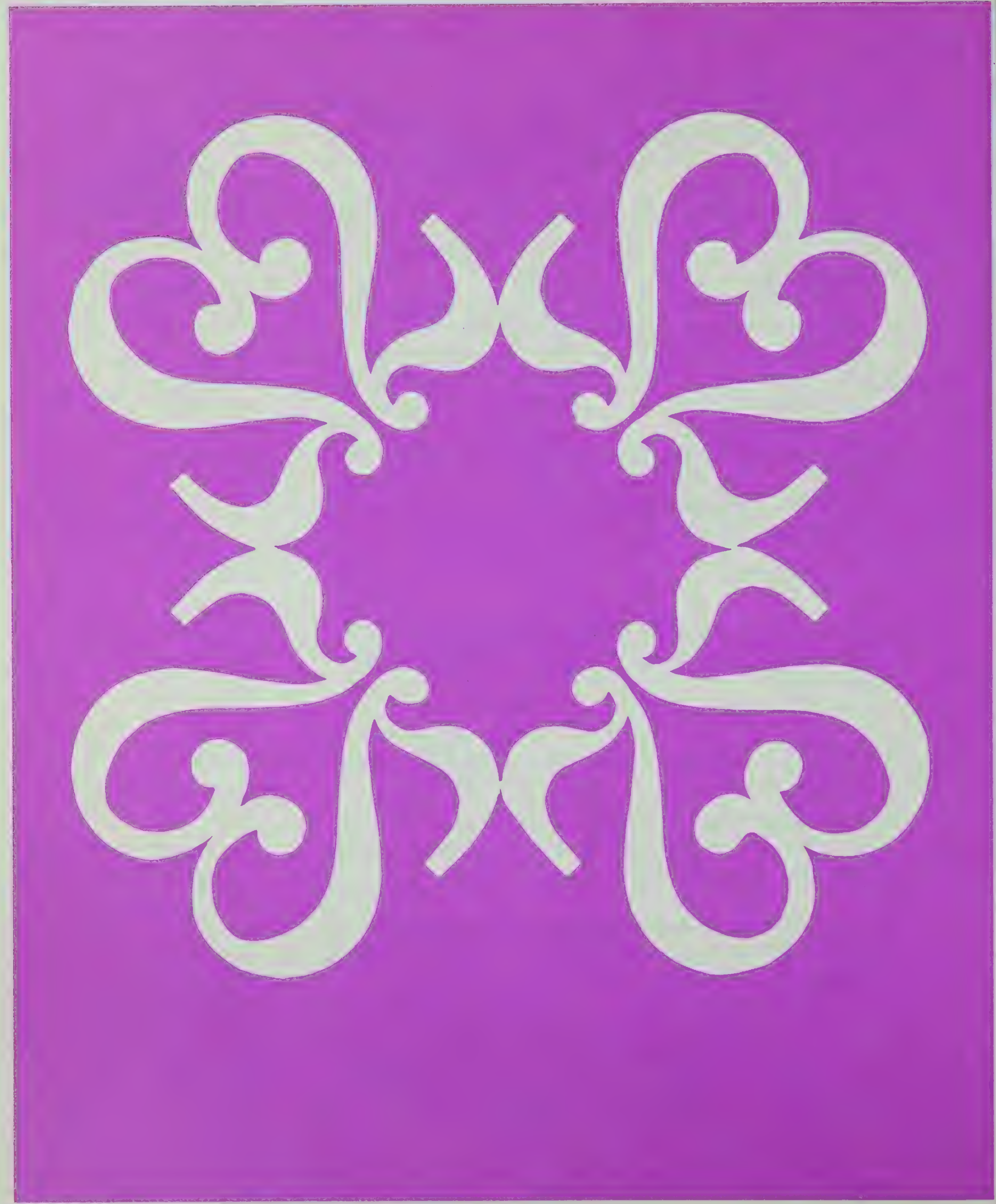
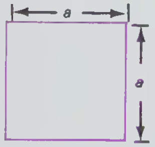


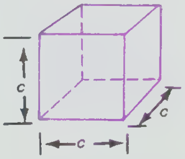


Unit 2
Operating with Power Terms

2.1 Multiplying and Dividing Powers



Area of the square
$= a \times a$
$= a^2$

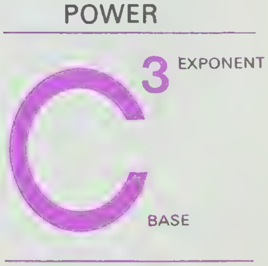


Volume of the cube
$= c \times c \times c$
$= c^3$

A Terms such as a^2 and c^3 are called *powers*. a^2 is the *second power of a*; a is the base, and 2 is the exponent.

1. State as a power of a base.
   (a) y^2 (c) $(2x)^4$ (e) $(10^2)^3$
   (b) 10^8 (d) $\left(\frac{a}{b}\right)^3$ (f) x

2. The expanded form of y^2 is $y \times y$. Express each part of Question 1 in expanded form.



We read:
"c cubed" or
"c to the third" or
"c to the exponent 3"
or "the third power of c"

B Study the following example to see how power terms are multiplied.

$$\begin{aligned} 10^2 \times 10^3 &= (10 \times 10) \times (10 \times 10 \times 10) \\ &= 10 \times 10 \times 10 \times 10 \times 10 \\ &= 10^5 \end{aligned}$$

1. How could we have obtained the exponent 5 immediately? Why does this short cut work? Would it work for any base?

To multiply powers with the same base, add the exponents.

2. State the product.
   - (a) $a^5 \times a^7$
   - (b) $3^2 \times 3^9$
   - (c) $3x^2 \times x^4$
   - (d) $y^8 \times 4y^7$
   - (e) $-2a^{10} \times 3a^{12}$
   - (f) $3^2 \times 10^3$

The short-cut works no matter how complicated the base.

$$\left(\frac{a}{b}\right)^5 \times \left(\frac{a}{b}\right)^7 = \left(\frac{a}{b}\right)^{5+7} = \left(\frac{a}{b}\right)^{12}$$

$$(2x)^3 \times (2x)^2 = (2x)^{3+2} = (2x)^5$$

3. State the product as a single power.
   - (a) $(-2y)^7 \times (-2y)^{10}$
   - (b) $(10^2)^3 \times (10^2)^8$

C Is there a rule for dividing powers in the same base? Let's see. In expanded form,

$$\begin{aligned} \frac{10^7}{10^3} &= \frac{\cancel{10} \times \cancel{10} \times \cancel{10} \times 10 \times 10 \times 10 \times 10}{\cancel{10} \times \cancel{10} \times \cancel{10}} \\ &= 10^4 \end{aligned}$$

Notice that

$$\begin{aligned} \frac{10^7}{10^3} &= 10^{7-3} \\ &= 10^4 \end{aligned}$$

To divide powers with the same base, subtract the exponents.

1. Find the quotient.
   - (a) $a^5 \div a^2$
   - (b) $10^8 \div 10^3$
   - (c) $10y^2 \div y$
   - (d) $12z^5 \div 3z^2$
   - (e) $-15x^{10} \div 5x^3$
   - (f) $35r^{12} \div (-7r^4)$

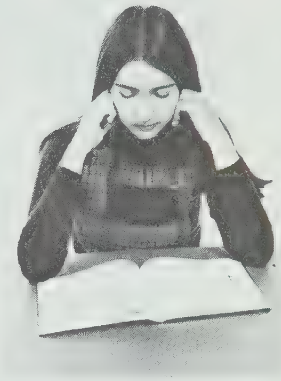
2. Find the quotient.
   - (a) $\left(\frac{3}{5}\right)^4 \div \left(\frac{3}{5}\right)^3$
   - (b) $(2x)^{10} \div (2x)^7$
   - (c) $10^5 \div 2^3$
   - (d) $a^3 \div a^8$

Careful in (c)!
(That's a "GOTCHA"!)

Why was the division different in 2(c)? This is how 2(d) appears in expanded form.

$$\frac{a^3}{a^8} = \frac{\cancel{a}\times\cancel{a}\times\cancel{a}}{\cancel{a}\times\cancel{a}\times\cancel{a}\times a\times a\times a\times a\times a}$$
$$= \frac{1}{a^5}$$

3. State the quotient.
   (a) $r^2 \div r^3$ (b) $(2x)^5 \div (2x)^8$ (c) $(-10)^3 \div (-10)^7$



Exercise 2.1

Discussion

1. State the product.
   (a) $x^2 \times x^5$
   (b) $3^5 \times 3^{10}$
   (c) $(5y)^{10} \times (5y)^4$
   (d) $10^8 \times 10^5$
   (e) $3^4 \times 5^2$
   (f) $(2a)^4 \times (2a)^3$
2. State the quotient.
   (a) $10^7 \div 10^5$
   (b) $b^6 \div b^2$
   (c) $(3a)^4 \div (3a)$
   (d) $x^5 \div x^9$
   (e) $a^5 \div b^2$
   (f) $(3x)^6 \div (3x)^2$

Careful !

3. Simplify.
   (a) $10^2 \times 10^5 \times 10^8$
   (b) $\dfrac{3^2 \times 3^5}{3^4}$
   (c) $(-2)^3 \div (-2)^5$
   (d) $\dfrac{a^5}{a^2 \times a}$

Remember?
$$\frac{10^x}{10^y} = 10^{x-y}$$

4. What condition must exist before you can multiply or divide two power terms?
5. What is the difference between $3x^2$ and $(3x)^2$?

Written Solutions

6. Simplify.
   (a) $10^5 \times 10^8 \times 10^2$
   (b) $x^3 \times x^4 \times x$
   (c) $(2a)^4 \div (2a)$
   (d) $y^4 \times y^5 \div y^3$
   (e) $3^{15} \div (3^2 \times 3^8)$
   (f) $(3t)^5 \times (3t)^2$
7. Find the product.
   (a) $4r^7 \times 5r^3$
   (b) $-3y \times 10y^4$
   (c) $4p^5 \times (-3p^2)$
   (d) $-12x^7 \times (-4x^3)$
   (e) $9m^2 \times (-3m^5)$
   (f) $-7y^5 \times 4y^8$

7(a)
$$4r^7 \times 5r^3 = 4\times 5 \times r^7 \times r^3$$
$$= 20r^{10}$$

6(a) 10^{15} (e) 3^5
(c) $(2a)^3$
7(c) $-12p^7$ (e) $-27m^7$

8. Find the quotient.

(*a*) $12s^5 \div 3s^2$ (*d*) $32f^3 \div (-8f^5)$

(*b*) $-2h^7 \div h^4$ (*e*) $-45v \div (-5v^7)$

(*c*) $7x^8 \div 14x^3$ (*f*) $48y^5 \div (-8y^4)$

9. A square measures $3x$ centimetres along each side.

(*a*) Find the area and perimeter of the square.

(*b*) Use (*a*) to find the area and perimeter when $x = 4.5$.

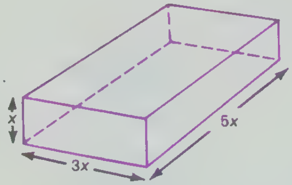
10. Find the area of the rectangles with dimensions given.

(*a*) $3x$ metres by $2x$ metres

(*b*) $4a^2$ centimetres by $2a$ centimetres

11. A rectangular room measures $5x$ metres by $3x$ metres by x metres. Find the volume of the room.

11.



$V = lwh$

8(b) $-2h^3$ **(f)** $-6y$

(d) $-4\frac{1}{f^2}$

9(a) $A = 9x^2$ cm², $P = 12x$ cm

(b) 182.25 cm², 54 cm

10(a) $6x^2$ m²

2.2 Powers of Products and Quotients

A The base of a power term frequently is a power term itself. $(a^2)^3$ represents the *third power of a-squared*.

1. Read the following terms.

(*a*) $(2x)^4$ (*b*) $(y^2)^7$ (*c*) $(10^3)^5$

Power terms like these can be simplified.
By expanding,

$$\begin{aligned}(a^2)^3 &= a^2 \times a^2 \times a^2\\ &= a^{2+2+2}\\ &= a^6\end{aligned}$$

2. Simplify.

(*a*) $(y^3)^4$ (*b*) $(t^2)^5$ (*c*) $(m^4)^7$

By expanding in (*c*),

$$\begin{aligned}(m^4)^7 &= m^4 \times m^4 \times \ldots \times m^4 \text{ to 7 factors}\\ &= m^{4+4+4+\ldots+4}\\ &= m^{7 \times 4}\\ &= m^{28}\end{aligned}$$

How many 4's?

Do you see that $(m^4)^7$ means 7 groups of 4 factors of m, or the product of 28 m's, which we write as m^{28}?

In general,

$$\boxed{(a^x)^y = a^{x \times y} = a^{xy}}$$

B The base may also be a product as in $(xy)^3$.
By expanding,

$$\begin{aligned}(xy)^3 &= xy \times xy \times xy\\ &= x \times x \times x \times y \times y \times y\\ &= x^3y^3\end{aligned}$$

1. Expand and write without brackets.
   (*a*) $(ab)^2$ (*b*) $(x^2y)^3$ (*c*) $(r^2s^3)^5$

Your solution to 1(*c*) should appear as follows.

$$\begin{aligned}(r^2s^3)^5 &= r^2s^3 \times r^2s^3 \times r^2s^3 \times r^2s^3 \times r^2s^3\\ &= (r^2 \times r^2 \times r^2 \times r^2 \times r^2) \times (s^3 \times s^3 \times s^3 \times s^3 \times s^3)\\ &= r^{5 \times 2} \times s^{5 \times 3}\\ &= r^{10}s^{15}\end{aligned}$$

Do you see a possible short cut?

2. Remove the brackets.
   (*a*) $(y^2z)^3$ (*b*) $(ab^4)^5$ (*c*) $(p^2q^3)^7$ (*d*) $(2x^5)^3$

C A quotient such as $\frac{x}{y}$ can be used as a base.

By expanding,

$$\begin{aligned}\left(\frac{x}{y}\right)^3 &= \frac{x}{y} \times \frac{x}{y} \times \frac{x}{y}\\ &= \frac{x^3}{y^3}\end{aligned}$$

1. Remove the brackets.
   (*a*) $\left(\frac{2}{3}\right)^2$ (*b*) $\left(\frac{a}{b}\right)^5$ (*c*) $\left(\frac{r^2}{s}\right)^5$

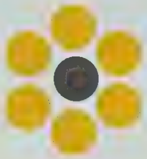
Example

Simplify. $\dfrac{(x^2y^3)^3}{(xy^2)^2}$

Solution

$$\begin{aligned}\frac{(x^2y^3)^3}{(xy^2)^2} &= \frac{x^{2 \times 3}\, y^{3 \times 3}}{x^{1 \times 2}\, y^{2 \times 2}}\\ &= \frac{x^6y^9}{x^2y^4} = x^4y^5\end{aligned}$$

Remember?
$(xy)^a = x^ay^a$



Exercise 2.2

Discussion

1. Simplify.
   (a) $(3^2)^5$ (b) $(a^2)^7$ (c) $y^3 \times y^5$

2. Simplify by removing brackets.
   (a) $(3x)^2$ (c) $(ab)^4$ (e) $(2xy)^2$
   (b) $(5y)^3$ (d) $(x^2y)^5$ (f) $(-3ab)^3$

3. Remove the brackets.
   (a) $\left(\frac{3}{5}\right)^2$ (c) $\left(\frac{r}{s}\right)^4$ (e) $\frac{(3 \times 2)^5}{(3 \times 2)^2}$
   (b) $\left(\frac{4}{7}\right)^2$ (d) $\left(\frac{a^2}{c}\right)^3$ (f) $\frac{(a \times b)^7}{(a \times b)^4}$

4. Complete the following statements.
   (a) $10^x \times 10^y = \square$ (c) $(ab)^x = \square$
   (b) $10^x \div 10^y = \square$ (d) $\left(\frac{a}{b}\right)^y = \square$

Written Solutions

5. Simplify.
   (a) $(2x)^2$ (c) $(4a^2)^4$ (e) $(3xy)^2$
   (b) $(-5a)^3$ (d) $(-ab)^2$ (f) $(8x^2y^3)^2$

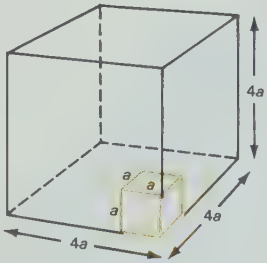
6. Simplify.
   (a) $\left(\frac{3}{5}\right)^2$ (c) $\left(\frac{2x}{3}\right)^3$ (e) $\left(-\frac{3}{10}\right)^3$
   (b) $\left(\frac{a^2}{b}\right)^3$ (d) $\left(-\frac{a}{2b}\right)^2$ (f) $\left(\frac{-x^2}{-2x}\right)^4$

7. Simplify.
   (a) $(3a)^2 \times 5a^2$ (d) $(3r)^2 \times (4r)^2$
   (b) $(-2c)^2 \times c^3$ (e) $(3.2 \times 10^2)(5 \times 10^3)$
   (c) $3y \times (5y^2)^2$ (f) $(4.6 \times 10^3)^2$

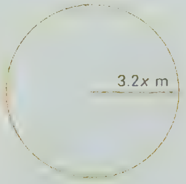
8. How many cubes whose edges are a centimetres long are contained in a cube whose edges are $4a$ centimetres long?

9. Find the area of a circle whose radius is $3.2x$ metres. ($\pi \doteq 3.14$)

8.



9.



$A = \pi r^2$

5(a) $4x^2$ (e) $9x^2y^2$
(c) $256a^8$
6(a) $\frac{9}{25}$ (e) $\frac{-27}{1000}$
(c) $\frac{8x^3}{27}$
7(a) $45a^4$ (e) 16×10^5
(c) $75y^5$
8. 64
9. $32.2x^2$ m^2

2.3 Powers with Zero and Negative Exponents

A a^3 means $a \times a \times a$; x^2 means $x \times x$. Is it possible to give meaning to a^0 and x^{-2}? Can the exponent refer to the number of times the base is written as a factor? Why not?

1. If 10^x means $\overbrace{10 \times 10 \times 10 \times 10 \times \ldots \times 10}^{x \text{ factors}}$, what kind of number must x be?

2. Are 0, -1, -2, ... counting numbers? Study the following example that shows a new meaning for an exponent.

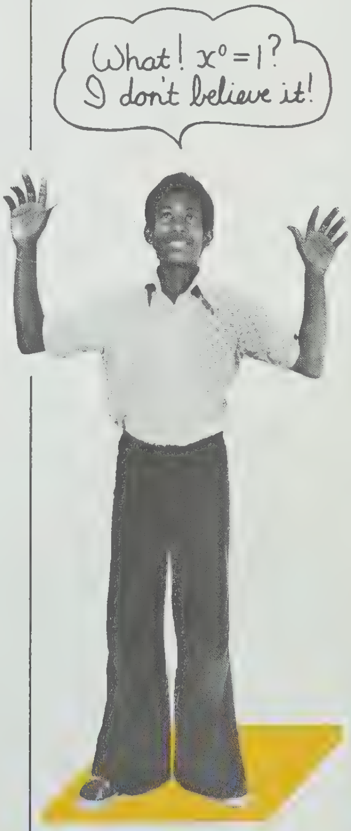
$$\frac{10^2}{10^2} = 10^{2-2} = 10^0 \qquad \text{and} \qquad \frac{10^2}{10^2} = \frac{100}{100} = 1$$

$$\frac{3^5}{3^5} = 3^{5-5} = 3^0 \qquad \frac{3^5}{3^5} = \frac{243}{243} = 1$$

Compare!

What conclusion can you draw?

In order that our rule for dividing powers will hold in every case, it appears that $10^0 = 1$ (even if you don't believe it!) We could use $x^2 \div x^2$ and the resulting statement would be that $x^0 = 1$.


What! $x^0=1$? I don't believe it!

B Negative exponents can result from dividing powers. Compare the two methods.

$$x^2 \div x^5 = x^{2-5} = x^{-3} \qquad x^2 \div x^5 = \frac{\cancel{x} \times \cancel{x}}{\cancel{x} \times \cancel{x} \times x \times x \times x} = \frac{1}{x^3}$$

A logical conclusion is that $x^{-3} = \frac{1}{x^3}$.

1. Express in a different form.

(*a*) a^{-2}

(*b*) 10^{-3}

(*c*) $\frac{1}{10^2}$

(*d*) $\frac{1}{x^2}$

(*e*) 3^{-5}

(*f*) $\frac{1}{3^4}$

If division of powers having the same base is to be performed by subtracting their exponents, then we must agree that

$$\boxed{x^0 = 1 \qquad \text{and} \qquad x^{-a} = \frac{1}{x^a}.}$$

Exercise 2.3

Discussion

1. Add.
   (*a*) $3+(-2)$ (*b*) $-1+(-4)$ (*c*) $-2+1$

2. Subtract.
   (*a*) $3-5$ (*b*) $-3-4$ (*c*) $5-(-2)$

3. State the product.
   (*a*) $10^2 \times 10^3$ (*c*) $x^2 \times x^{-4}$ (*e*) $a^5 \times a^{-1}$
   (*b*) $10^4 \times 10^{-2}$ (*d*) $10^{-1} \times 10^{-3}$ (*f*) $y^{-2} \times y^{-3}$

4. State the quotient.
   (*a*) $3^5 \div 3^2$ (*c*) $10^3 \div 10^{-1}$ (*e*) $x^{-5} \div x^2$
   (*b*) $10^2 \div 10^5$ (*d*) $5^{-2} \div 5^3$ (*f*) $a^{-2} \div a^{-5}$

5. Simplify.
   (*a*) $3^0 \times 4$ (*c*) 4×10^{-1} (*e*) $5^0 + 1$
   (*b*) 6×10^0 (*d*) 2×3^{-2} (*f*) $10^0 \div 10^{-1}$

6. Express as a decimal.
   (*a*) 10^{-1} (*b*) 10^{-2} (*c*) 10^{-3} (*d*) 10^{-4}

Written Solutions

7. Find the product.
   (*a*) $10^2 \times 10^3 \times 10^{-4}$ (*d*) $y^0 \times y^2 \times y^{-3}$
   (*b*) $a^{-1} \times a^5 \times a^{-2}$ (*e*) $10 \times 10^{-2} \times 10^3$
   (*c*) $10^4 \times 10^{-7} \times 10^{-1}$ (*f*) $10^5 \times 10^0 \times 10^{-5}$

8. Simplify.
   (*a*) $\dfrac{a^2 \times a^3}{a^4}$ (*d*) $\dfrac{r^{-1} \times r^4}{r^{-3}}$
   (*b*) $\dfrac{10^2 \times 10^{-1}}{10^3}$ (*e*) $\dfrac{8 \times 10^3}{2 \times 10^{-1}}$
   (*c*) $\dfrac{x^2}{x^3 \times x^4}$ (*f*) $\dfrac{(3 \times 10^{-2})(8 \times 10^3)}{4 \times 10^2}$

9. Evaluate.
   (*a*) 2^{-1} (*d*) $(0.2)^{-2}$ (*g*) $10^0 + 2^0$
   (*b*) 3^{-2} (*e*) $\left(\dfrac{2}{3}\right)^{-3}$ (*h*) $\dfrac{1}{4^{-1}}$
   (*c*) $5^{-1} + 10^{-1}$ (*f*) $(0.5)^{-3}$ (*i*) $3^{-2} - 2^{-3}$

7(a) 10 (e) 10^2
(c) 10^{-4}

8(a) a (e) 4×10^4
(c) x^{-5}

9(a) $\frac{1}{2}$ (g) 2
(c) $\frac{3}{10}$ (i) $-\frac{1}{72}$
(e) $\frac{27}{8}$

2.4 Numbers in Standard Form

A Powers of 10 are present whenever numbers are written in standard form. The standard form of 3250 is $\underline{3.25} \times 10^3$.

This number will be between 1 and 10.

1. Express in standard form.
   (*a*) 53 (*b*) 236 (*c*) 23.1 (*d*) 1746

Smaller numbers such as 0.000 032 can also be expressed as

$$3.2 \times \frac{1}{100\,000} \quad \text{or} \quad 3.2 \times 10^{-5}.$$

2. Express in standard form.
   (*a*) 0.051 (*b*) 0.003 64 (*c*) 0.592 (*d*) 0.000 073

B Conversion to standard form helps in estimating answers.

Example 1

Estimate the product.
(*a*) 563×297 (*b*) 0.0035×0.0197

Solution

(*a*)
$$\begin{aligned} & 563 \times 297 \\ &= 5.63 \times 10^2 \times 2.97 \times 10^2 \\ &\doteq 6 \times 3 \times 10^4 \\ &= 18 \times 10^4 \\ &= 180\,000 \end{aligned}$$

(*b*)
$$\begin{aligned} & 0.0035 \times 0.0197 \\ &= 3.5 \times 10^{-3} \times 1.97 \times 10^{-2} \\ &\doteq 3.5 \times 2 \times 10^{-5} \\ &= 7 \times 10^{-5} \\ &\doteq 0.000\,07 \end{aligned}$$

Example 2

Estimate the quotient.
(*a*) $48.6 \div 0.0062$ (*b*) $0.003\,82 \div 28.4$

Solution

(*a*)
$$\begin{aligned} & 48.6 \div 0.0062 \\ &= \frac{48.6}{0.0062} \\ &= \frac{4.86 \times 10}{6.2 \times 10^{-3}} \\ &= \frac{4.86}{6.2} \times \frac{10}{10^{-3}} \\ &\doteq 0.8 \times 10^{1-(-3)} \\ &= 0.8 \times 10^4 \\ &= 8000 \end{aligned}$$

(*b*)
$$\begin{aligned} & 0.003\,82 \div 28.4 \\ &= \frac{0.003\,82}{28.4} \\ &= \frac{3.82 \times 10^{-3}}{2.84 \times 10} \\ &= \frac{3.82}{2.84} \times \frac{10^{-3}}{10} \\ &\doteq 1.3 \times 10^{-3-1} \\ &= 1.3 \times 10^{-4} \\ &= 0.000\,13 \end{aligned}$$

$$\frac{3.82}{2.84} \doteq \frac{4}{3} \doteq 1.3$$

Exercise 2.4

Discussion

1. Express in standard form.
   (*a*) 48.6 (*c*) 0.003 01 (*e*) 5280
   (*b*) 297 (*d*) 0.79 (*f*) 0.000 054

2. Express in decimal notation.
   (*a*) 6.2×10^2 (*c*) 4.61×10^{-3}
   (*b*) 3.8×10^{-1} (*d*) 7.61×10^3

3. Restate as a product of decimal numbers and a product of powers of 10.
   (*a*) $(3.6 \times 10^{-1}) \times (4.8 \times 10^4)$
   (*b*) $(2.61 \times 10^{-5}) \times (3.87 \times 10^{-2})$

4. Restate as a quotient of decimal numbers multiplied by a quotient of powers of 10.
   (*a*) $(3.62 \times 10^{-1}) \div (4.81 \times 10^2)$ (*c*) $\dfrac{4.15 \times 10^5}{2.87 \times 10^2}$
   (*b*) $(1.6 \times 10^4) \div (7.6 \times 10^{-1})$

Written Solutions

5. Estimate the following products in standard form.
   (*a*) $(5.91 \times 10^2) \times (3.1 \times 10)$
   (*b*) $(8.17 \times 10^{-1}) \times (4.69 \times 10^{-3})$
   (*c*) $(7.54 \times 10) \times (2.16 \times 10^{-1})$
   (*d*) $(1.64 \times 10^3) \times (8.39 \times 10^{-2})$

6. Estimate the following quotients in standard form.
   (*a*) $(3.81 \times 10^5) \div (1.94 \times 10^2)$
   (*b*) $(7.61 \times 10^2) \div (3.92 \times 10^4)$
   (*c*) $(9.58 \times 10^{-1}) \div (5.13 \times 10^2)$
   (*d*) $(6.05 \times 10^{-2}) \div (2.97 \times 10^{-1})$

7. Estimate the following quotients in standard form.
   (*a*) $(2.68 \times 10^4) \div (6.37 \times 10^2)$
   (*b*) $(3.02 \times 10^{-2}) \div (8.95 \times 10^3)$
   (*c*) $(6.17 \times 10) \div (7.93 \times 10^{-3})$
   (*d*) $(1.75 \times 10^{-1}) \div (5.16 \times 10^{-4})$

8. Evaluate Questions 5 to 7 to 2 decimal places. (A calculator is a big timesaver here!) Express your answers in standard form.

For Question 8:
5(a) 1.84×10^4
(c) 1.63×10

6(a) 1.96×10^3
(c) 1.87×10^{-3}

7(a) 4.21×10
(c) 7.78×10^3

2.5 Calculating with Numbers in Standard Form

Complicated expressions can be readily estimated when the numbers are converted to standard form.

Example

Estimate the answer in each case.

(*a*) $\dfrac{635 \times 0.000\,397}{0.236}$ (*b*) $\dfrac{748}{0.063 \times 83.7}$

Solution

(*a*)

$$\begin{aligned}
&\frac{635 \times 0.000\,397}{0.236}\\
&= \frac{6.35 \times 10^2 \times 3.97 \times 10^{-4}}{2.36 \times 10^{-1}}\\
&\doteq \frac{6 \times 4 \times 10^{2+(-4)}}{2 \times 10^{-1}}\\
&= 12 \times 10^{-2-(-1)}\\
&= 12 \times 10^{-1}\\
&= 1.2
\end{aligned}$$

(*b*)

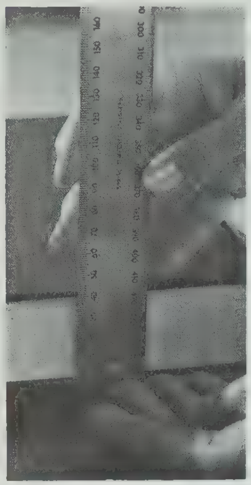
$$\begin{aligned}
&\frac{748}{0.063 \times 83.7}\\
&= \frac{7.48 \times 10^2}{6.3 \times 10^{-2} \times 8.37 \times 10^1}\\
&\doteq \frac{7 \times 10^2}{6 \times 8 \times 10^{-2+1}}\\
&= \frac{7 \times 10^2}{48 \times 10^{-1}}\\
&\doteq \frac{7 \times 10^2}{5 \times 10^1 \times 10^{-1}} \longleftarrow\\
&= 1.4 \times 10^2\\
&= 140
\end{aligned}$$

$10^1 \times 10^{-1} = 10^0 = 1$

Exercise 2.5

Discussion

1. Express in standard form.
   (*a*) 328 (*b*) 0.0279 (*c*) 48 960 (*d*) 15
2. Express as a power of 10.
   (*a*) $10^2 \times 10^3 \times 10^{-4}$ (*b*) $10^{-2} \times 10^{-5}$ (*c*) $\dfrac{10^{-1}}{10^4}$ (*d*) $\dfrac{10^2}{10^{-3}}$
3. Estimate in standard form.
   (*a*) $\dfrac{2.6 \times 10^3}{1.2 \times 10}$ (*b*) $\dfrac{8.75 \times 10^{-3}}{3.1 \times 10^2}$ (*c*) $\dfrac{5.37 \times 10^{-1}}{1.32 \times 10^{-2}}$ (*d*) $\dfrac{4.83 \times 10^5}{1.61 \times 10^{-2}}$



Written Solutions

In Questions 4, 5, 6, and 7, estimate the answers by expressing each number in standard form. Then evaluate the expressions and compare your answers.

4. (*a*) $\dfrac{63\,200 \times 296}{3460}$ (*b*) $\dfrac{5280 \times 2320}{432\,000}$

5. (*a*) $\dfrac{456 \times 0.000\,372}{61.4}$ (*b*) $\dfrac{28 \times 1590}{0.0791}$

6. (*a*) $\dfrac{328}{641 \times 152}$ (*b*) $\dfrac{0.782}{541 \times 0.000\,392}$

7. (*a*) $\dfrac{0.261 \times 845}{273 \times 0.0059}$ (*b*) $\dfrac{3870 \times 0.0429}{3.62 \times 0.000\,174}$

8. The annual quantity of wood harvested from Canada's forests is approximately 121 433 000 m³. If the total volume that could be cut is 17 825 000 000 m³, what percentage of our usable forests are we cutting annually?

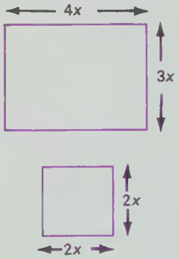
9. The sun is approximately 150 000 000 km from the earth, and light travels at the rate of 298 000 km/s. Find how long it takes light to go from the sun to the earth.

10. Find the thickness of one sheet of paper used in (*a*) a paperback, (*b*) a hardcover textbook, (*c*) an encyclopedia, by measuring carefully the thickness of the book and finding the number of pages in the book. Check your work by measuring the total thickness of 100 pages.

11. How many squares whose sides measure 2*x* centimetres does it take to produce the same area as a rectangle that measures 4*x* centimetres by 3*x* centimetres?

12. If 10 000 000 people die of starvation throughout the world each year, determine the average number of people that die each hour.

11.



4(a) 5.41×10^3
(b) 2.84×10

5(a) 2.76×10^{-3}
(b) 5.63×10^5

6(a) 3.37×10^{-3}
(b) 3.69

7(a) 1.37×10^2
(b) 2.64×10^5

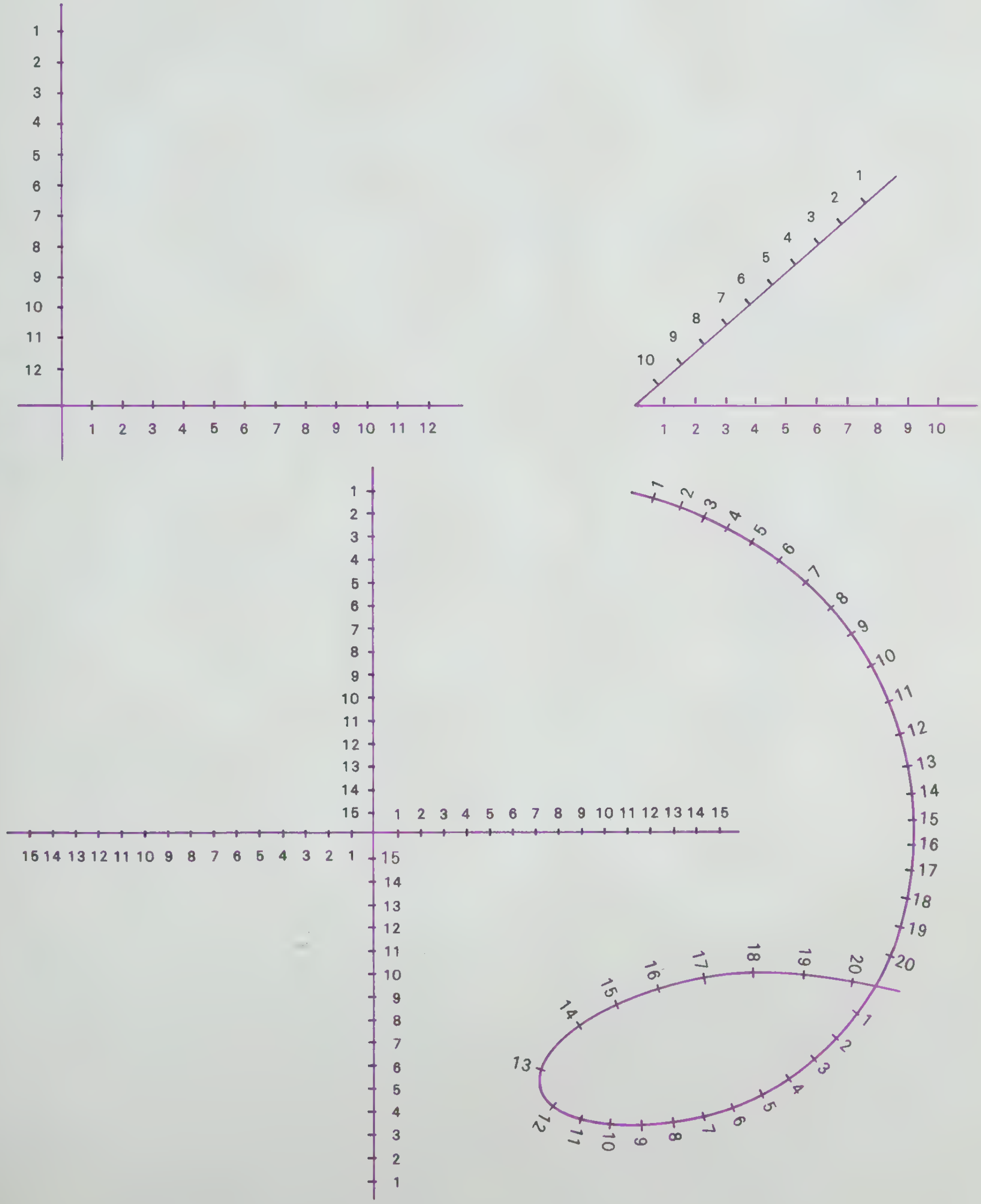
8. 0.681%

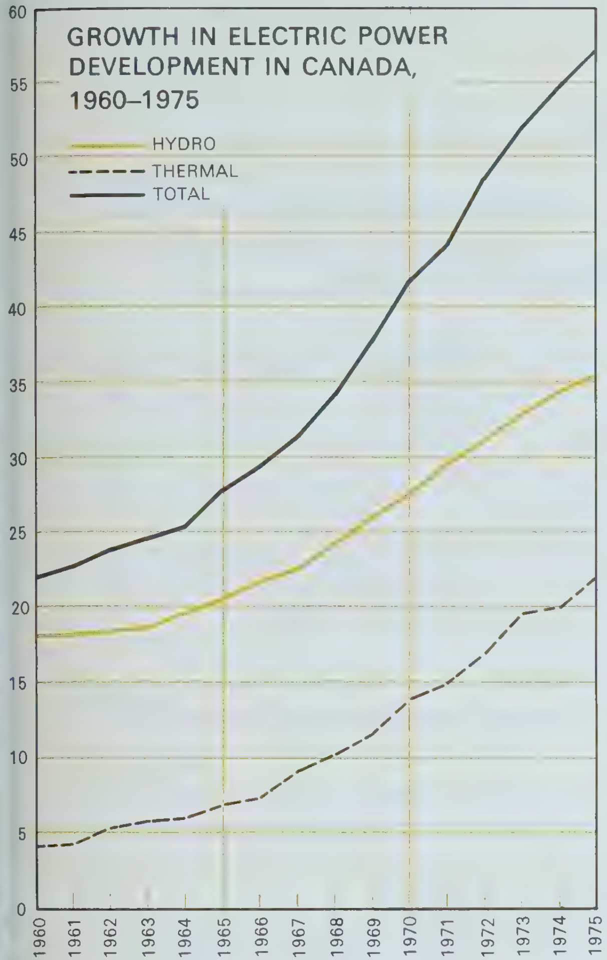
9. 8.4 min

11. 3

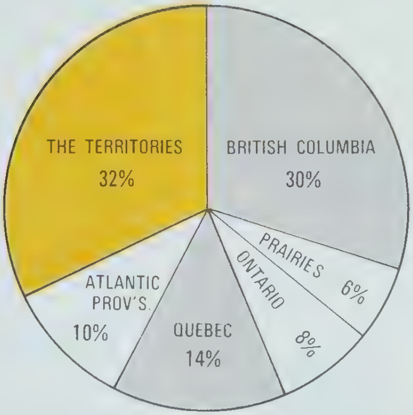
Number Activity

Copy each diagram on a large sheet of paper. Join the same numbers using straight lines and watch the result. Try using various colors as well!

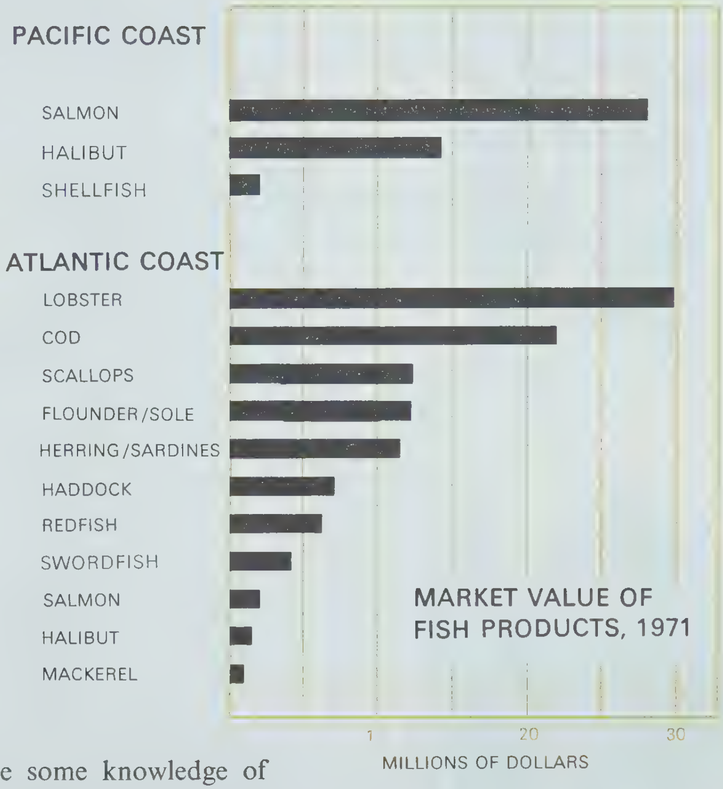

1 2 3 4 5 6 7 8 9 10 11 12
1 2 3 4 5 6 7 8 9 10 11 12
1 2 3 4 5 6 7 8 9 10
1 2 3 4 5 6 7 8 9 10
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20



GROWTH IN ELECTRIC POWER DEVELOPMENT IN CANADA, 1960–1975



PROPORTIONS OF STREAM FLOW
IN EACH REGION OF CANADA



MARKET VALUE OF FISH PRODUCTS, 1971

Unit 3
Statistics

Today it is important that everyone have some knowledge of statistics since numbers and graphs are often used to convey information clearly and forcefully. In this age of computers we are bombarded with more and more factual information, and the need to make meaningful evaluation of such data becomes increasingly important.

"Data" means "information".

3.1 Representing Data

A Information can be gathered on a wide variety of topics. Listed below is an array of data showing the individual heights (in centimetres) of 100 elementary school students.

125	108	150	160	138	107	118	132	97	143
135	142	103	147	88	109	103	123	113	120
104	159	145	100	115	138	97	108	142	115
160	98	90	105	131	134	85	94	165	117
95	112	115	123	128	141	101	142	109	96
140	115	140	136	102	110	137	163	98	161
128	133	110	108	150	103	91	155	132	143
92	156	93	99	97	152	88	108	123	131
103	102	170	131	141	139	106	123	119	106
150	135	109	119	106	122	117	135	102	94

The table in this form conveys little. We *could* arrange the heights in order from the shortest to the tallest, but even that wouldn't help us much.

It *does* help to organize the data into a table like this.

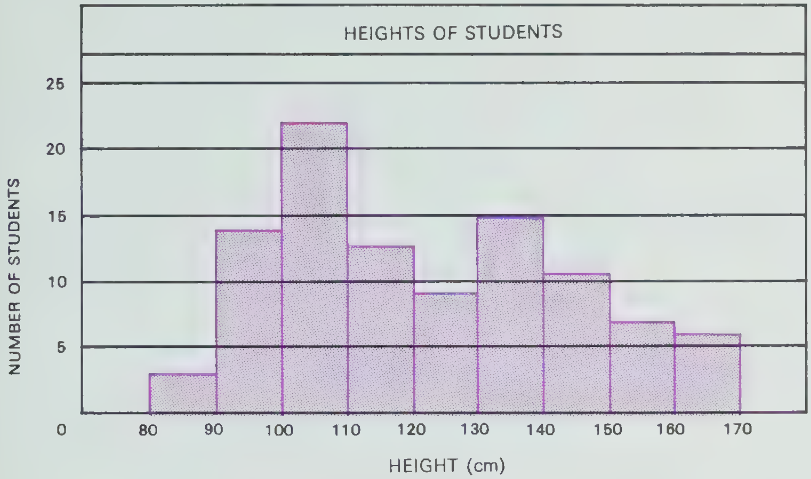
Heights of Students

Height (cm)	*Tally*	*Number of Students*
less than 90	|||	3
90-99	𝍸 𝍸 ||||	14
100-109	𝍸 𝍸 𝍸 𝍸 ||	22
110-119	𝍸 𝍸 |||	13
120-129	𝍸 ||||	9
130-139	𝍸 𝍸 𝍸	15
140-149	𝍸 𝍸 |	11
150-159	𝍸 ||	7
160 or greater	𝍸 |	6

This table is called a *frequency distribution*. At a glance, the tally shows the grouping of the data. Notice that there is a concentration of students in the height intervals 100–109 and 130–139. Can you account for this?

B Using the frequency distribution, we can graph the data as shown below.

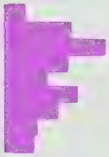
Ancient proverb says, "A picture is worth a thousand words."



This graph is a vertical bar graph.

The frequency distribution of the heights contains 9 *groupings*. The number of groupings depends on the *range* of the values in the array. Here the range was 80–170, and it was convenient to use groupings that included heights in intervals of 10 cm.

The graph could have been drawn showing the height on the vertical axis and the number of students across the horizontal axis. This result would be called a *horizontal bar graph*.



Well, natch!

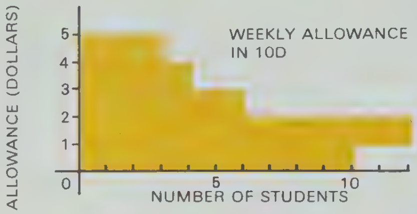
Exercise 3.1

Discussion

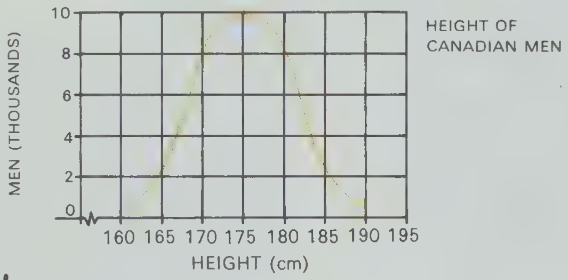
1. State the range for each of the following sets of data.
   (*a*) 18, 16, 12, 25, 42, 5, 3
   (*b*) 168, 227, 560, 132, 1024, 102
2. Study the table at the right and answer the following questions.
   (*a*) How many students are in the 10 G Science class?
   (*b*) How many students got 50 marks or more?
   (*c*) What would you estimate the class average to be?

SCIENCE MARKS — 10G

MARK	NUMBER OF STUDENTS
0-39	3
40-49	5
50-59	8
60-69	10
70-79	6
80-89	3
90-100	1



3. The 35 students in 10 D receive allowances as shown in the diagram.
   (*a*) What is the most common allowance?
   (*b*) How many students receive an allowance of more than $3 per week?
   (*c*) Can you tell from the graph if anyone receives no allowance?



This curve is a "normal distribution".

4. If a large enough group is sampled, the frequency distribution of a physical characteristic, such as height, appears as in the graph.
   (*a*) What is the average height of a Canadian man?
   (*b*) In this example, how many men have a height of 170 cm?

Written Solutions

5. The following table contains the marks achieved by 75 Grade 10 students in English.
   (*a*) Prepare a tally sheet using intervals of 10 marks.
   (*b*) Make a frequency distribution table.
   (*c*) Make a bar graph to show the distribution.

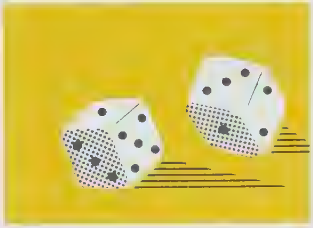
82	52	62	35	40	50
65	75	58	67	81	42
46	63	72	53	76	55
59	60	70	59	58	64
73	85	45	72	62	73
38	78	80	62	66	61
69	56	61	56	51	57
90	53	66	72	43	59
56	65	47	63	92	87
30	78	55	84	63	62
59	65	57	63	62	
56	62	67	61	50	
70	58	72	60	53	

6. Using your class as a sample, gather data and prepare a frequency distribution showing
   - (*a*) the age of the family car,
   - (*b*) the allowance received or money earned by members of the class,
   - (*c*) the age of the members of the class,
   - (*d*) the heights of the (i) girls, (ii) boys.

7. Using two pages from a book, construct a frequency distribution showing the number of times each letter of the alphabet appears.

8. (*a*) Visit a shoe store in your community and set up a frequency distribution of shoe sizes from the manager's monthly order list.
   (*b*) Form a frequency distribution of shoe sizes for the students in your Mathematics class and compare it to the distribution in (*a*).

9. Visit several clothing shops near you and make a frequency distribution of the number of each size of jeans available.



10. If you roll two dice, what are the possible sums you can get? Estimate the number of times *each* sum will appear in 100 rolls and then try it. Make a frequency distribution of the 100 sums that you get. Compare your work to that of other students in the class.

11. Select an intersection of two streets or highways in your community and make a frequency distribution showing the number of cars observed during ten equal time intervals. In a city during the rush hour the time intervals might be as small as 2 min or 3 min.

12. Pick a sport like hockey, football, or baseball and use the game summaries from the newspapers for as many games as possible to investigate if scoring occurs more often in some periods of the game than in others. Use a frequency distribution to analyze your data.

Does the average have to be one of your marks?

3.2 Mode, Median, and Mean

Average is a word that is much used (and misused) to-day. We hear of average marks, average wages, the average cost of a house, the average age of the members of a team, and so on.

In this section you will study several special types of numbers that may be used as the representative of a group of numbers.

A There are 3 ways of selecting a single number to represent a set of data.

The *mode* of a group of numbers is the number that occurs most often.

Example 1

Arnold Parbreaker's golf scores for 18 holes during one summer were as follows. Find the mode for this list of scores. 83, 85, 79, 82, 80, 85, 90, 81, 83, 80, 82, 83, 84.

Solution

The mode of this distribution is 83 since it occurs three times and no other value occurs more than twice.

B The *median* of a set of numbers is the number in the middle when the numbers have been put in order.

Example 2

The property taxes paid by eleven residents of Green Meadows were $430, $360, $515, $623, $465, $602, $508, $750, $475, $325, $538. Find the median tax.

Solution

The taxes in order are $325, $360, $430, $465, $475, $508, $515, $538, $602, $623, $750.

The middle number in the set is $508.

∴ Median tax on the 11 residents of Green Meadows is $508.

When there is an even number of items in the group, the median is the average of the 2 middle items, as in the following example.

Example 3

The weekly earnings of 8 office workers were \$118, \$124, \$112, \$136, \$125, \$132, \$115, and \$120. Find the median weekly wage.

Solution

The wages in order are

\$112, \$115, \$118, \$120, \$124, \$125, \$132, \$136

$\therefore$ The median wage is $\dfrac{\$120 + \$124}{2} = \$122.$

2

middle numbers!

Does the median have to be one of the given wages?

C The most common representative of a group of numbers is the average. We will call this the *arithmetic mean*. You find it just as you do when averaging your marks.

That's a-rith-MÉ-tic, not a-RÍTH-me-tic.

Example 4

Suppose that 72, 56, 47, 63, 60, and 59 were your marks on a report card. Compute your average mark.

Solution

$$\text{Your average is } \frac{72 + 56 + 47 + 63 + 60 + 59}{6}$$

$$\text{or } \frac{357}{6} = 59.5$$

Thus the arithmetic mean is found by adding the numbers involved and dividing the sum by the number of items in the group.

Does your average have to be one of the marks?

D Each of these special statistics—the mode, the median, and the mean—can be used to represent a set of data. It takes experience to determine which of these is the most representative.

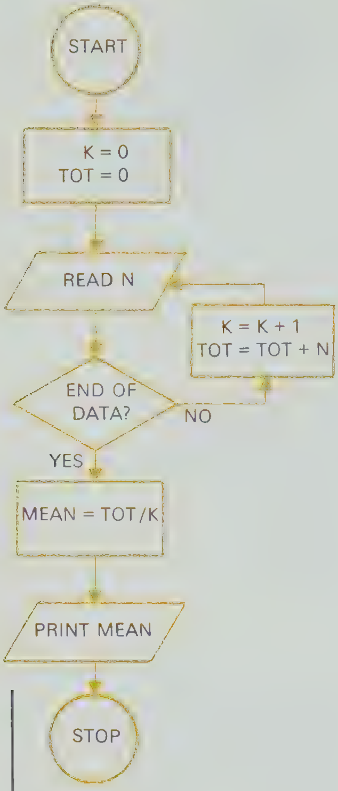
MEAN → AVERAGE
MEDIAN → MIDDLE
MODE → MOST OFTEN

The arithmetic mean and the median are more widely used than the mode. In practice the mean is used most frequently since it can readily be found and doesn't require that the data be in order. However the mean can be greatly affected by one or two numbers that differ radically from the other numbers in the set. When this happens it is best to use the median.

The advantage!

The disadvantage!

The most important factor is the number of items included. You can rely on a mean or a median when you know that a large number of representative values were used to find it.



Example 5

The following amounts were collected in a community charity drive: \$30, \$25, \$50, \$10, \$5, \$20, \$300, \$15, \$20, \$25. Calculate the most representative donation.

Solution

(1) The mean is

$$\frac{\$30 + \$25 + \$50 + \$10 + \$5 + \$20 + \$300 + \$15 + \$20 + \$25}{10}$$

$$= \frac{\$500}{10} = \$50$$

(2) The median is $\frac{\$20 + \$25}{2} = \$22.50$ Since there are two middlenumbers.

The \$300 donation has a great effect on the mean. By including the \$300 we obtain the false picture of a \$50 mean. This group of donations is best represented by the median of \$22.50.

The other approach is to throw out the extreme value. The average of the remaining donations represents the group. Notice that if the extreme value of \$300 is omitted, the remaining nine donations have a mean of $\frac{\$200}{9}$ or \$22.22.

E Finding a representative value for data in a frequency distribution requires a different method.

Example 6

In one apartment complex in a large metropolitan area, the monthly rents being charged for 2-bedroom apartments are summarized as follows.

Monthly Rent	*Number of Apartments*
\$210	7
215	10
220	15
225	12
230	20
235	18
240	13
245	15
250	9
255	6

Find (*a*) the mode, (*b*) the median rent, and (*c*) the mean rent.

Solution

(*a*) The mode by inspection is $230.

(*b*) To find the median, we note that there are 125 apartments listed, so the median rent is for the $\frac{125 + 1}{2}$ or 63rd apartment in the list. By addition, we find that the middle apartment is one of the 20 listed at $230.

∴ The median rent is $230.

(*c*) To find the arithmetic mean rent, it would be wrong to average the rents from $210 to $255, as some rents appear more frequently than others. One method of finding the mean is to list the 125 rents, total them, and divide by 125. Effort can be saved by means of a table.

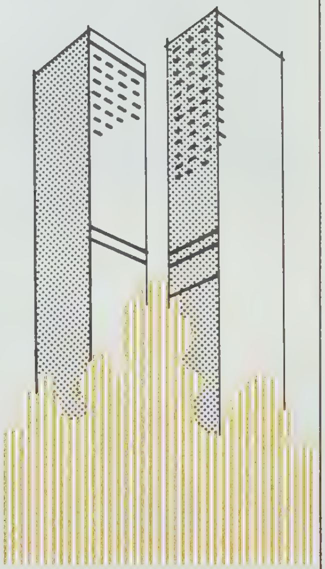
Monthly Rent ①	*Number of Apartments* ②	①×②
$210	7	$ 1470
215	10	2150
220	15	3300
225	12	2700
230	20	4600
235	18	4230
240	13	3120
245	15	3675
250	9	2250
255	6	1530
	125	$29 025

∴ The average rent is $\frac{\$29\ 025}{125}$ or $232.20.

This average is called a *weighted average*.

44 apartments so far! →

7
10
15
12
20
18
⋮



In what way is this average weighted?

OVER 17
FULL TIME
MANY OPENINGS

$148-$184 per wk.

For 2 Units
NO EXPERIENCE NECESSARY

A new division of large publishing company is opening many new offices. On the job training provided. Must be able to start immediately.

362-1019
BETWEEN 8:30 A.M.-3:30 P.M.

Exercise 3.2

Discussion

1. In finding the median of a group of numbers what step must you take first?

2. An ad in the paper says, "Work at Zobble Manufacturing Co. Average weekly salary is $180." Not bad, you say?

Job Classification	*Weekly Payroll*
Manager	$ 150
Gripper	100
Griper	85
Stripper	95
Striper	80
Pusher	85
Runner	80
Designer	135
Secretary	90
President	900
	$1800

$$\text{Average} = \frac{\$1800}{10} = \$180$$

Comment on the use of "average" in the advertisement.

3. Which of *mode*, *median*, and *mean* can be read immediately from a frequency distribution?

4. When there is a large array of data it becomes very difficult to calculate the mean. Why? In such a case how would *you* find the average?

5. (*a*) Use the frequency distribution at the right to find the median mark.
   (*b*) State the mode for this set of data.
   (*c*) How would you find the arithmetic mean of the data?

MARK (OUT OF 10)	NUMBER OF STUDENTS
0	1
1	3
2	4
3	3
4	2
5	9
6	5
7	12
8	8
9	6
10	3

Written Solutions

6. The batting averages of the Ninesville Peewees were 0.263, 0.309, 0.350, 0.207, 0.256, 0.278, 0.378, 0.283, 0.274, and 0.229. Find: (*a*) the median batting average,
(*b*) the arithmetic mean of these averages.
Would this necessarily be the *team* batting average?

7. In the *Podunk Junction Times*, 2-year old cars were advertised at the following prices.

   $1750, $1230, $1575, $2400, $3200, $1850,
   $1475, $1695, $2150, $3800, $2595, $2160

   Find the average cost of a 2-year old car.

8. In Science, class 10E was learning how to use the equal-arm balance to find the mass of objects. Objects A and B were weighed by 10 different students, with these results.

   Object A 235 g, 241 g, 250 g, 229 g, 235 g, 350 g, 243 g, 248 g, 236 g, 232 g

   Object B 650 g, 642 g, 672 g, 655 g, 660 g, 525 g, 720 g, 645 g, 658 g, 628 g

   Find
   (*a*) the arithmetic mean, and
   (*b*) the median of the masses for each object. How would you find the best approximation to the true mass of each object?

9. The students of Lowville High took a survey to find the hourly earnings of students in the school having part-time jobs. They summarized their results using a table.

Hourly Rate	*Number of Students*
$1.90	16
1.95	25
2.00	30
2.05	53
2.10	75
2.15	125
2.20	62
2.25	80
2.30	50
2.35	45
2.40	20

For this distribution of hourly earnings, find
(*a*) the mode,
(*b*) the median, and
(*c*) the arithmetic mean.

6(a) 0.276 (b) 0.283
7. $2157

8.	Object A	Object B
(a)	250 g	646 g
(b)	239 g	653 g

9(a) $2.15 (c) $2.17
(b) $2.15

Remember weighted average?

10. Sara Saver decided to invest in shares of the R and M Supermarket. She was able to buy 50 shares at \$4.75, 125 shares at \$5.60, and 75 shares at \$6.10. Find the average cost per share.

11. (*a*) The table below summarizes the costs of material for the Flimsy Fabric Shoppe. Find the average cost per metre.
    (*b*) If the material is sold at the prices indicated, find the average profit per metre.

Amount (metres)	*Cost per Metre*	*Selling Price per Metre*	*Amount (metres)*	*Cost per Metre*	*Selling Price per Metre*
200	\$0.50	\$0.99	75	\$1.80	\$3.50
150	0.82	1.25	50	2.10	4.00
75	0.90	1.49	125	2.50	4.50
125	1.00	1.75	75	2.75	5.00
90	1.10	2.25	25	3.00	5.50

12. If you have computer programming experience, write a program to determine the mean of a set of data.

10. **\$5.58**
11(a) **\$1.36**
(b) **\$1.12**

3.3 Statistics Illustrated by Graphs

A You have already seen how a frequency distribution can be illustrated by a bar graph. Bar graphs have other uses as well.

Example 1

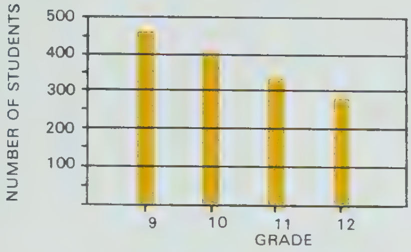
The numbers of students in the grades of Union High School were as follows.

Grade	*Number of Students*
9	450
10	400
11	325
12	280

Illustrate by a bar graph.

Solution

Since 450 is the largest number, we make 1 unit on the vertical axis represent 50 students.



DISTRIBUTION OF
DELTA HIGH STUDENTS
BY GRADE

How many students leave school between Grades 10 and 12?

B Two or more quantities can be compared by the use of a *comparison bar graph.*

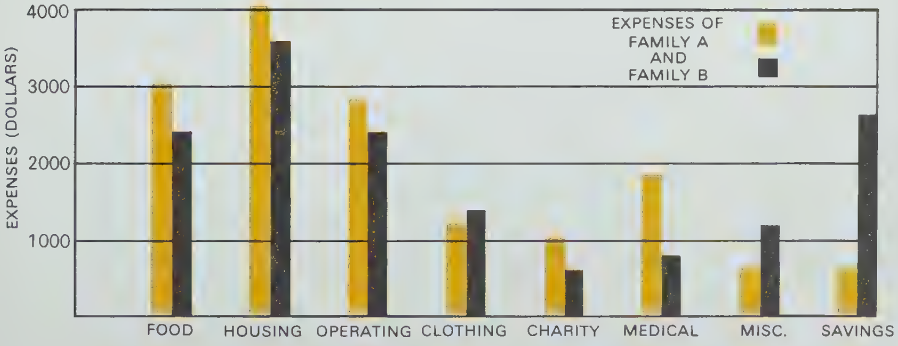
Example 2

Two families have the following division of expenses of $15 000 during one year.

	Family A	*Family B*
Food	$3000	$2400
Housing	4000	3600
Operating expenses	2800	2400
Clothing	1200	1400
Charitable donations	1000	600
Medical expenses	1800	800
Miscellaneous	600	1200
Savings	600	2600

Illustrate using a comparison bar graph.

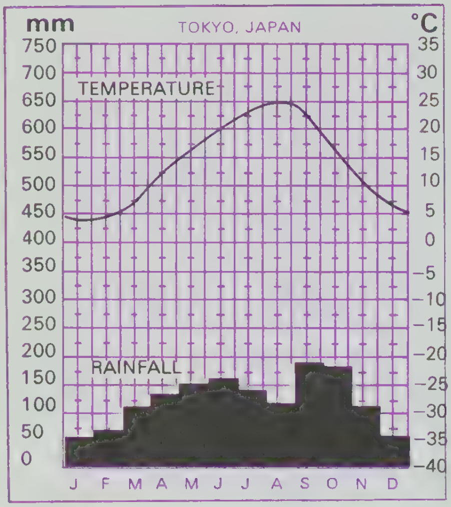
Solution



FOOD
HOME
EDUCATION
CLOTHES
VACATION
CAR
DOUBLE WORD SCORE
DOUBLE LETTER SCORE

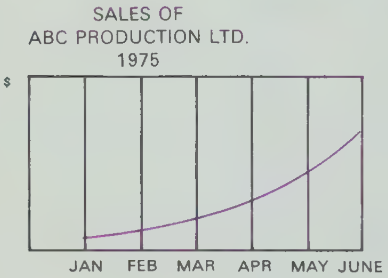
Exercise 3.3

Discussion

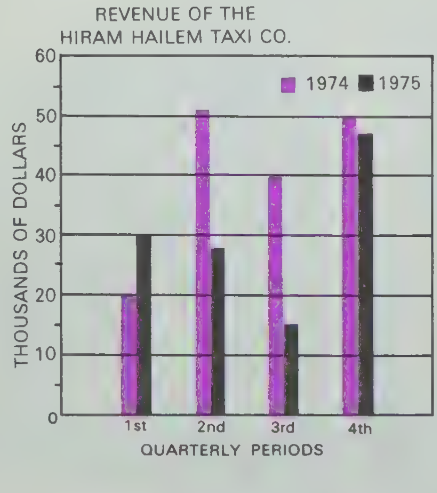


1. Weather information for the months of the year is often displayed as in the diagram. The letters across the bottom refer to the months of the year. The line graph shows the average monthly temperature (degrees Celsius are on the vertical axis at the right) and the bar graph shows the average monthly rainfall (millimetres of rainfall are on the vertical axis on the left).

   (*a*) State the rainfall in the wettest and driest months.
   (*b*) State the hottest and coldest months.
   (*c*) Estimate the average monthly rainfall and then check by actually listing the rainfall by month.



2. The accompanying diagram seems to give an optimistic picture of ABC Productions Ltd. Why is it misleading?



3. The accompanying chart compares the gross revenues of the Hiram Hailem Taxi Company by quarter for the calendar years 1974 and 1975.

   (*a*) State the revenues for the first quarter of each year.
   (*b*) What is the drop in revenues in the third quarter of 1975 from the previous year? Give a possible reason for such a sharp decline.
   (*c*) Why might the fourth quarter be a time of high revenues?

Written Solutions

4. The results of a survey of teen-age weekly spending habits are shown below.

	Boys	*Girls*
Entertainment	$ 3.00	$ 1.75
School	1.00	1.00
Savings	1.00	1.00
Food	3.50	2.75
Clothing	2.00	2.75
Sports equipment	1.25	0.75
Gasoline	4.00	2.50
Records, tapes	1.25	1.25
Cosmetics	0.25	1.75
Other	3.50	3.25
	$20.75	$18.75

Construct a comparison bar graph illustrating this survey.

5. The total value of goods and services produced in Canada is called Canada's Gross National Product (GNP). The following list shows the growth of the GNP since 1935.

Year	*GNP*	*Year*	*GNP*
1935	$ 4 301 000 000	1955	$28 528 000 000
1940	6 713 000 000	1960	38 359 000 000
1945	11 863 000 000	1965	55 369 000 000
1950	18 491 000 000	1970	85 610 000 000

Construct a bar graph showing this growth.

Does this mean that in 1970 20 times as much was produced as in 1935?

6. In 1971 the values of certain mineral products produced in Canada were as follows.

Mineral	*Value*	*Mineral*	*Value*
Petroleum	$1 356 943 000	Cement	$191 244 000
Nickel	800 064 000	Sand and gravel	152 628 000
Copper	760 016 000	Potash	134 955 000
Iron ore	555 136 000	Coal	121 727 000
Zinc	418 161 000	Lead	109 488 000
Natural gas	342 549 000	Stone	96 537 000
Asbestos	203 999 000	Gold	79 903 000
Gas by-products	193 191 000	Silver	71 797 000

Draw a bar graph illustrating this information.

7. The following table shows the percentage of Canada's exports shipped to the countries named for selected years since Confederation.

	1910	*1920*	*1930*	*1940*	*1950*	*1960*	*1970*
United Kingdom	50	36	27	41	15	17	9
Commonwealth	6	6	9	10	7	6	4
United States	37	40	45	35	65	56	65
Others	7	18	20	14	13	21	22

Illustrate by a comparison bar graph.
What conclusions can you draw concerning Canada's pattern of exports?

8. The average monthly temperature and rainfall for the cities of Calgary, Alberta, and St. John's, Newfoundland are listed.

Calgary, Alberta

Month	J	F	M	A	M	J	J	A	S	O	N	D
Temp (°C)	−11	−9	−4	4	9	13	16	15	11	6	−2	−7
Rainfall (mm)	13	15	18	20	58	74	66	64	33	18	18	13

St. John's, Newfoundland

Month	J	F	M	A	M	J	J	A	S	O	N	D
Temp (°C)	−4	−5	−2	2	6	11	15	16	12	7	3	−2
Rainfall (mm)	137	127	117	109	97	91	97	94	97	137	152	137

(*a*) Calculate the mean temperature over a full year.
(*b*) Construct a line graph showing the mean monthly temperatures on the same diagram.
(*c*) Construct a comparison bar graph illustrating the monthly rainfalls.
(*d*) Compare the weather in the two cities in a brief paragraph.

9. In 1971, Canadians belonged to the following ethnic groups: British Isles 9 624 120, French 6 180 120, German 1 317 200, Italian 730 820, Ukrainian 580 655, Netherlander 425 945, Scandinavian 384 795, Polish 316 425, Indian/Eskimo 312 765, Jewish 296 940, Hungarian 131 890, Greek 124 475, Chinese 118 820, Other European 650 520, Other Asian 166 720, Other/Unknown 206 100.

Convert these numbers to percents and illustrate this information by a bar graph.

3.4 Standard Deviation

A Dianne Dark manages a factory producing light bulbs. Dianne knows that to be sure she is producing quality bulbs at all times, she must make frequent tests. The table shows the life of 50 bulbs tested in each of 3 weeks of production.

Life of Bulb (h)	*Number of Bulbs*		
	Week 1	*Week 2*	*Week 3*
500-599	0	1	0
600-699	0	2	0
700-799	2	8	2
800-899	10	10	5
900-999	15	15	8
1000-1099	9	6	10
1100-1199	8	4	9
1200-1299	6	2	7
1300-1399	0	1	5
1400-1499	0	1	4
Total	50	50	50

1. State the grouping containing the median life of the bulbs tested for each week.
2. State the range of hours for the bulbs tested for each week.
3. Which week produced the best bulbs? The worst?
4. In which week was quality most consistent?

B Ideally all bulbs would last approximately the 1000 h that Dianne uses in her advertising. That is, the *range* should be as narrow as possible around a mean of 1000.

In Section 3.2 you studied the mode, the median, and the average (mean). These numbers can be used to represent the items in a distribution. The *range* gives us some idea of the *spread* of values in the data. This spread is important in industry. When producing items whose dimensions must be correct to within a very small error, the range of measurements of a dimension must be as small as possible.

'Deviation' means "difference."

Sometimes it is useful to eliminate values near the ends of the range when measuring the spread.

Example 1

On a test out of 100, a class of 20 students received the following marks. Comment on the range of achievement in this class.

15	50	65	72
30	52	67	75
33	57	67	81
38	57	68	85
40	60	70	96

Solution

The range of these data is 15 to 96. However, if we eliminate the highest and lowest mark, or 10% of the cases, the range is narrowed to 30 to 85 and still includes 90% of the class.

We can comment more precisely by finding the *average deviation* of the marks from the mean.

$$\text{The mean mark} = \frac{\text{sum of all the marks}}{20} = \frac{1178}{20} \doteq 59$$

The deviations from the mean are

$$59 - 15,\ 59 - 30, \ldots, 85 - 59,\ 96 - 59.$$

$\therefore$ Average deviation

$$= \frac{(59 - 15) + (59 - 30) + (59 - 33) + \ldots + (85 - 59) + (96 - 59)}{20}$$

$$= \frac{44 + 29 + 26 + \cdots + 26 + 37}{20}$$

$$= \frac{316}{20}$$

$$\doteq 16$$

Now we can say that the mean mark is 59 and the average student received within 16 marks of this.

C There is a third way of measuring the spread of a group of numbers from their mean called the *standard deviation.* The formula for it is

$$\text{Standard deviation} = \sqrt{\frac{\text{sum of the squares of the deviations}}{\text{the number of items}}}$$

Example 2

Given the numbers 4, 8, 10, 12, 13, 16, 20, and 25, find (*a*) the mean and (*b*) the standard deviation.

Solution

(*a*) $$\text{Mean} = \frac{4 + 8 + 10 + \ldots + 25}{8} = \frac{108}{8} \doteq 14$$

(*b*)

Number	*Mean*	*Deviation*	*Square of Deviation*
4	14	$14 - 4 = 10$	$10^2 = 100$
8	14	6	36
10	14	4	16
12	14	2	4
13	14	1	1
16	14	2	4
20	14	6	36
25	14	11	121
108			318

$$\therefore \text{Standard deviation} = \sqrt{\frac{\text{Sum of the squares of the deviations}}{\text{Number of items}}}$$

$$= \sqrt{\frac{318}{8}}$$

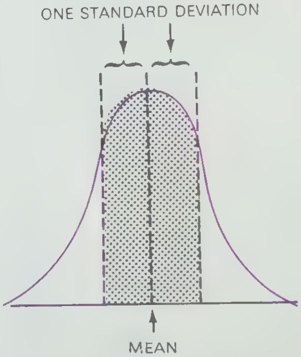
$$= \sqrt{39.8}$$

$$\doteq 6$$

The standard deviation has a property that makes it quite important in advanced work in mathematics. If a set of data is normally distributed about its mean, then just over two thirds of the items are within 1 standard deviation of the mean. (See the margin diagram.)

Not only is this true, but we also know the percentage of items within 2, 3, and so on, standard deviations of the mean and can find the range that includes 80%, 90%, or 95% of all items.

The calculation of the standard deviation is a basic step in determining the quality-control limits when testing production in a factory. A shop manager knows whether to scrap some production and adjust the machinery if too many items tested do not fall within given tolerances.



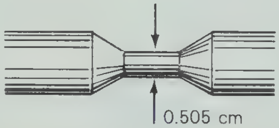
The shaded portion includes about $\frac{2}{3}$, and the unshaded portion $\frac{1}{3}$ of the total of the items in the distribution.

5. **Average deviation = 15.7**

6.	*Range*	*Mean*
(a)	**(0.498 cm–0.510 cm**	**0.505 cm**
(b)	**$235–$4135**	**$2152**
(c)	**48 kg–59 kg**	**53 kg**

	Av. Deviation
(a)	**0.0024 cm**
(b)	**$811.87**
(c)	**3.1 kg**

6(*a*)



6 (*b*)

BELGIUM	$2590
CANADA	3230
DENMARK	2795
FEDERAL REPUBLIC OF GERMANY	2935
FRANCE	2655
GREECE	855
ICELAND	1560
ITALY	1630
LUXEMBOURG	1970
NETHERLANDS	2415
NORWAY	2550
PORTUGAL	670
TURKEY	235
UNITED KINGDOM	2055
UNITED STATES OF AMERICA	4135

Exercise 3.4

Discussion

1. In a distribution of 40 items a 90% range was used to measure the spread. How many items were eliminated from the range?

2. 15 is the mean of a distribution that includes the numbers 8, 12, 16, and 20. State the deviation of each of these items from the mean.

3. State the mean of the distribution that consists of 5, 8, 10 and 17. How would you continue if you wanted to find the average deviation?

4. Before finding the standard deviation of a set of data, what must be calculated?

Written Solutions

5. Copy and complete the following table to find the average deviation of the data.

Number	*Mean*	*Deviation*
15		
18		
25		
42		
51		
59		

6. Calculate the range, the mean, and the average deviation for each of the following sets of data.

   (*a*) The diameters, in centimetres, of 10 test items machined on a lathe: 0.504, 0.503, 0.505, 0.507, 0.506, 0.498, 0.510, 0.502, 0.505, and 0.508.

   (*b*) The per capita values of production of the citizens of the 15 NATO countries listed in the margin.

   (*c*) The masses, in kilograms, of 8 teenagers entered in the Healthy Teen Contest of Upper Podunk Junction: 48, 53, 49, 59, 57, 54, 55, and 50.

7. The test results out of 100 in a class of 24 Home Economics students are as follows.

65	50	63	45	73	65	40	52
72	78	59	55	64	42	62	60
56	61	85	70	57	80	75	68

(*a*) What range includes 75% of the marks?
(*b*) Using a table, find the standard deviation of the marks.
(*c*) Find the number of items that are within one standard deviation of the mean and express this as a percentage of the total.

8. The Traffic Department of Suburbia conducted a survey to determine the need for a traffic light at a busy intersection.

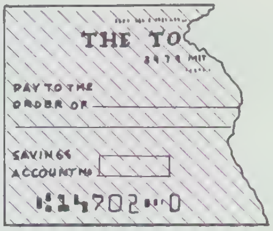
Time	*Vehicles at Street A*	*Vehicles at Street B*
06:00–07:00	500	450
07:00–08:00	750	625
08:00–09:00	860	720
09:00–10:00	415	250
10:00–11:00	120	90
11:00–12:00	350	240
12:00–13:00	560	410
13:00–14:00	200	130
14:00–15:00	150	100
15:00–16:00	600	540
16:00–17:00	800	620
17:00–18:00	875	745
18:00–19:00	420	360
19:00–20:00	510	405

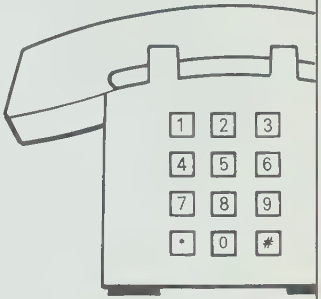
(*a*) Find the mean of the number of vehicles per hour on each street.
(*b*) Determine the standard deviation for these sets of data.
(*c*) As a class project count the number of vehicles passing through a busy intersection during an hour period for several days and calculate the mean and standard deviation for your data.

7(a) 50-75
(b) 11.5
(c) 16, 66.7%

8.	**Street A**	**Street B**
(a)	**508**	**406**
(b)	**243**	**216**

Unit 4
The Number System

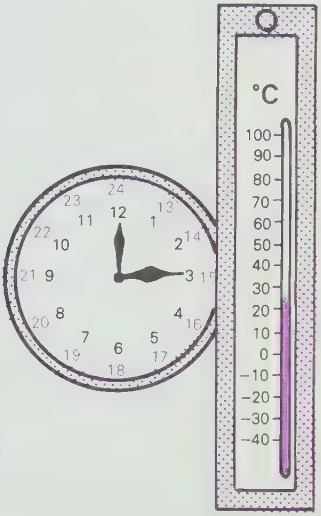

THE TO
PAY TO THE ORDER OF
SAVINGS ACCOUNT No
1:14702"0


1
2
3
4
5
6
7
8
9
*
0
#

4.1 Review of the Rational Numbers

At any minute of the day (even outside math class!) we can come into contact with one of the most useful sets of symbols: *numbers.* Numbers are encountered everywhere. Every act of counting or measuring involves us with numbers.

Since we cannot avoid contact with numbers, it is important that we know something of the make-up of our number system. The chart below shows the relationship among the various sets of numbers.


°C
100
90
80
70
60
50
40
30
20
10
0
−10
−20
−30
−40
12
1
2
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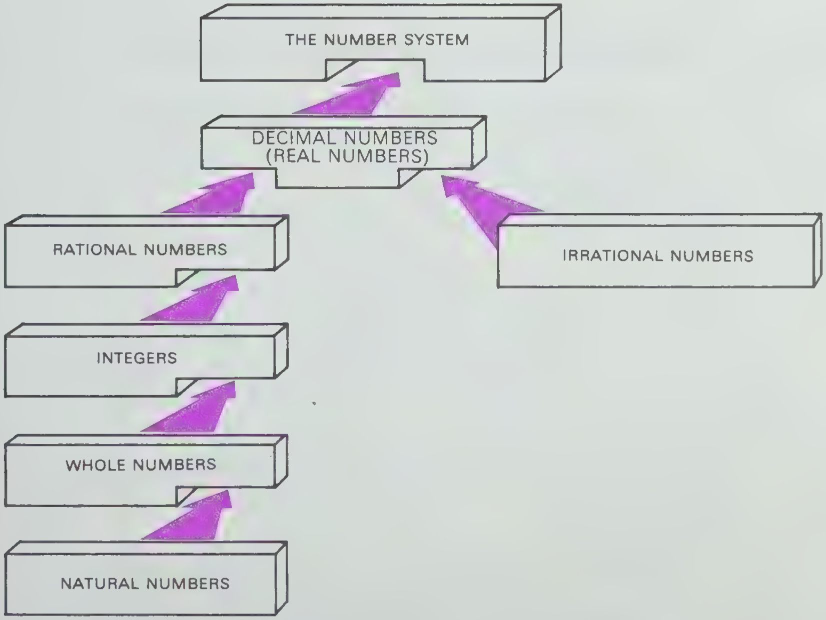
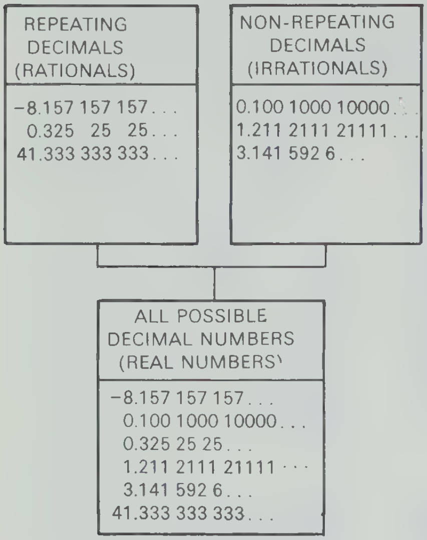
23


THE NUMBER SYSTEM
DECIMAL NUMBERS (REAL NUMBERS)
RATIONAL NUMBERS
IRRATIONAL NUMBERS
INTEGERS
WHOLE NUMBERS
NATURAL NUMBERS


REPEATING DECIMALS (RATIONALS)
−8.157 157 157...
0.325 25 25...
41.333 333 333...
NON-REPEATING DECIMALS (IRRATIONALS)
0.100 1000 10000...
1.211 2111 21111...
3.141 592 6...
ALL POSSIBLE DECIMAL NUMBERS (REAL NUMBERS)
−8.157 157 157...
0.100 1000 10000...
0.325 25 25...
1.211 2111 21111...
3.141 592 6...
41.333 333 333...

Exercise 4.1

Discussion

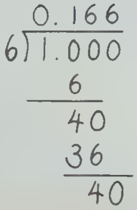
1. State whether each number is rational or irrational.
   - (*a*) 17.13 13 13 13...
   - (*b*) −0.8531 8531 8531...
   - (*c*) 1.12 13 14 15 16 17 18...
   - (*d*) 93.687 500 000...
   - (*e*) 215.21 221 2221 22221...
   - (*f*) 0.123 456 789 10 11 12...
   - (*g*) 7.392 568 714 504 177...
   - (*h*) 0.000 406 406 406...
   - (*i*) 3294.000 000 000...
   - (*j*) 64.2468 1012 1416 18...
2. (*a*) Is 5 a rational number? An integer? A whole number? A natural number? Explain.
   (*b*) Is −3 a rational number? An integer? A whole number?
   (*c*) Is 5.261 261 ... a rational number? An integer? A whole number? Explain.
3. A number could belong to more than one set. The number 5, for example, may be classified as a counting number, a whole number, an integer, or a real number. State the sets of numbers to which each of the following could belong.
   - (*a*) 19
   - (*b*) $\frac{3}{4}$
   - (*c*) −7
   - (*d*) 2563
   - (*e*) 0
   - (*f*) −5.92
   - (*g*) $-1\frac{1}{2}$
   - (*h*) 1.27 27...

4.2 Fractional Numbers and Periodic Decimals

A You may recall from previous work that it is possible to find a periodic decimal for every fractional number, and a fractional number for every periodic decimal.

Consider the fraction $\frac{1}{6}$. By simple division, we find that

$$\frac{1}{6} = 0.166\ldots = 0.1\dot{6} \text{ (abbreviated form)}$$


0.166
6)1.000
6
40
36
40

The *period* of $0.1\dot{6}$ is 6 and its *length* is 1.

Period is the set of repeated digits.

Similarly, $4\frac{1}{7} = \frac{29}{7} = 4.\dot{1}4285\dot{7}$. Here the *period* is 142857, and the *length* of the period is 6.

Length of a period is the number of digits in 1 period.

1. State the period and the length of each of the following periodic decimals.
   - (*a*) 5.324 324...
   - (*b*) $-7.0\dot{1}56\dot{3}$
   - (*c*) $0.8\dot{0}$
   - (*d*) 942.659 59 59...
   - (*e*) $0.0\dot{0}41\dot{2}$
   - (*f*) $-11.\dot{3}$

B The reverse procedure (finding the fractional equivalent of a rational number) can be somewhat more involved.

Example 1

Convert to fractional form.

(*a*) 0.25 (*b*) -2.375

Solution

(*a*) $$0.25 = 0.25 \times \frac{100}{100} = \frac{25}{100} = \frac{1}{4}$$

(*b*) $$-2.375 = -2.375 \times \frac{1000}{1000} = \frac{-2375}{1000} = -2\frac{3}{8}$$

The period of $0.25\dot{0}$ is 0. Decimals with period 0 are also called terminating decimals.

Example 2

Convert to fractional form.

(*a*) $0.\dot{2}\dot{7}$ (*b*) $2.\dot{3}0\dot{6}$

Notice that the period is 306.

Solution

(*a*) $$\begin{aligned} \text{Let} \quad x &= 0.\dot{2}\dot{7} = 0.27\,27\,27\,27\ldots \\ \therefore\ 100x &= 27.27\,27\,27\ldots \\ \text{but} \quad 1x &= \underline{\ 0.27\,27\,27\ldots\ } \\ \text{Subtract.} \quad 99x &= 27 \\ \therefore\ x &= \frac{27}{99} \\ &= \frac{3}{11} \\ \therefore\ 0.\dot{2}\dot{7} &= \frac{3}{11} \end{aligned}$$

Why $100x$? What happens to the repeating decimal portion?

(*b*) $$\begin{aligned} \text{Let} \quad k &= 0.\dot{3}0\dot{6} = 0.306\,306\,306\ldots \\ \therefore\ 1000k &= 306.306\,306\ldots \\ \text{but} \quad 1k &= \underline{\ 0.306\,306\ldots\ } \\ \text{Subtract.} \quad 999k &= 306 \\ \therefore\ k &= \frac{306}{999} \\ &= \frac{34}{111} \\ \therefore\ 2.\dot{3}0\dot{6} &= 2\frac{34}{111} \end{aligned}$$

Why $1000k$?

Exercise 4.2

Discussion

1. State two examples of each.
   (*a*) fractional numbers
   (*b*) periodic decimals
   (*c*) terminating decimals
   (*d*) non-periodic decimals

2. Express as fractions in lowest terms.
   (*a*) 0.3
   (*b*) 0.5
   (*c*) 0.009
   (*d*) 0.125
   (*e*) 2.25
   (*f*) 1.375

3. State the number you would use to multiply each of the following in finding its fractional equivalent.
   (*a*) $0.\dot{9}\dot{3}$
   (*b*) $0.\dot{5}1\dot{6}$
   (*c*) 0.3333 . . .
   (*d*) 0.012 012 012. . .
   (*e*) $0.\dot{4}$
   (*f*) $5.\dot{6}5\dot{2}$

Written Solutions

4. Convert the following to decimal fractions.
   (*a*) $\frac{3}{5}$ (*b*) $\frac{7}{20}$ (*c*) $3\frac{3}{4}$ (*d*) $5\frac{7}{8}$ (*e*) $8\frac{19}{50}$

5. Change to fractional form in lowest terms.
   (*a*) 3.35 (*b*) 14.675 (*c*) 25.32 (*d*) 0.4875 (*e*) 0.003 75

6. Find the decimal equivalent of each of the following fractional numbers and state the period and its length.
   (*a*) $\frac{2}{3}$
   (*b*) $\frac{5}{7}$
   (*c*) $\frac{2}{9}$
   (*d*) $\frac{4}{5}$
   (*e*) $\frac{1}{13}$
   (*f*) $\frac{3}{13}$

7. Express in fractional form in lowest terms.
   (*a*) 0.04
   (*b*) 2.35
   (*c*) 0.72
   (*d*) 1.875
   (*f*) 0.68 68 68. . .
   (*g*) −1.36 36 36. . .
   (*g*) −1.363636 . . .
   (*h*) $0.\dot{1}0\dot{2}$

8. Convert 0.263 63 63. . . to fractional form.

Surprised? →

9. Find the fractional equivalent of $0.5\dot{0}$ and $0.4\dot{9}$.

4(a) 0.6 (e) 2.046 875
(c) 3.75

5(a) $3\frac{7}{20}$ (e) $\frac{3}{800}$
(c) $25\frac{8}{25}$

6.

	Decimal	*Period*	*Length*
(a)	$0.\dot{6}$	6	1
(c)	$0.\dot{2}$	2	1
(e)	$0.\dot{0}7692\dot{3}$	076923	6

7(a) $\frac{1}{25}$ (e) $\frac{8}{9}$
(c) $\frac{18}{25}$ (g) $-1\frac{4}{11}$

8. $\frac{29}{110}$

9. $\frac{1}{2}$

4.3 Working with Rational Numbers

Study carefully the following examples.

Example 1

Evaluate.

(*a*) $\frac{1}{3}+\frac{3}{4}-\frac{5}{2}$ (*b*) $3\frac{1}{2}\times 5\frac{1}{3}$ (*c*) $2\frac{1}{4}\div\frac{5}{8}$

Solution

(*a*)
$$\begin{aligned}\frac{1}{3}+\frac{3}{4}-\frac{5}{2} &= \frac{4}{12}+\frac{9}{12}-\frac{30}{12}\\ &= \frac{13}{12}-\frac{30}{12}\\ &= -\frac{17}{12}\end{aligned}$$

Fractions must have a common denominator before they can be added or subtracted!

*

(*b*)
$$\begin{aligned}3\frac{1}{2}\times 5\frac{1}{3} &= \frac{7}{\cancel{2}}\times\frac{\cancel{16}^{\,8}}{3}\\ &= \frac{56}{3}\end{aligned}$$

*

Change to improper fractions first!

(*c*)
$$\begin{aligned}2\frac{1}{4}\div\frac{5}{8} &= \frac{9}{4}\div\frac{5}{8}\\ &= \frac{9}{\cancel{4}}\times\frac{\cancel{8}^{\,2}}{5}\\ &= \frac{18}{5}\end{aligned}$$

To divide by a fraction, multiply by its reciprocal.

Example 2

If $a = 0.2$, $b = -0.1$, $c = 3$, evaluate:

(*a*) $4a - 3bc$ (*b*) $b^2 - a^2$

Solution

(*a*)
$$\begin{aligned}4a-3bc &= 4(0.2)-3(-0.1)(3)\\ &= 0.8+0.9\\ &= 1.7\end{aligned}$$

$$\boxed{\begin{array}{l} a = 0.2\\ b = -0.1\\ c = 3\end{array}}$$

(*b*)
$$\begin{aligned}b^2-a^2 &= (-0.1)^2-(0.2)^2\\ &= 0.01-0.04\\ &= -0.03\end{aligned}$$

Recall substitution

1. Copy expression and values.
2. Replace variables with ( ) and substitute.
3. Evaluate.

Remember

BODMAS !

Exercise 4.3

Discussion

1. State the value of each of the following.
   - (*a*) $0.22 + 0.17$
   - (*b*) $0.19 - 0.24$
   - (*c*) $0.01 + 0.001$
   - (*d*) $\frac{5}{6} \times \frac{3}{2}$
   - (*e*) 0.5×0.7
   - (*f*) $(0.3)^2$
   - (*g*) $\frac{3}{4} \div \frac{5}{4}$
   - (*h*) $0.250 - \frac{1}{4}$
   - (*i*) $1\frac{1}{4} + 5\frac{3}{8}$
   - (*j*) $\frac{3}{4} - \frac{1}{4} \times 5$
   - (*k*) $7.5 + 3 \times 1.5$
   - (*l*) $2(\frac{3}{5} - \frac{2}{5}) - 1$

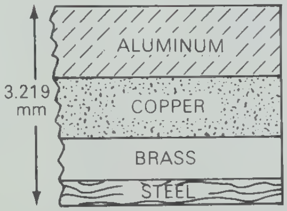
Written Solutions

2. Evaluate.
   - (*a*) $\frac{4}{5} + \frac{5}{6} - \frac{1}{2}$
   - (*b*) $2\frac{1}{4} - 3\frac{1}{8}$
   - (*c*) $\frac{4}{5} + (\frac{-3}{10}) - \frac{1}{20}$
   - (*d*) $-5\frac{1}{2} + 2\frac{1}{4} - 1\frac{7}{8}$
   - (*e*) $(-\frac{4}{5})(2\frac{1}{2})(-\frac{3}{7})$
   - (*f*) $2\frac{5}{8} \times 3\frac{3}{7}$
   - (*g*) $(3\frac{1}{2}) \div (\frac{14}{3}) \times \frac{4}{9}$
   - (*h*) $\left(\dfrac{1\frac{3}{4}}{-2\frac{5}{8}}\right) - 3\frac{1}{6}$
   - (*i*) $\frac{1}{2} - \frac{2}{3} \times \frac{5}{8}$
   - (*j*) $\frac{3}{4}$ of $1\frac{3}{5} \div \frac{2}{15}$

3. A piece of metal 7 cm long is cut into 4 equal lengths. Express the length of each segment as a fraction and as a decimal.

4. A piece of testing material is constructed of 4 different types of metal, as shown. Find the thickness of the brass if the other thicknesses are steel 0.188 mm, copper 1.125 mm, and aluminum 1.156 mm.

4.



5. Polly Titian makes 4 banners each requiring 2.3 m of material from a bolt of cloth 10.4 m long. What length of cloth does Polly have left over?

6. Find the total cost of Hannah and Henry's groceries.

   9 loaves of bread @ 42¢/loaf
   8 ℓ of milk @ 41¢/ ℓ
   250 g of bacon @ $2.80/kg
   3 dozen eggs @ 94¢/dozen
   500 g of butter @ $2.00/kg

2(a) $\frac{17}{15}$ (g) $\frac{1}{3}$
(c) $\frac{9}{20}$ (i) $\frac{1}{12}$
(e) $\frac{6}{7}$

3. $\frac{7}{4}$ cm, 1.75 cm
4. 0.750 mm
5. 1.2 m
6. $11.58

4.4 Percent

A Recall that percent means "per hundred." Fractions or decimal numbers are easily converted to percents as shown below.

Percent may also be considered as a ratio with 100 as the second term.
$5 \text{ percent} = \frac{5}{100} = 5\%$

Example 1

Convert to percent.

(*a*) $\frac{3}{8}$ (*b*) 0.675

Solution

(*a*) Let $\frac{3}{8} = \frac{x}{100}$ *

$$\therefore 8x = 300$$
$$\therefore x = \frac{300}{8}$$
$$= 37.5$$
$$\therefore \frac{3}{8} = \frac{37.5}{100} = 37.5\%$$

(*b*) $0.675 = \frac{67.5}{100} = 67.5\%$

* Remember equivalent ratios!

Remember cross-multiplication!

B To change percents to decimals or fractions, we simply reverse the procedure.

That makes sense!

Example 2

Convert to fractional and decimal form.

(*a*) 75% (*b*) 112.5%

Solution

(*a*) $75\% = \frac{75}{100} = \frac{3}{4}$ or $75\% = \frac{75}{100} = 0.75$

(*b*) $112.5\% = \frac{112.5}{100} = \frac{112.5}{100} \times \frac{10}{10} = \frac{1125}{1000} = \frac{9}{8}$ or $112.5\% = \frac{112.5}{100} = 1.125$

Multiply numerator and denominator by 10!

Exercise 4.4

Discussion

1. State as a percent.
   (*a*) $\frac{1}{2}$ (*c*) $\frac{4}{5}$ (*e*) 3
   (*b*) $\frac{1}{10}$ (*d*) 0.25 (*f*) 1.4

2. State in decimal form.
   (*a*) 80% (*c*) 9.2% (*e*) 260%
   (*b*) 27.5% (*d*) 42.3% (*f*) 0.5%

3. State in fractional form in lowest terms.
   (*a*) 60% (*c*) 8% (*e*) 55%
   (*b*) $33\frac{1}{3}$% (*d*) 150% (*f*) 0.7%

Written Solutions

4. If the sales tax is 5% and an item sells for $7.80 what is the total cost to a buyer?

Cost = Selling Price
+ Sales Tax

5. In machine shop class, Hydraulic Jack is asked to set up a table of decimal equivalents and percents for the rational numbers shown. Copy and complete the table.

Fraction	*Decimal*	*Percent*
$\frac{1}{4}$	■	■
$\frac{2}{5}$	■	■
$\frac{3}{8}$	■	■
$\frac{5}{8}$	■	■
$\frac{1}{6}$	■	■
$\frac{2}{3}$	■	■

6. What percent is 12 of each of the following numbers?
   (*a*) 24 (*c*) 16 (*e*) 12
   (*b*) 30 (*d*) 20 (*f*) 4

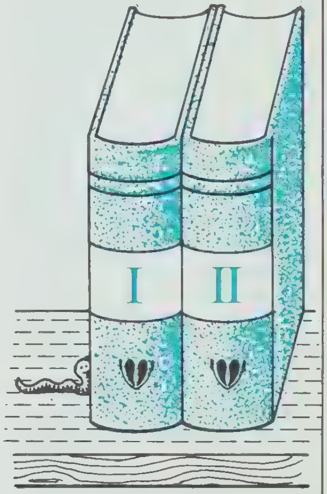
7. A bank charges 8% interest on money it lends for a one-year period. Calculate the interest on each of the following loans at this rate.
   (*a*) $100 (*c*) $17
   (*b*) $350 (*d*) $2575

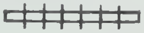
8. Over a 6-month period, a teen-ager's height increased from 120 cm to 132 cm. What was the percent increase in height?

4. $8.19
6(a) 50% (e) 100%
(c) 75%
7(a) $8 (c) $1.36
8. 10%

Nutty Math

1. How can you make 7 even?
2. Why is twice 10 like 2 times 11?
3. How many times can 19 be subtracted from 190?
4. Jack is carrying a bag of apples. Jill is carrying 10 bags. All the bags are the same size, but Jack's load is 30 times heavier than Jill's. How can this be?
5. What two whole numbers multiply together to give 31?
6. Six ears of corn are in a hollow stump. How long will it take a squirrel to remove them if he takes out 3 ears a day?
7. How can you divide 12 in half and get 7?
8. If it takes 1 min to make each cut, how long will it take to cut a 7 m pole into 7 equal pieces?
9. How many cubic metres of dirt are there in a hole 45 cm wide, 60 cm long, and 22.5 cm deep?
10. Two volumes of an encyclopedia are standing side by side on a bookshelf, as shown. A hungry bookworm starts eating at page 1 of volume I and eats its way in a straight line to the last page of volume II. If each cover is 2.5 mm thick and each book (without covers) is 30 mm thick, how far does the bookworm travel?



1. Cross out the "s". seven
2. Twice 10 is 20, and 2 times 11 is *twenty, too*!
3. Only once! Because then you subtract 19 from 171!
4. Jill's bags are empty!
5. 31 and 1. Easy, eh?
6. 6 days! Each day it carries out one ear of corn, as well as its own 2 ears, and that's 3 ears a day!
7. ~~XII~~ ∴ VII
8.  6 cuts therefore 6 min!
9. None! There's no dirt in a *hole*, you blockhead!
10. Precisely 5 mm.

Maintaining Basic Skills

Exercise 1

1. Add.

(*a*)	(*b*)	(*c*)	(*d*)
423	7148	$4\frac{1}{8}$	5.83
129	2997	$3\frac{2}{5}$	72.091
68	1563	$1\frac{7}{10}$	6.042
429	8411	$2\frac{3}{4}$	0.91
805	7652		23.4

2. Subtract.

(*a*)	(*b*)	(*c*)	(*d*)
4932	25.913	635	$48\frac{5}{6}$
1568	17.087	927	$29\frac{7}{8}$

3. Evaluate.

(*a*) $-5-(-7+2)$
(*b*) $(-3)(-7)(-2)$
(*c*) $-15 \div (-3+2)$
(*d*) $(-8)^0$
(*e*) 2^3
(*f*) $(\frac{3}{4})^{-2}$
(*g*) $2^{-1}+3^{-2}$
(*h*) $5(-8) \div (14-7 \times 2)$

4. Determine the total cost for the groceries listed below.

5 kg apples	at 69¢/kg
6 ℓ milk	at 41¢/ℓ
5 kg bananas	at 29¢/kg
500 g tomatoes	at \$1.20/kg
2.5 kg roast	at \$3.89/kg
5 tubes of toothpaste	at 98¢/tube

5. State the number of significant digits in each number.

(*a*) 43.1
(*b*) 0.152
(*c*) 46 200
(*d*) 7.200
(*e*) 1.73
(*f*) 0.0005

6. By estimation, determine which answer is correct.

(*a*)	47.6×11.2	5.3312,	533.12,	0.533 12
(*b*)	$95.3 \div 7.9$	120.6,	1.206,	12.06
(*c*)	$4.31 \times 116 \div 2.4$	208.3,	2.083,	2083

1(a)	**1854**	**(c)**	**$11\frac{39}{40}$**
2(a)	**3364**	**(c)**	**−292**
3(a)	**0**	**(e)**	**8**
(c)	**15**	**(g)**	**$\frac{11}{18}$**
4.	**\$22.59**		
5(a)	**3**	**(c)**	**5**

7. Find the product to 3 significant digits.
(*a*) 14.3×7.20 (*c*) $21.9 \times 52.5 \times 16.7$
(*b*) 412×83.0 (*d*) $4.39 \times 0.642 \times 95.7 \times 20.3$

8. Evaluate to 3 significant digits.
(*a*) $42.3 \div 9.40$ (*c*) $10.6 \div 114$ (*e*) $6.30 \div 7.10$
(*b*) $72.5 \div 11.7$ (*d*) $1.25 \div 0.985$ (*f*) $0.573 \div 3.14$

9. Evaluate to the nearest whole number.
(*a*) $\dfrac{14.3 \times 7.1}{9.2}$ (*c*) $\dfrac{140.3}{71 \times 0.94}$ (*e*) $\dfrac{22.4 \times 11.5^2}{67 \times 14.2}$
(*b*) $\dfrac{219 \times 63}{41}$ (*d*) $\dfrac{16 \times 17.3}{42 \times 1.95}$ (*f*) $\left(\dfrac{1.94 \times 31.6}{12.9}\right)^2$

10. Multiply.
(*a*) $x^2 \times x^7$ (*c*) $a^{-1} \times a^3$ (*e*) $h^{-1} \times h^{-2}$
(*b*) $y^0 \times y^5$ (*d*) $c^{-4} \times c^2$ (*f*) $5^3 \times 5^{-1}$

11. Divide.
(*a*) $x^5 \div x^2$ (*c*) $a^0 \div a^2$ (*e*) $h^{-4} \div h^5$
(*b*) $y^{-4} \div y^3$ (*d*) $c^{-1} \div c^{-2}$ (*f*) $4^3 \div 4^{-1}$

12. Simplify.
(*a*) $15x^3 \div (-3x)$ (*c*) $5y^0 \div y^{-2}$ (*e*) $8a^2 \div 2a^{-1}$
(*b*) $3a^5 \times (-2a)^{-1}$ (*d*) $(\frac{2}{3}) \div (\frac{2}{3})^3$ (*f*) $(\frac{3}{5})^{-1}$

13. Simplify.
(*a*) $(x^2)^3$ (*c*) $(2^3)^{-1}$ (*e*) $10^{-3} \times 10^5$
(*b*) $(y^{-1})^4$ (*d*) $(x^2y^3)^5$ (*f*) $4^{-1} + 2^{-1}$

14. Estimate and then evaluate.
(*a*) $(2.35 \times 10^2) \times (3.16 \times 10^{-1})$ (*c*) $\dfrac{6.23 \times 10^2 \times 4.26}{1.95 \times 10^4}$
(*b*) $(4.82 \times 10^{-3}) \div (5.18 \times 10^{-5})$ (*d*) $\dfrac{5.14 \times 10^{-1} \times 6.32}{3.76 \times 10^5 \times 1.79}$

15. Find the median and mean.
(*a*) 26, 84, 42, 78, 54, 79, 52 (*b*) 212, 256, 240, 260, 275, 225

16. Find the mean, and the average deviation.
(*a*) 42, 48, 50, 32, 56, 54 (*b*) 15, 22, 26, 24, 18, 14, 16

17. Find the standard deviation for the following marks.
58, 62, 78, 42, 67, 52, 85, 46, 70, 66

7(a)	**103**	**(c)**	**19 200**
8(a)	**4.50**	**(c)**	**0.0930**
9(b)	**337**	**(d)**	**3**
10(b)	y^5	**(d)**	c^{-2}
11(a)	x^3	**(c)**	a^{-2}
12(a)	$-5x^2$	**(c)**	$5y^2$
13(b)	y^{-4}	**(d)**	$x^{10}y^{15}$
14(a)	7.43×10		
15(a)	**54, 59**		
16(a)	**47, 6.7**		
17.	**12.9**		

Unit 5
Operating with Square Root

A guy wire is used to steady a television tower. How long is the guy wire in the diagram?

$$x^2 = 40^2 + 30^2$$ Remember Pythagoras!

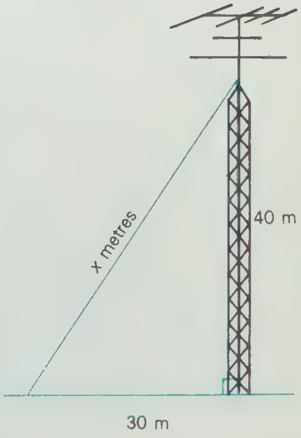
$$= 1600 + 900$$

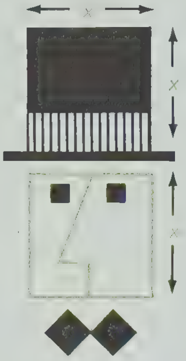
$$= 2500$$

But x^2 means $x \times x$. Can we find a number which multiplied by itself gives 2500?

We know that $50 \times 50 = 2500$.

Thus $x = 50$ and the guy wire is 50 m long.





THE PERFECT SQUARE

We say that 2500 is the square of 50. Similarly 4 is the square of 2 and 49 is the square of 7. Numbers such as 2500, 4, and 49 are called *perfect squares.*

The square of 7 is 49. The square of -7 is 49. We say that 7 and -7 are *square roots* of 49.

We write the symbol $\sqrt{}$ to mean square root and $\sqrt{}$ is called a *radical sign.* Whenever we use the radical sign we shall expect you to give only the positive square root of the number.

For example,

$$\sqrt{64} = 8$$
$$\sqrt{81} = 9$$

Discussion

Express each of the following as a whole number.

(*a*) $\sqrt{25}$
(*b*) $\sqrt{36}$
(*c*) $\sqrt{1}$
(*d*) $\sqrt{81}$
(*e*) $\sqrt{16}$
(*f*) $\sqrt{100}$
(*g*) $\sqrt{144}$
(*h*) $\sqrt{400}$
(*i*) $\sqrt{900}$
(*j*) $\sqrt{1600}$
(*k*) $\sqrt{4900}$
(*l*) $\sqrt{0}$

5.1 Multiplying and Dividing Radicals

A A number expressed with the radical sign is called a *radical.* The following examples illustrate how we may multiply and divide with radicals.

Example 1

Evaluate $\sqrt{25} \times \sqrt{25}$.

Solution

$$\begin{aligned}\sqrt{25} \times \sqrt{25} &= 5 \times 5\\ &= 25\end{aligned}$$

This simple result can be expressed as

$$(\sqrt{25})^2 = 25$$

1. Show that $(\sqrt{9})^2 = 9$

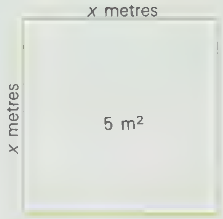
B We shall agree that for any whole number a,

$$\boxed{(\sqrt{a}\,)^2 = a}$$

The rule may seem obvious for numbers like $\sqrt{25}$ and $\sqrt{9}$, but is it obvious for $\sqrt{5}$? Is $\sqrt{5}$ a number? If so, is $\sqrt{5}$ a *useful* number?

Example 2

The area of a square room is 5 m². What is the length of each side of the room?



Solution

Let the length of one side be x metres.

$$\therefore \quad x \times x = 5$$
$$x^2 = 5$$
$$\therefore \quad x = \sqrt{5} \qquad (\text{since } (\sqrt{5}\,)^2 = 5)$$

$\therefore$ The length of each side of the room is $\sqrt{5}$ m.

Since numbers such as $\sqrt{5}$ *can* be used to express length, it is necessary to know how to calculate with such numbers.

C How do we multiply radicals? Is there a special rule?

Example 3

Express $\sqrt{25} \times \sqrt{4}$ as a radical.

Solution

$$\begin{aligned}\sqrt{25} \times \sqrt{4} &= 5 \times 2 \\ &= 10 \\ &= \sqrt{100}\end{aligned} \qquad \text{or} \qquad \begin{aligned}\sqrt{25} \times \sqrt{4} &= \sqrt{25 \times 4} \\ &= \sqrt{100}\end{aligned}$$

In general,

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$
$$= \sqrt{ab}$$

1. Express each of the following as a radical. Justify your answer.
   - (*a*) $\sqrt{4} \times \sqrt{9}$
   - (*b*) $\sqrt{9} \times \sqrt{16}$
   - (*c*) $\sqrt{1} \times \sqrt{49}$
   - (*d*) $\sqrt{100} \times \sqrt{16}$

2. Express each of the following as a radical.
   - (*a*) $\sqrt{2} \times \sqrt{5}$ *
   - (*b*) $\sqrt{3} \times \sqrt{7}$
   - (*c*) $\sqrt{5} \times \sqrt{9}$
   - (*d*) $\sqrt{8} \times \sqrt{11}$
   - (*e*) $\sqrt{12} \times \sqrt{10}$
   - (*f*) $\sqrt{6} \times \sqrt{20}$
   - (*g*) $\sqrt{0} \times \sqrt{7}$
   - (*h*) $\sqrt{3} \times \sqrt{21}$
   - (*i*) $\sqrt{586} \times \sqrt{1}$

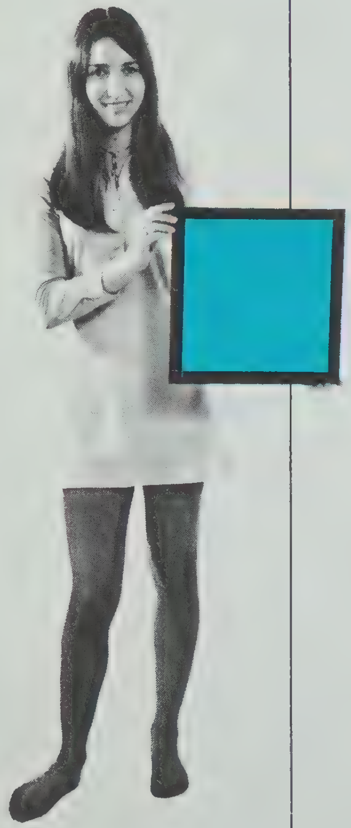
*

Like this!

$$\sqrt{2} \times \sqrt{5} = \sqrt{2 \times 5}$$
$$= \sqrt{10}$$

In general,

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$



D 1. Express each of the following as a radical.

(a) $\dfrac{\sqrt{36}}{\sqrt{9}} = \dfrac{6}{3} = \square = \sqrt{\square}$

(b) $\dfrac{\sqrt{100}}{\sqrt{4}} = \dfrac{\square}{\square} = \square = \sqrt{\square}$

(c) $\dfrac{\sqrt{900}}{\sqrt{9}} = \dfrac{\square}{\square} = \square = \sqrt{\square}$

2. Give a rule for division of radicals.

Exercise 5.1

Discussion

1. Which of the following are perfect squares?
   - (a) 99
   - (b) 81
   - (c) 141
   - (d) 121
   - (e) 104
   - (f) 90
   - (g) 25
   - (h) 250
   - (i) 2500
   - (j) 25 000
   - (k) 250 000
   - (l) 1 000 000
2. Square each of the following numbers.
   - (a) 10
   - (b) 1
   - (c) 0.1
   - (d) 20
   - (e) 2
   - (f) 0.2
   - (g) 60
   - (h) 6
   - (i) 0.6
   - (j) 90
   - (k) 9
   - (l) 0.9
3. Express each of the following without radical signs.
   - (a) $\sqrt{400}$
   - (b) $\sqrt{4}$
   - (c) $\sqrt{0.04}$
   - (d) $\sqrt{4900}$
   - (e) $\sqrt{49}$
   - (f) $\sqrt{0.49}$
   - (g) $\sqrt{100}$
   - (h) $\sqrt{1}$
   - (i) $\sqrt{0.01}$
4. Find each of the following products.
   - (a) $\sqrt{5} \times \sqrt{5}$
   - (b) $\sqrt{735} \times \sqrt{735}$
   - (c) $\sqrt{0.31} \times \sqrt{0.31}$
5. Express each of the following as a radical.
   - (a) $\sqrt{21} \times \sqrt{3}$
   - (b) $\sqrt{32} \times \sqrt{3}$
   - (c) $\sqrt{9} \times \sqrt{7}$
   - (d) $\sqrt{8} \times \sqrt{20}$
   - (e) $\sqrt{0.11} \times \sqrt{0.5}$
   - (f) $\sqrt{0.1} \times \sqrt{0.2}$
6. Express each of the following as a radical.
   - (a) $\dfrac{\sqrt{99}}{\sqrt{9}}$
   - (b) $\dfrac{\sqrt{80}}{\sqrt{2}}$
   - (c) $\dfrac{\sqrt{240}}{\sqrt{20}}$
   - (d) $\dfrac{\sqrt{500}}{\sqrt{25}}$
   - (e) $\dfrac{\sqrt{0.12}}{\sqrt{0.2}}$
   - (f) $\dfrac{\sqrt{0.33}}{\sqrt{0.03}}$

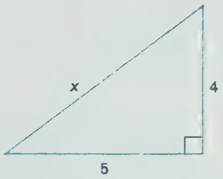
Written Solutions

7. We goofed! We put the decimal point in the wrong place. Put the decimal point in the correct place so that each of the following can be expressed as a whole number.

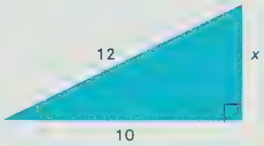
(*a*) $\sqrt{90.0}$ (*c*) $\sqrt{14.4}$ (*e*) $\sqrt{0.10}$
(*b*) $\sqrt{8.10}$ (*d*) $\sqrt{640.0}$ (*f*) $\sqrt{1.21}$

8. Express as a radical the length of the unknown side of each triangle.

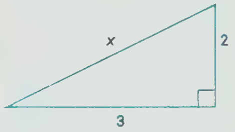
(*a*)



(*b*)



(*c*)

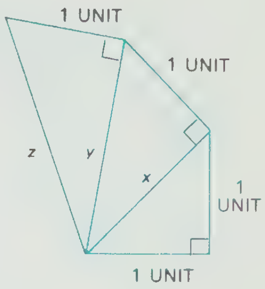


9. Use a right-angled triangle to construct a length that represents $\sqrt{29}$.

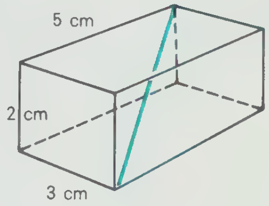
10. (*a*) Express *x*, *y*, and *z* as radicals. (See diagram.)
(*b*) Using drawing instruments and a convenient scale, construct the figure and continue the pattern until you arrive at a length that represents $\sqrt{8}$.
(*c*) Measure $\sqrt{8}$ and $\sqrt{2}$ and determine how many times longer $\sqrt{8}$ is than $\sqrt{2}$.

11. A small block of wood has dimensions as given in the diagram. Express the diagonal length as a radical.

10.



11.



8(a) $x = \sqrt{41}$
(b) $x = \sqrt{44}$
(c) $x = \sqrt{13}$
10(a) $x = \sqrt{2}$
$y = \sqrt{3}$
$z = \sqrt{4}$
(c) **Twice as long.**
11. $\sqrt{38}$ **cm**

5.2 Adding and Subtracting Radicals

A Is it possible to express $\sqrt{8} + \sqrt{2}$ as a single radical? In general this cannot be done, but for special cases it can. To help us perform such operations we shall have to distinguish between mixed and entire radicals.

Entire Radicals	*Mixed Radicals*
$\sqrt{3}$, $\sqrt{2}$, $\sqrt{5}$, $\sqrt{27}$,	$2\sqrt{3}$, $5\sqrt{2}$, $7\sqrt{5}$, $3\sqrt{27}$,
$\sqrt{200}$, $\sqrt{97}$, $\sqrt{1972}$	$4\sqrt{200}$, $2\sqrt{97}$, $13\sqrt{1972}$

$2\sqrt{3}$ means
$2 \times \sqrt{3}$

Square the number.

Example 1

Express as entire radicals.

(*a*) $2\sqrt{7}$ (*c*) $x\sqrt{2}$
(*b*) $5\sqrt{2}$ (*d*) $3\sqrt{x}$

Solution

(*a*) $$2\sqrt{7} = 2 \times \sqrt{7} = \sqrt{4} \times \sqrt{7} = \sqrt{28}$$

(*b*) $$5\sqrt{2} = 5 \times \sqrt{2} = \sqrt{25} \times \sqrt{2} = \sqrt{50}$$

(*c*) $$x\sqrt{2} = x \times \sqrt{2} = \sqrt{x^2} \times \sqrt{2} = \sqrt{2x^2}$$

(*d*) $$3\sqrt{x} = 3 \times \sqrt{x} = \sqrt{9} \times \sqrt{x} = \sqrt{9x}$$

Be careful how you write the answer. $\sqrt{5}y$ might look like $\sqrt{5y}$ if you get careless!

Example 2

Express as mixed radicals.

(*a*) $\sqrt{27}$ (*c*) $\sqrt{5y^2}$
(*b*) $\sqrt{12}$ (*d*) $\sqrt{50x}$

Solution

(*a*) $$\sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \times \sqrt{3} = 3 \times \sqrt{3} = 3\sqrt{3}$$

(*b*) $$\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2 \times \sqrt{3} = 2\sqrt{3}$$

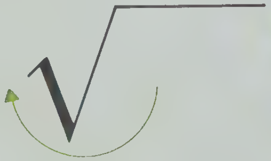
(*c*) $$\sqrt{5y^2} = \sqrt{5 \times y^2} = \sqrt{5} \times \sqrt{y^2} = \sqrt{5} \times y = y\sqrt{5}$$

(*d*) $$\sqrt{50x} = \sqrt{25 \times 2x} = \sqrt{25} \times \sqrt{2x} = 5 \times \sqrt{2x} = 5\sqrt{2x}$$

1. Express as entire radicals.
   (*a*) $2\sqrt{5}$ (*d*) $x\sqrt{3}$
   (*b*) $3\sqrt{2}$ (*e*) $6\sqrt{2k}$
   (*c*) $4\sqrt{3}$ (*f*) $m\sqrt{n}$

2. Express as mixed radicals.
   (*a*) $\sqrt{20}$ (*d*) $\sqrt{3x^2}$
   (*b*) $\sqrt{18}$ (*e*) $\sqrt{72k}$
   (*c*) $\sqrt{48}$ (*f*) $\sqrt{p^2q}$

Take the square root of the number.



B This example will show you how to add and subtract radicals.

Example 3

Simplify.

(*a*) $3x + 4x$	(*d*) $6\sqrt{5} - 2\sqrt{5}$
(*b*) $3\sqrt{2} + 4\sqrt{2}$	(*e*) $8\sqrt{x} + 5\sqrt{x}$
(*c*) $6y - 2y$	(*f*) $a\sqrt{7} + b\sqrt{7}$

Solution

(*a*) $3x + 4x = 7x$	(*d*) $6\sqrt{5} - 2\sqrt{5} = 4\sqrt{5}$
(*b*) $3\sqrt{2} + 4\sqrt{2} = 7\sqrt{2}$	(*e*) $8\sqrt{x} + 5\sqrt{x} = 13\sqrt{x}$
(*c*) $6y - 2y = 4y$	(*f*) $a\sqrt{7} + b\sqrt{7} = (a + b)\sqrt{7}$

Remember: You may add only like radicals!

1. Simplify.

(*a*) $2\sqrt{5} + 10\sqrt{5}$	(*d*) $3\sqrt{2} - 14\sqrt{2}$
(*b*) $9\sqrt{2} - 3\sqrt{2}$	(*e*) $5\sqrt{x} + 3\sqrt{x} - 6\sqrt{x}$
(*c*) $17\sqrt{2} - 8\sqrt{2} + 5\sqrt{2}$	(*f*) $2\sqrt{y} - 5\sqrt{y} - \sqrt{y}$

C The next example shows how seemingly “unlike” radicals may be converted to like radicals to make simplification possible.

Example 4

Simplify.

(*a*) $\sqrt{2} + \sqrt{8} - \sqrt{98}$
(*b*) $2\sqrt{3} - \sqrt{48} + 5\sqrt{12}$

Solution

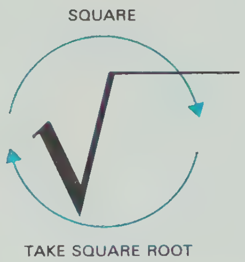
(*a*)
$$\begin{aligned}\sqrt{2} + \sqrt{8} - \sqrt{98} &= \sqrt{2} + \sqrt{4 \times 2} - \sqrt{49 \times 2}\\ &= \sqrt{2} + 2\sqrt{2} - 7\sqrt{2}\\ &= -4\sqrt{2}\end{aligned}$$

(*b*)
$$\begin{aligned}2\sqrt{3} - \sqrt{48} + 5\sqrt{12} &= 2\sqrt{3} - \sqrt{16 \times 3} + 5\sqrt{4 \times 3}\\ &= 2\sqrt{3} - 4\sqrt{3} + 5 \times 2\sqrt{3}\\ &= 2\sqrt{3} - 4\sqrt{3} + 10\sqrt{3}\\ &= 8\sqrt{3}\end{aligned}$$

Remember conversion of entire to mixed radicals?

1. Change to like radicals and simplify.

(*a*) $3\sqrt{5} + \sqrt{45} - \sqrt{80}$ (*b*) $\sqrt{20} - \sqrt{125}$



Exercise 5.2

Written Solutions

1. Convert to entire radicals.
   (*a*) $2\sqrt{5}$ (*b*) $3\sqrt{7}$ (*c*) $4\sqrt{2}$ (*d*) $a\sqrt{3}$
2. Convert to mixed radicals.
   (*a*) $\sqrt{20}$ (*b*) $\sqrt{12}$ (*c*) $\sqrt{27}$ (*d*) $\sqrt{5b^4}$
3. Simplify.
   (*a*) $2\sqrt{7}+4\sqrt{7}$ (*c*) $3\sqrt{10}+4\sqrt{10}-5\sqrt{10}$
   (*b*) $3\sqrt{2}-9\sqrt{2}$ (*d*) $4\sqrt{13}-2\sqrt{13}+2\sqrt{6}$
4. Convert to mixed radicals.
   (*a*) $\sqrt{1000}$ (*c*) $\sqrt{1500}$ (*e*) $\sqrt{20\,000}$
   (*b*) $\sqrt{500}$ (*d*) $\sqrt{3700}$ (*f*) $\sqrt{93\,000}$
5. Convert to entire radicals.
   (*a*) $13\sqrt{3}$ (*c*) $5\sqrt{5}$ (*e*) $2a\sqrt{5}$
   (*b*) $9\sqrt{2}$ (*d*) $x\sqrt{3}$ (*f*) $7x\sqrt{5y^2}$
6. Convert to mixed radicals.
   (*a*) $\sqrt{75}$ (*c*) $\sqrt{98a^2}$ (*e*) $\sqrt{80a^2b}$
   (*b*) $\sqrt{84}$ (*d*) $\sqrt{50x^2}$ (*f*) $\sqrt{72m^5n^3}$
7. Perform the operations indicated.
   (*a*) $3\sqrt{7}-5\sqrt{7}+8\sqrt{7}$ (*c*) $3\sqrt{10}-4\sqrt{10}+2\sqrt{10}$
   (*b*) $2\sqrt{6}+4\sqrt{6}-5\sqrt{6}$ (*d*) $2\sqrt{x}+7\sqrt{x}-3\sqrt{x}$
8. Simplify.
   (*a*) $3\sqrt{2}+4\sqrt{2}+7\sqrt{3}-4\sqrt{3}$
   (*b*) $2\sqrt{5}-7\sqrt{6}+3\sqrt{5}-2\sqrt{6}$
   (*c*) $6\sqrt{a}+5\sqrt{a}-3\sqrt{b}+4\sqrt{b}$
   (*d*) $4\sqrt{x}-3\sqrt{y}+\sqrt{x}+5\sqrt{y}$
9. Simplify. *
   (*a*) $\sqrt{48}+2\sqrt{3}$ (*d*) $\sqrt{63}-\sqrt{28}-\sqrt{7}$
   (*b*) $\sqrt{48}+\sqrt{12}+\sqrt{27}$ (*e*) $\sqrt{80}+\sqrt{45}-\sqrt{20}$
   (*c*) $\sqrt{98}-\sqrt{8}+\sqrt{32}$ (*f*) $\sqrt{50}-\sqrt{72}+\sqrt{18}$
10. Simplify.
    (*a*) $2\sqrt{8}+4\sqrt{50}+3\sqrt{18}$ (*c*) $\sqrt{125}+2\sqrt{80}-3\sqrt{20}$
    (*b*) $6\sqrt{54}-3\sqrt{24}-2\sqrt{6}$ (*d*) $-\sqrt{75}-2\sqrt{48}-5\sqrt{12}$
11. Which of the following are correct?
    (*a*) $\sqrt{810}=10\sqrt{8.1}$ (*c*) $\sqrt{900\,000}=30\sqrt{100}$
    (*b*) $\sqrt{1500}=10\sqrt{15}$ (*d*) $\sqrt{85\,000}=100\sqrt{8.5}$

1(a) $\sqrt{20}$ **(c)** $\sqrt{32}$
2(a) $2\sqrt{5}$ **(c)** $3\sqrt{3}$
3(a) $6\sqrt{7}$ **(c)** $2\sqrt{10}$
(d) $2\sqrt{13}+2\sqrt{6}$
4(a) $10\sqrt{10}$ **(e)** $100\sqrt{2}$
(c) $10\sqrt{15}$
5(a) $\sqrt{507}$ **(e)** $\sqrt{20a^2}$
(c) $\sqrt{125}$
6(a) $5\sqrt{3}$ **(e)** $4a\sqrt{5b}$
(c) $7a\sqrt{2}$
7(a) $6\sqrt{7}$ **(d)** $6\sqrt{x}$
8(a) $7\sqrt{2}+3\sqrt{3}$
(c) $11\sqrt{a}+\sqrt{b}$

* Remember to change to like radicals.

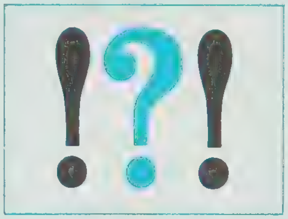
9(a) $6\sqrt{3}$ **(e)** $5\sqrt{5}$
(c) $9\sqrt{2}$
10(a) $33\sqrt{2}$ **(c)** $7\sqrt{5}$
11(a) Correct
(b) Correct
(c) Incorrect
(d) Correct

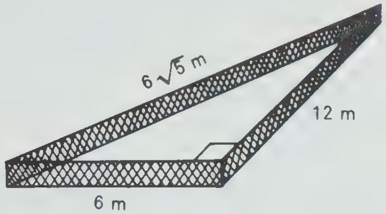
5.3 Approximating a Square Root

A Since lengths obtained by calculation often involve square roots, we find it necessary to express radicals as rational numbers that have approximately the same value.

I'll have
$18+6\sqrt{5}$ m
of fencing, please.

HARDWARE STORE





The easiest method of determining square root is by means of a table constructed for this very purpose.

Square Root Table

n	$\sqrt{n}$	n	$\sqrt{n}$	n	$\sqrt{n}$	n	$\sqrt{n}$	n	$\sqrt{n}$
1	1.00	21	4.58	41	6.40	61	7.81	81	9.00
2	1.41	22	4.69	42	6.48	62	7.87	82	9.06
3	1.73	23	4.80	43	6.56	63	7.94	83	9.11
4	2.00	24	4.90	44	6.63	64	8.00	84	9.17
5	2.24	25	5.00	45	6.71	65	8.06	85	9.22
6	2.45	26	5.10	46	6.78	66	8.12	86	9.27
7	2.65	27	5.20	47	6.86	67	8.19	87	9.33
8	2.83	28	5.29	48	6.93	68	8.25	88	9.38
9	3.00	29	5.39	49	7.00	69	8.31	89	9.43
10	3.16	30	5.48	50	7.07	70	8.37	90	9.49
11	3.32	31	5.57	51	7.14	71	8.43	91	9.54
12	3.46	32	5.66	52	7.21	72	8.49	92	9.59
13	3.61	33	5.74	53	7.28	73	8.54	93	9.64
14	3.74	34	5.83	54	7.35	74	8.60	94	9.70
15	3.87	35	5.92	55	7.42	75	8.66	95	9.75
16	4.00	36	6.00	56	7.48	76	8.72	96	9.80
17	4.12	37	6.08	57	7.55	77	8.77	97	9.85
18	4.24	38	6.16	58	7.62	78	8.83	98	9.90
19	4.36	39	6.24	59	7.68	79	8.89	99	9.95
20	4.47	40	6.32	60	7.75	80	8.94	100	10.00

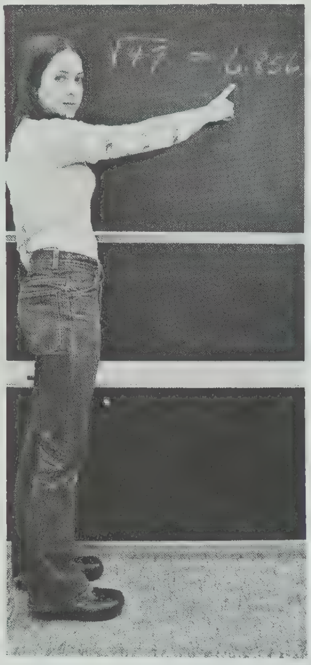
Notice that most of these are only approximate. They have been rounded off to 2 decimals. Hence we should write either
$\sqrt{7} \doteq 2.65$ or
$\sqrt{7} = 2.65$
(to 2 decimals.)

You will find a more complete table on pages 444 and 445.

1. Find the following values, using the table.

(*a*) $\sqrt{29}$ (*c*) 6.16^2 (*e*) $\sqrt{78\frac{1}{3}}$

(*b*) $\sqrt{76}$ (*d*) $\sqrt{35.5}$ (*f*) 9.75^2



The table may be used to find square roots of numbers not listed if we make use of the ideas of mixed radicals.

Example 1

Express $\sqrt{1900}$ as $x\sqrt{y}$, where y is a number you can find in the table, and evaluate.

Solution

$$\begin{aligned}\sqrt{1900} &= \sqrt{100 \times 19}\\ &= \sqrt{100} \times \sqrt{19}\\ &= 10 \times \sqrt{19}\\ &\doteq 10 \times 4.36\\ &= 43.6\end{aligned}$$

$\therefore \sqrt{1900} \doteq 43.6$

Example 2

Evaluate $\sqrt{0.47}$ using the table.

Solution

$$\begin{aligned}\sqrt{0.47} &= \sqrt{\frac{47}{100}}\\ &= \sqrt{47} \div \sqrt{100}\ ^{*}\\ &\doteq 6.86 \div 10\\ &= 0.686\end{aligned}$$

$\therefore \sqrt{0.47} \doteq 0.686$

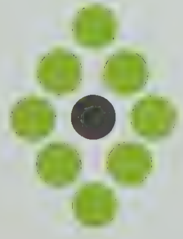
Do you see why there must be an even number of zeros here?

2. Evaluate using the table.
   (*a*) $\sqrt{500}$
   (*b*) $\sqrt{3500}$
   (*c*) $\sqrt{0.56}$
   (*d*) $\sqrt{0.03}$

Exercise 5.3

Written Solutions

1. Use the square root table on pages 444 and 445 to evaluate.
   (*a*) $\sqrt{7}$
   (*b*) $\sqrt{23}$
   (*c*) $\sqrt{46}$
   (*d*) $\sqrt{53}$
   (*e*) $\sqrt{72}$
   (*f*) $\sqrt{91}$
   (*g*) $\sqrt{243}$
   (*h*) $\sqrt{190}$
   (*i*) $\sqrt{113}$
   (*j*) $\sqrt{316}$
   (*k*) $\sqrt{409}$
   (*l*) $\sqrt{493}$



2. Use the square root table to evaluate the following.
   (*a*) $\sqrt{3500}$ (*c*) $\sqrt{0.24}$ (*e*) $\sqrt{390\,000}$
   (*b*) $\sqrt{6700}$ (*d*) $\sqrt{0.47}$ (*f*) $\sqrt{0.0019}$

3. Use the tables to show that each of the following is true.
   (*a*) $\sqrt{8} = 2\sqrt{2}$ (*b*) $\sqrt{27} = 3\sqrt{3}$

4. Evaluate, correct to 2 decimal places.
   (*a*) $\sqrt{345}$ (*c*) $\sqrt{407}$ (*e*) $\sqrt{87}$
   (*b*) $\sqrt{163}$ (*d*) $\sqrt{142}$ (*f*) $\sqrt{220}$

2(a) 59.2 (e) 624
(c) 0.49 (f) 0.0436
4(a) 18.57 (e) 9.33
(c) 20.17

5.4 Other Methods of Finding Square Root

A If you have access to a calculating machine, you can quite easily do square root by *Newton's Method.*

Example 1

Find $\sqrt{5}$ to 2 decimal places, using Newton's Method.

Solution

Use 2 as a trial divisor.*

$$\frac{5}{2} = 2.5 \quad \text{and average these: } \frac{2 + 2.5}{2} = 2.25$$

Now use 2.25 as a new trial divisor.

$$\frac{5}{2.25} = 2.22 \quad \text{and average again: } \frac{2.25 + 2.22}{2} = 2.235$$

This method is best carried out in a table.

$\sqrt{5}$	*1st Trial*	*2nd Trial*	*3rd Trial*
Trial Divisor	2	2.25	2.235
Trial	$\frac{5}{2} = 2.25$	$\frac{5}{2.25} = 2.22$	$\frac{5}{2.235} = 2.237$
Average	$\frac{2 + 2.5}{2} = 2.25$	$\frac{2.25 + 2.22}{2} = 2.235$	$\therefore \sqrt{5} = 2.23$ (2 decimals)

*

Since $2^2 = 4$

On a calculator it would look like this!

$5 \ \boxed{\div} \ 2 \ \boxed{+} \ 2 \ \boxed{\div} \ 2 \ \boxed{=} \ 2.25$

$5 \ \boxed{\div} \ 2.25 \ \boxed{+} \ 2.25 \ \boxed{\div} \ 2 \ \boxed{=} \ 2.235$

$5 \ \boxed{\div} \ 2.235 \ \boxed{+} \ 2.235 \ \boxed{\div} \ 2 \ \boxed{=} \ 2.237$

Then $\boxed{\times}$ $\boxed{=}$ as a check!

EXTRACTION OF THE SQUARE ROOT

The root of your first
  period you
Must place in quote
  if you work true:
Whose square from your
  said period then
You must subtract:
  and to th' remain
Another period being
  brought,
You must divide as
  here is taught,
By the double of your
  quote, but see
Your unit's place you
  do leave free;
Which place will be
  supply'd by th' square
Of your next quoted
  figure there:
Next multiply, subtract,
  and then
Repeat your work unto
  the end:
And if your number
  be irrational
Add pairs of cyphers
  for a decimal.

From Arithmetic
Both in the
Theory and Practice
by John Hill, Gent.
Edinburgh

MDCCLX

(Courtesy of
Mr. P. Templin)

---

1. Evaluate (to 2 decimals) each of the following using Newton's Method.
   (*a*) $\sqrt{2}$ (*c*) $\sqrt{114}$
   (*b*) $\sqrt{37}$ (*d*) $\sqrt{40}$

B If no mechanical aids are available, the Formal Method of finding square root is used. Study the following examples.

Example 2

Find $\sqrt{13|69}$.

Solution

$$\sqrt{13|69.}$$

Step 1 "Pair off" digits from the decimal point.

$$\begin{array}{r} 3\quad . \\ \sqrt{13|69.} \end{array}$$

Step 2 Find the largest digit (3) which when squared is less than or equal to the first pair (13). This gives the first digit in the answer.

$$\begin{array}{r|r} & 3\quad . \\ & \sqrt{13|69.} \\ & 9 \\ \cline{2-2} & 469 \end{array}$$

Step 3 Square this digit and put the result under the first pair. Then subtract, and bring down the next pair.

$$\begin{array}{r|r} (\times 2)\longrightarrow & 3\quad . \\ \downarrow & \sqrt{13|69.} \\ & 9 \\ \cline{2-2} 6■ & 469 \end{array}$$

Step 4 To find a *trial divisor*, double the answer so far (3■), and put this trial divisor (6■) opposite the 469.

$$\begin{array}{r|r} & 3\ 7. \\ & \sqrt{13|69.} \\ & 9 \\ \cline{2-2} 67 & 469 \end{array}$$

Step 5 Divide the trial divisor (6■) into the 469. It goes 7 times. The final divisor is then 67 and the next digit in the answer is 7.

$$\begin{array}{r|r} & 3\ 7. \\ & \sqrt{13|69.} \\ & 9 \\ 67 & 469 \\ & 469 \\ \cline{2-2} & 0 \end{array}$$

Step 6 Finally, multiply 7 × 67 and subtract the answer (469) from 469. Since the result is zero, we find that $\sqrt{1369} = 37$.

By repeating Steps 4, 5, and 6, we can find the square root of a number to any required accuracy. The division goes on indefinitely for numbers like $\sqrt{3}, \sqrt{5}, \ldots$

Example 3

Find $\sqrt{506}$ to 2 decimal places.

Solution

Step 1

$$\begin{array}{r|l} & 2\ \ . \\ & \sqrt{506.0000} \\ & 4 \\ & \overline{106} \end{array}$$

Step 2 (×2)

$$\begin{array}{r|l} & \boxed{2}\,2. \\ & \sqrt{506.0000} \\ & 4 \\ \boxed{42} & \overline{106} \\ & \ \ 84 \\ & \overline{\ \ 2200} \end{array}$$

Step 3 (×2)

$$\begin{array}{r|l} & \boxed{2\ 2.}\ 4 \\ & \sqrt{506.0000} \\ & 4 \\ 42 & \overline{106} \\ & \ \ 84 \\ 444 & \overline{\ \ 2200} \\ & \ \ 1776 \\ & \overline{\ \ \ 42400} \end{array}$$

Step 4 (×2)

$$\begin{array}{r|l} & \boxed{2\ 2.\ 4}\,9 \\ & \sqrt{506.0000} \\ & 4 \\ 42 & \overline{106} \\ & \ \ 84 \\ 444 & \overline{\ \ 2200} \\ & \ \ 1776 \\ \boxed{4489} & \overline{\ \ \ 42400} \\ & \ \ \ 40401 \\ & \overline{\ \ \ \ 1999} \end{array}$$

Since the remainder, 1999, is less than half the trial divisor, 4489, the next digit in the answer will be less than 5.

Hence, $\sqrt{506} \doteq 22.49$ *

*

Why do we need ≐ ?

1. Find the square root using the Formal Method, to the accuracy indicated.
   (*a*) $\sqrt{2209}$ (nearest whole number)
   (*b*) $\sqrt{684.2}$ (1 decimal)
   (*c*) $\sqrt{7}$ (2 decimals)
   (*d*) $\sqrt{0.413}$ (2 decimals)

Exercise 5.4

Discussion

1. State which method (tables, Newton's Method, Formal Method) you prefer to use in finding square root, and why.

2. Would you like to use the Formal Method to find the square root of a number to 10 decimal places? Explain.

3. In your own words, tell how you would go about finding $\sqrt{11.6}$
   (*a*) by Newton's Method,
   (*b*) by the Formal Method.

Written Solutions

Use tables to check answers!

4. Evaluate without using tables (all answers to 3 digits).

(*a*)	$\sqrt{23}$	(*c*)	$\sqrt{158}$
(*b*)	$\sqrt{5}$	(*d*)	$\sqrt{2310}$

5. Evaluate without using tables, to the accuracy required.

(*a*)	$\sqrt{11.2}$ (1 decimal)	(*d*)	$\sqrt{1.352}$ (2 decimals)
(*b*)	$\sqrt{129}$ (1 decimal)	(*e*)	$\sqrt{3651.3}$ (1 decimal)
(*c*)	$\sqrt{8.76}$ (1 decimal)	(*f*)	$\sqrt{0.065\,13}$ (3 decimals)

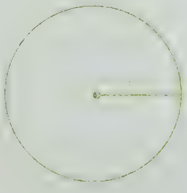
6. In helping to decorate the gym for the Sadie Hawkins Dance, Myles Kilometre must cut out a circular piece of tinfoil whose area is 2800 cm². Calculate the radius of the piece of tinfoil. (Use $\pi \doteq 3.14$.)

7. The area of a triangle is given by the formula
$$A = \sqrt{s(s-a)(s-b)(s-c)} \text{ where}$$
a, b, and c are the sides of the triangle, and $s = \dfrac{a+b+c}{2}$.
Determine the area of the triangle whose dimensions are as given in the margin diagram.

8. If you have some computer background, write a program for finding the square root of a number by Newton's Method.

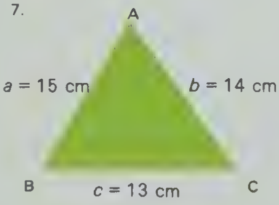
4(a)	4.80	(c)	12.6
(b)	2.24	(d)	48.1
5(a)	3.3	(d)	1.16
(b)	11.4	(e)	60.4
(c)	3.0	(f)	0.255
6.	29.9 cm		
7.	84 cm²		

6.



$A = \pi r^2$

7.



5.5 Square Root Problems

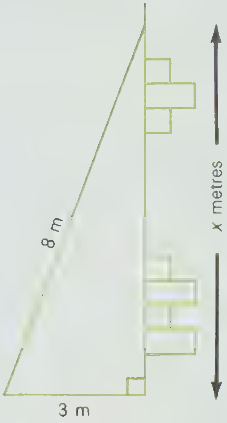
In practical problems, it is often convenient to use rational number approximations of irrational numbers. The table of square roots on pages 444 and 445 give us rational approximations (to 2 decimals) of some irrational numbers. From the table,

$$\sqrt{2} \doteq 1.41 \quad \text{and} \quad \sqrt{3} \doteq 1.73$$

≐ "is approximately equal to."

Example 1

A ladder 8 m long leans against a vertical wall and has its foot 3 m from the base of the wall. How far up the wall will the ladder reach? (Answer to 2 decimal places.)



Solution

Let the distance up the wall be represented by x metres.

By the Law of Pythagoras,

$$\begin{aligned} 8^2 &= 3^2 + x^2 \\ \text{or } 64 &= 9 + x^2 \\ \therefore x^2 &= 64 - 9 \\ &= 55 \\ \therefore \quad x &= \sqrt{55} \\ &\doteq 7.42 \end{aligned}$$

From the tables.

∴ The ladder reaches 7.42 m up the wall.

Example 2

The electrical power in an electric circuit is given by the formula

$$P = I^2R$$

where P is the power, in watts
I is the current, in amperes
R is the resistance, in ohms

Find the current when the resistance is 80.0 Ω and the power is 1000 W. (Answer to 3 significant digits)

W means "watts"
A means "amperes"
Ω means "ohms"

Solution

$$\begin{aligned} \because \quad P &= I^2R \\ \therefore I^2 &= \frac{P}{R} \\ &= \frac{1000}{80} \\ &= 12.5 \\ \therefore \quad I &= \sqrt{12.5} \\ &\doteq 3.54 \end{aligned}$$

∴ The current is 3.54 A.

Exercise 5.5

Written Solutions

Note: Give all answers to 3 significant digits.

1. Determine the length of the longest straight piece of spaghetti that will lie flat in the bottom of a 23 cm square cooking dish.

2. The area of a circle is given by the equation $A = \pi r^2$, where $\pi \doteq 3.14$ and r is the radius of the circle. Find the radius (to 3 digits) of circles whose areas are given below.
   (*a*) $A = 87\ \text{cm}^2$
   (*b*) $A = 19.56\ \text{m}^2$
   (*c*) $A = 4716\ \text{mm}^2$
   (*d*) $A = 190.3\ \text{cm}^2$

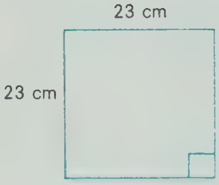
3. Sally Sidearm, star of the South City Starlets softball team, can throw to 2nd base from a crouched position behind home plate. If the distances from home plate to 1st base and from 1st base to 2nd base are each 27.4 m, find the length of Sally's throw, to the nearest 0.1 m.

4. Neglecting air resistance, an object dropped from an aircraft falls according to the formula $d = 0.5gt^2$, where d is the distance (metres) above the ground, g is the acceleration due to gravity (metres per second per second), and t is the time (seconds) taken to reach the ground. How long will it take an object to reach the ground when dropped from a height of 4000 m, if the acceleration due to gravity is 9.8 m/s^2?

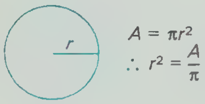
5. An oil storage tank 3 m high contains 73.9 kl of oil. Find the radius of the tank.

6. Fiona Fashionwhiz folds a square cloth into a triangular kerchief for her head. If the area of the triangle is 1100 cm^2, what are the dimensions of the cloth?

1.

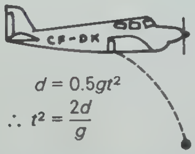


2.

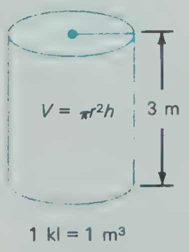


1. 32.5 cm
2(a) 5.26 cm (c) 38.8 mm
3. 38.7 m
4. 28.6 s
5. 2.8 m
6. 46.9 cm by 46.9 cm

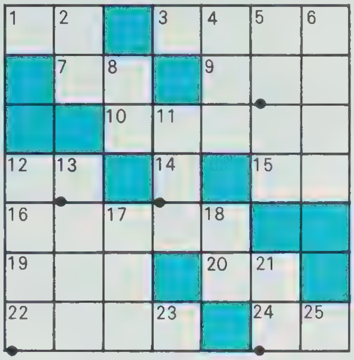
4.



5.



For "Square-Shooters" Only!



Horizontal

1. A perfect square between 30 and 40.
3. $(12)^3$
7. Find the square root of 3136.
9. $\sqrt{7}$ (correct to 2 decimal places)
10. $(3 \times 2)^6$
12. $\sqrt{3.24}$
14. $\sqrt{0.25}$
15. $3^2 + 4^2$
16. Find 2^{15}
19. $\sqrt{82\,944}$
20. $(4)^2 + (2)^2 - (1)^2$
22. $(0.75)^2$
24. $(0.2)^2$

Vertical

1. $\sqrt[3]{27}$
2. Find the square root of 4225.
3. $(-1)^6$
4. $3^6 - 3$
5. $\sqrt{703.31}$ (correct to 2 decimal places)
6. $(92)^2 + (10)^2 + (1)^2$
8. 2^6
11. $\sqrt{4303.36}$
12. $(35)^2 + (10)^2$
13. $\sqrt{68.66}$ (correct to 3 decimal places)
17. Find the square root of 611 524
18. 3^4
21. $(10)^2 - (10)^1$
23. Find the sum of 2 squared and 1 squared.
25. $\sqrt{(5)^2 - (3)^2}$

Unit 6
Introduction to Variable Expressions

6.1 Review of Operations with Variable Expressions

A Adding Expressions

1. Find the sum.
   (*a*) $2x + 3x$ (*d*) $3x + y + 5x$
   (*b*) $4a + (-2a)$ (*e*) $-3h + 4s + (-2s)$
   (*c*) $-5b + 7b$ (*f*) $3c + 5d + (-4c) + (-2d)$
2. Select three groups of like terms from the following list.
   $4a,\ 3x^2,\ 14ab,\ 2x^2,\ -5a,\ -ab,\ -3x^2,\ a$
3. Simplify if possible.
   (*a*) $2x^2 + 3x^2$ (*d*) $-4a + 3b + 2a + (-10b)$
   (*b*) $3a + 5b$ (*e*) $8r + (-10t) + 7r$
   (*c*) $-2y^3 + 5y^3$ (*f*) $3x^2 + 5x + 1$

Recall $\overbrace{2x+3x}^{\text{LIKE TERMS}}$
$= (2+3)x$
$= 5x$

What kind of terms can be added to form a single term?

Recall $2x - 5x$
$= 2x + (-5x)$
$= -3x$

B Subtracting Expressions

1. Find the difference.
(*a*) $10a - 5a$ (*c*) $4x^2 - x^2$ (*e*) $5xy - 2xy$
(*b*) $18r - 20r$ (d) $-2z - 3z$ (*f*) $14a - 7a - 2a$

Like terms:

2. Simplify if possible.
(*a*) $3a - 8a$ (*c*) $4xyz - 6xyz$ (*e*) $2a - 7b + 4b - 5a$
(*b*) $10r - 3s$ (*d*) $5x^2 - 3x + 1$ (*f*) $10x - 3y - 5x + 8y$

C Naming Expressions

Some expressions are given special names depending on the number of terms they contain.

Expression	*Number of Terms*	*Names*
$2x$	1	monomial
$3a - 5bc$	2	binomial
$2x^2 - 5x + 7$	3	trinomial
$5r^3 - 2r^2 + 4r - 1$	4	polynomial with 4 terms

"Polynomial" is generally used to name any expression having 4 or more terms.

1. Divide the following list into groups of monomials, binomials, trinomials, or polynomials.
$3x - 5y$ $-3t^2$ $3x^3 - 2x + 4y - 1$
$4xyz$ 5 $7x^2 - 3y^2$
$2a^2 - 5a + 4$ $4a + 7b$ $7b^2 - 3b + 1$

$$\underbrace{3x-2y}_{\text{COEFFICIENTS}}$$

2. List the coefficients of each term.
(*a*) $2x^2 - 5x$ (*b*) $-4abc$ (*c*) $36x^2 - 25y^2$

D Multiplying Terms

1. State the product.
(*a*) $3x \times 5$ (*c*) $7a \times 3b$ (*e*) $4a \times 3b \times 2c$
(*b*) $-4 \times 7b$ (*d*) $-2x \times (-5y)$ (*f*) $-6x \times 5x$

2. Multiply the following *powers*.*
(*a*) $a^2 \times a^3$ (*c*) $5r^3 \times r^2$ (*e*) $-3a^2 \times (-5a^7)$
(*b*) $3b^2 \times b^4$ (*d*) $y^3 \times 2y^5$ (*f*) $2x^2 \times 3x^3 \times 4x^4$

Pattern for Multiplying Powers $x^a \times x^b = x^{a+b}$

*
Recall that
a^2 means $a \times a$
and that
$a^2 \times a^3$
$= a^{2+3}$
$= a^5$

E Dividing Terms

1. State the quotient.
   (*a*) $28x \div 4$
   (*b*) $(-12x^2) \div 3$
   (*c*) $14a \div 7a$
   (*d*) $(-32ab) \div ab$
   (*e*) $(-42x) \div (-7)$
   (*f*) $6t \div (-3t)$

2. Divide the following powers. *
   (*a*) $x^5 \div x^2$
   (*b*) $r^7 \div r^3$
   (*c*) $16t^3 \div 8t$
   (*d*) $-20n^3 \div 4n$
   (*e*) $24x^4 \div (-6x)$
   (*f*) $(-36x^2) \div (-4x)$

* Recall that

$y^5 \div y^2$ or $\frac{y^5}{y^2}$
$= y^{5-2}$
$= y^3$

Pattern for Dividing Powers $x^a \div x^b = x^{a-b}$

Exercise 6.1

Discussion

1. State one example of
   (*a*) a monomial
   (*b*) a binomial
   (*c*) a trinomial
   (*d*) a 5-term polynomial

2. Name (i) the coefficient (ii) the exponent of the variable part of each of the following terms.
   (*a*) $3x^2$
   (*b*) $-5y$
   (*c*) $52a^3$

3. (*a*) Add $3a + 5a$. What parts of each term did you really add?
   (*b*) Why can $5a + 7b$ not be added as in (*a*)?

Add like terms and you get another like term!

$3a + 5a$
$= (3+5)a$
$= 8a$

4. Are the following statements true or false? Explain.
   (*a*) $5a - 3b$ is a binomial.
   (*b*) The coefficient of the term $-5xyz$ is 5.
   (*c*) $4y - y = 4$
   (*d*) $5x^2 \times 3x = 15x^3$
   (*e*) $4a + b = 4ab$
   (*f*) $4a \times b = 4ab$
   (*g*) $x^2 \times x^3 = x^6$
   (*h*) $r^5 \times s^2 = rs^7$
   (*i*) $h^8 \div h^2 = h^4$
   (*j*) $6^3 \div 6^2 = 6$

5. Perform the indicated operation.

(a) $5a + 8a$
(b) $-3b \times 5b$
(c) $7y - 4y$
(d) $6r \times 5s$
(e) $-12a^2 \div 4a$
(f) $(-6t)(-3t)$
(g) $(\frac{1}{2}x)(-8x^2)$
(h) $(-6yz) \div 3yz$

6. What terms could have been multiplied to give the following products?

(a) $6x^2$
(b) $4ab$
(c) $15xy$
(d) $-20x$
(e) $-14y^3$
(f) $12rst$

$3x \times 2x$?
$3 \times 2x^2$?
$6 \times x^2$?

Written Solutions

Simplify? I guess that here it means to add like terms.

7. Simplify.

(a) $4x + 2y - 7x - 5y$
(b) $-6a + 7b - 3a + 4b$
(c) $14mn - 3mn + 5mn$
(d) $3x^2 - 5x^2 + 6y^2 - 2y^2$
(e) $4yz - 7yz + 3yz$
(f) $10x - 38y - 15x + 25y$

8. Find the product.

(a) $-4x \times 5$
(b) $3x \times (-7y)$
(c) $-12a \times 5a$
(d) $(10h)(-3h^2)$
(e) $4t \times (-3t) \times \frac{1}{3}t$
(f) $(-3x^2)(-8x^3)$
(g) $4t \times (-11s)$
(h) $15xy \times 2y$

9. Simplify.

(a) $6x^2 - 2x + 1 - 4x^2 + 5x + 7$
(b) $3mn - 5nq - 8mn + 7nq$
(c) $3b^2 - 7b + 4a^2 - 3b$
(d) $7a^2 - 3a + 2 - 10a^2 + 5a - 4$

10. Find the quotient.

(a) $12a^3 \div 3a^2$
(b) $-15r^7 \div 5r^3$
(c) $20t^5 \div (-4t)$
(d) $-30xy \div (-6x)$
(e) $6x^3 \div 12x$
(f) $28rst \div (-4rs)$

11. Simplify.

(a) $6ab \div 2a + 5b$
(b) $-3x + 6x^2 \times \dfrac{2}{x}$
(c) $20x^2y - (4x)(3xy)$
(d) $3a + 6a^2 \div (-2a) - a$
(e) $(6x^2 - 2x^2)(5y - 8y)$
(f) $\dfrac{16x^3 - 5 \times 2x^3}{7x^2 - 3x^2}$

7(a) $-3x - 3y$ (e) 0
(c) $16mn$

8(a) $-20x$ (e) $-4t^3$
(c) $-60a^2$ (g) $-44st$

9(a) $2x^2 + 3x + 8$
(c) $4a^2 + 3b^2 - 10b$

10(a) $4a$ (e) $\frac{1}{2}x^2$
(c) $-5t^4$

11(a) $8b$ (d) $-a$
(b) $9x$ (e) $-12x^2y$
(c) $8x^2y$ (f) $\frac{3}{2}x$

Recall

bodmas

6.2 Multiplying and Factoring

A A steel plant uses limestone, iron ore and coal to produce steel. These raw materials are *factors* of production and the steel is the *product*.

In mathematics you multiply terms to produce a *product*. The terms are *factors* of that product.

$$\underset{\text{FACTORS}}{\underbrace{2a \times 5a}} = \underset{\text{PRODUCT}}{10a^2}$$

In Section 6.1 you reviewed how to multiply terms. Study Examples 1 and 2 in which binomials are used.

Example 1

Multiply the binomial $x + y$ by the term 4.

Recall the pattern

$$\underset{\xrightarrow[\text{EXPAND}]{}}{a(b+c)} = ab+ac$$

Solution

$$\begin{aligned} 4(x+y) &= (4 \times x) + (4 \times y) \\ &= 4x + 4y \end{aligned}$$

Example 2

Factor the binomial $2x + 2y$.

Solution

$$\begin{aligned} 2x + 2y &= (2 \times x) + (2 \times y) \\ &= 2(x+y) \end{aligned}$$

Does this work with trinomials? Let's see...
$2a+2b+2c$
$= 2(a+b+c)$

1. Expand.
   - (*a*) $3(a+b)$
   - (*b*) $-5(x+y)$
   - (*c*) $4(x+2)$
   - (*d*) $2(r-s)$
   - (*e*) $-7(k-t)$
   - (*f*) $-3(3x+5)$
   - (*g*) $2(x+y+z)$
   - (*h*) $-5(2a-3b+c)$
2. Factor.
   - (*a*) $4x+4y$
   - (*b*) $3r-3s$
   - (*c*) $6x+12$
   - (*d*) $5c-5d$
   - (*e*) $-8h+8k$
   - (*f*) $10a+15b$
   - (*g*) $2x+2y+2z$
   - (*h*) $3x-6y+9$

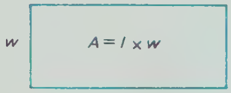
Pattern for Expanding $a(x+y) = ax + ay$
Pattern for Factoring $pq + pr = p(q+r)$

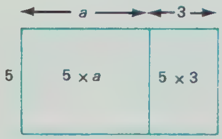
p is the common factor.

hcf

2 is a common factor of 8 and 12, but is not the greatest number that divides into 8 and 12.

What larger number is a factor of 8 and 12? We call 4 the highest common factor of 8 and 12.*





What is the area of a rectangle that measures 5 by a + 3? What dimensions are possible for a rectangle whose area is 4x + 4y?

B Finding a common factor of the terms of a variable expression takes practice. As you do each part of Question 1 study the answer and compare it to the given question.

1. Expand.
   - (*a*) $3x(a + b)$
   - (*b*) $2a(a + 3)$
   - (*c*) $5y(y + 1)$
   - (*d*) $6a(2a + 5)$
   - (*e*) $3s(4t - 5)$
   - (*f*) $7b(5c - 3d + 2)$

2. State the highest common factor (HCF) in each case.
   - (*a*) $12x + 12y$
   - (*b*) $5a + 15$
   - (*c*) $14d + 7e + 7$
   - (*d*) $8a^2 + 12a$ *
   - (*e*) $10xy - 15x$
   - (*f*) $16ab - 20b$

3. Factor each expression in Question 2.

Exercise 6.2

Discussion

1. Expand.
   - (*a*) $3(a + b)$
   - (*b*) $2(x + 3y)$
   - (*c*) $-4(2c + d - 2)$
   - (*d*) $5(3x - 1)$
   - (*e*) $7(5b + 4c - 3d)$
   - (*f*) $-3(8r - 3s)$

2. State the HCF.
   - (*a*) 8, 6
   - (*b*) $10x$, $15y$
   - (*c*) $14a$, $21b$
   - (*d*) 20, 12
   - (*e*) $30a$, $24x$
   - (*f*) $40x$, $32y$

3. Factor.
   - (*a*) $5a + 5b$
   - (*b*) $10c - 10d$
   - (*c*) $4a + 2b + 2$
   - (*d*) $3x + 12$
   - (*e*) $7a - 42b + 21c$
   - (*f*) $6r + 8$

4. Is $3x + 3 = 3(x + 1)$? Explain where the 1 comes from.

5. Compare the operations of expanding and factoring.

6. Correct the following statements.
   - (*a*) $3x(x + 5) = 3x^2 + 5$
   - (*b*) $14x^2y - 21xy^2 = 7x^2y^2(2y - 3x)$

Written Solutions

7. Expand.
   - (a) $5(2x + 3y)$
   - (b) $-4(6a - 3)$
   - (c) $15(4c + d - 2e)$
   - (d) $7(3x - 5)$
   - (e) $8(2x + y - z)$
   - (f) $-10(8x^2 - 3y^2)$

8. Factor.
   - (a) $2a + 2b$
   - (b) $5c - 5d$
   - (c) $8r + 16s - 24t$
   - (d) $3r^2 + 6$
   - (e) $15h + 5k + 25$
   - (f) $24t - 6$

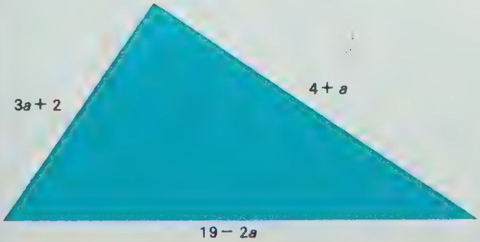
9. Expand.
   - (a) $2x(x + 5)$
   - (b) $3a(a - 7)$
   - (c) $-4y(2y + 1)$
   - (d) $5t(3t + 4)$
   - (e) $6x(3y - 5)$
   - (f) $8a(9b - 7c)$

10. Factor.
    - (a) $5x^2 + 15x$
    - (b) $3a^2 - 6a$
    - (c) $-8y^2 - 4y$
    - (d) $15ab + 25a$
    - (e) $14x^2 - 7xy$
    - (f) $33xy + 121y$

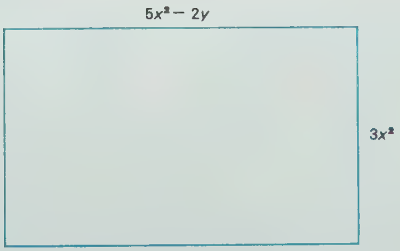
11. Simplify.
    - (a) $x(x + 3) + 5(x + 2)$
    - (b) $y(y - 3) + 4(y - 1)$
    - (c) $b(b + 2) - 3(b + 6)$
    - (d) $x(x - 5) - 4(x - 3)$
    - (e) $c(c + 3) - 3(c + 3)$
    - (f) $m(a + b) - m(a - b)$

12. Factor.
    - (a) $(x + 2)x + (x + 2)3$
    - (b) $a(a + 3) + 2(a + 3)$
    - (c) $(b - 4)b + (b - 4)2$
    - (d) $x(x - 1) - 5(x - 1)$

13. Find the perimeter of the triangle.



14. Find the area of the rectangle.



7(a) $10x + 15y$
(c) $60c + 15d - 30e$
(e) $16x + 8y - 8z$

8(a) $2(a + b)$
(c) $8(r + 2s - 3t)$
(e) $5(3h + k + 5)$

9(a) $2x^2 + 10x$
(c) $-8y^2 - 4y$
(e) $18xy - 30x$

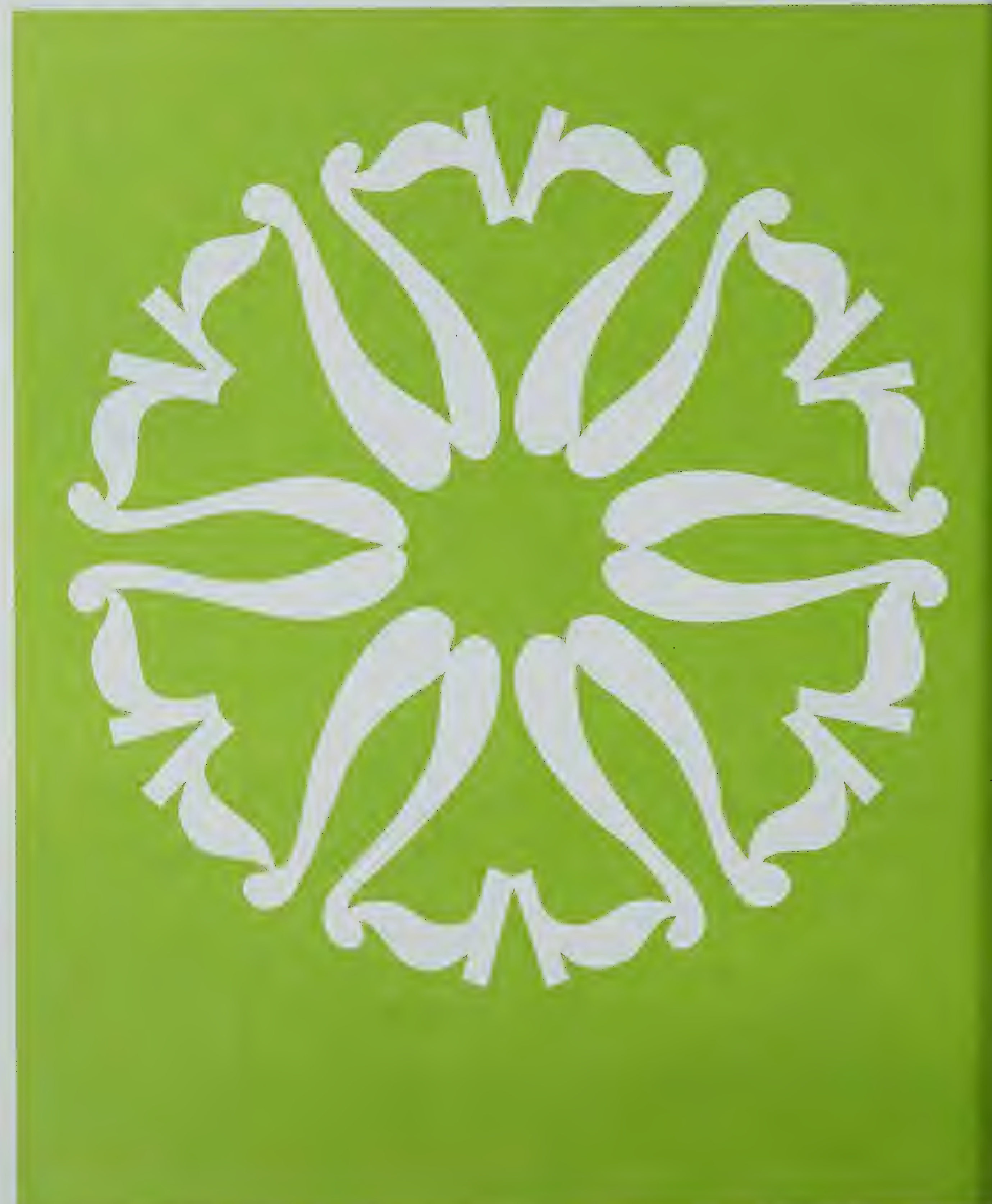
10(a) $5x(x + 3)$
(c) $-4y(2y + 1)$
(e) $7x(2x - y)$

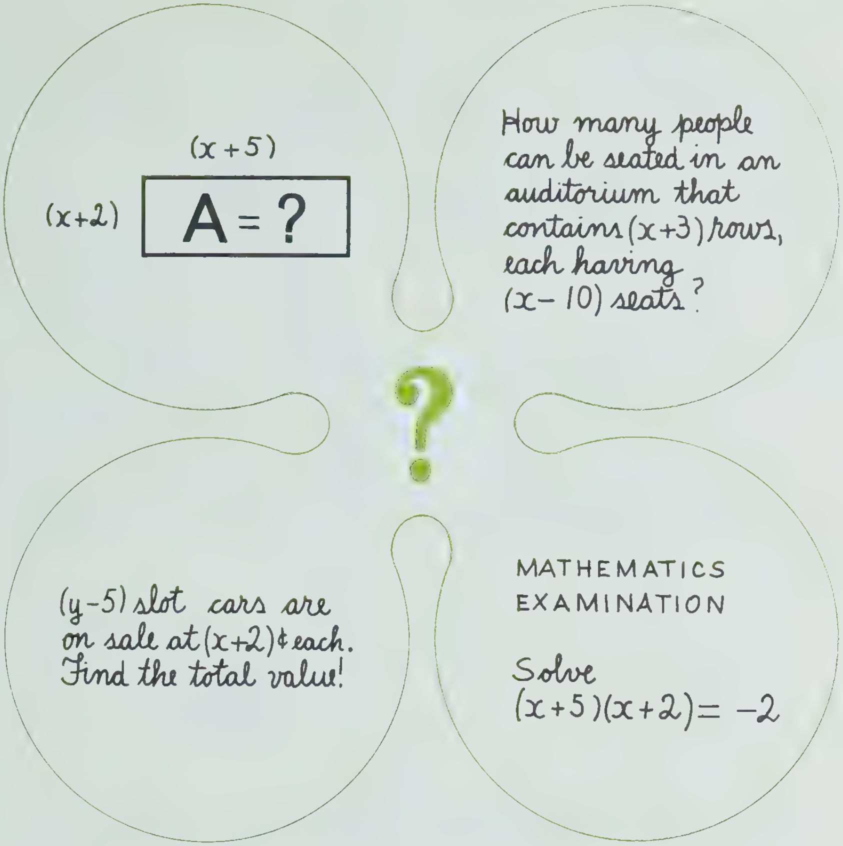
11(a) $x^2 + 8x + 10$
(c) $b^2 - b - 18$
(e) $c^2 - 9$

12(a) $(x + 2)(x + 3)$
(d) $(x - 1)(x - 5)$

13. $2a + 25$

14. $15x^4 - 6x^2y$





How many people can be seated in an auditorium that contains $(x+3)$ rows, each having $(x-10)$ seats?

$(y-5)$ slot cars are on sale at $(x+2)$¢ each. Find the total value!

MATHEMATICS EXAMINATION

Solve $(x+5)(x+2) = -2$

Unit 7
Multiplying Expressions

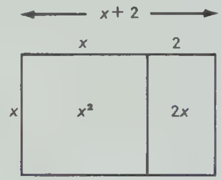
Each of the problems shown here can be solved if you know how to multiply two binomials. You also need to know how to factor expressions such as $x^2 + 7x + 12$ as a product of two binomials.

7.1 Multiplying Binomials

A The idea of the area of a rectangle (length × width) can be used to assist us in multiplying variable expressions.

1. State the area of a rectangle that measures:
   - (*a*) n by w
   - (*b*) x by y
   - (*c*) 15 by 5
   - (*d*) x by $(x + 2)$
   - (*e*) y by $(y - 1)$
   - (*f*) $(x + 1)$ by $(x + 3)$

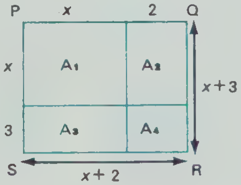
1 (d)



$x(x + 2) = x^2 + 2x$

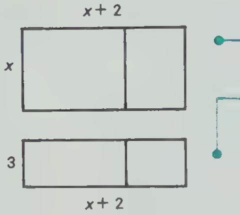
2. For the diagram in the margin state
   - (*a*) the dimensions of the large rectangle PQRS,
   - (*b*) the product that represents the area of rectangle PQRS,
   - (*c*) the area of each of the four small rectangles.
   - (*d*) Write the product $(x + 2)(x + 3)$ as the sum of the four terms from (*c*).

2.



In Question 2 the total area $= (x + 3)(x + 2)$

$$\begin{aligned} &= \boxed{x(x+2)} + \boxed{3(x+2)} \\ &= x^2 + 2x + 3x + 6 \\ &= x^2 + 5x + 6 \end{aligned}$$



3. Use a rectangle to find the following products.
   - (*a*) $(y + 1)(y + 6)$
   - (*b*) $(a + 5)(a + 3)$

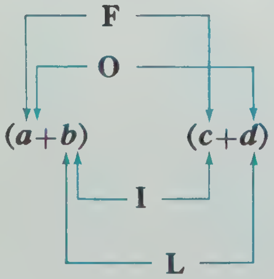
   You should have found in 3(*a*) that

   $$\begin{aligned} (y + 1)(y + 6) &= y^2 + 6y + 1y + 6 \\ &= y^2 + 7y + 6 \end{aligned}$$

4. Multiply.
   - (*a*) $(b + 2)(b + 5)$
   - (*b*) $(t + 1)(t + 4)$
   - (*c*) $(d + 6)(d + 3)$
   - (*d*) $(x + 1)(x - 4)$

Pattern For Multiplying Binomials

$$\begin{aligned} (x + 3)(x + 5) &= x^2 + 5x + 3x + 15 \\ &= x^2 + 8x + 15 \end{aligned}$$



Remember

FOIL ?

In words: *Each term of one factor is multiplied by each term of the other.*

B The area diagram for multiplying can be used when negative terms are present, but it's rather complicated, so we will use the FOIL pattern instead.

1. Find the product.
   (*a*) $(y+3)(y-4)$
   (*b*) $(h-2)(h+8)$
   (*c*) $(z-5)(z+8)$
   (*d*) $(a-4)(a-7)$

Example

Expand $(3x+5)(4x+7)$.

Solution

F O I L

$$\begin{aligned}(3x+5)(4x+7) &= (3x)(4x)+(3x)(7)+(5)(4x)+(5)(7)\\ &= 12x^2+21x+20x+35\\ &= 12x^2+41x+35\end{aligned}$$

2. Multiply.
   (*a*) $(2x+3)(8x+1)$
   (*b*) $(3x-4)(5x+2)$
   (*c*) $(5+4a)(3-2a)$
   (*d*) $(6-2y)(7-3y)$

Exercise 7.1

Discussion

1. State the product.
   (*a*) $(x+3)(x+4)$
   (*b*) $(t+2)(t+9)$
   (*c*) $(y-3)(y-7)$
   (*d*) $(w+6)(w-4)$

Do you notice any short cut?

2. Expand.
   (*a*) $(3x+4)(x+2)$
   (*b*) $(d+1)(3d-5)$
   (*c*) $(2s+6)(3s-1)$
   (*d*) $(5t+1)(4t+7)$

Does your short cut from Question 1 still work?

3. State whether each of the following is true or false.
   (*a*) $(x+2)(x+3) = x^2+2x+6$
   (*b*) $(x+4)(x-4) = x^2-16$
   (*c*) $(3x+5)(4x+1) = 12x^2+6x+5$
   (*d*) $(2x+7)(3x-5) = 6x^2+11x-35$

4. Express as the product of two binomials.
   (*a*) $(a+5)^2$
   (*b*) $(b-4)^2$

Written Solutions

5. Expand.
   (*a*) $(x+3)(x+2)$
   (*b*) $(y-4)(y-3)$
   (*c*) $(t+5)(t+7)$
   (*d*) $(3+a)(5+a)$
   (*e*) $(4-b)(6-b)$
   (*f*) $(m+10)(m-2)$

6. Expand.
   (*a*) $(y+8)(y-4)$
   (*b*) $(c-7)(c+2)$
   (*c*) $(t+10)(t-15)$
   (*d*) $(y+8)(y-8)$
   (*e*) $(5+x)(4-x)$
   (*f*) $(-x+6)(x-3)$

7. Rewrite and then expand.
   (*a*) $(y+2)^2$
   (*b*) $(x-4y)^2$
   (*c*) $(3+x)^2$
   (*d*) $(5-h)^2$
   (*e*) $(a+b)^2$
   (*f*) $(a-b)^2$

8. Find the product.
   (*a*) $(x+3)(x-5)$
   (*b*) $(d-6e)(d+7e)$
   (*c*) $(4-p)(-6+p)$
   (*d*) $(a+4)(a-8)$
   (*e*) $(x+9)(x-9)$
   (*f*) $(3+y)(3-y)$
   (*g*) $(y+4z)(10z-y)$
   (*h*) $(6-x)(x+9)$

9. Expand.
   (*a*) $(2x+1)(5x+8)$
   (*b*) $(3a+7b)(a+9b)$
   (*c*) $(2x-4)(5x-1)$
   (*d*) $(3-5x)(4-7x)$
   (*e*) $(2+7p)(4+3p)$
   (*f*) $(8s+3t)(7s-5t)$

10. Expand.
   (*a*) $(7x-2)(8x+5)$
   (*b*) $(3z+5)(3z-5)$
   (*c*) $(4h-8)(5h+1)$
   (*d*) $(3-6x)(5+9x)$
   (*e*) $(10+7p)(10-7p)$
   (*f*) $(5x-4)^2$

11. Expand and simplify.
   (*a*) $(x+4)^2+(x+2)^2$
   (*b*) $(5y+4)^2+7y+1$
   (*c*) $3(b+4)+(b+6)(b-5)$
   (*d*) $(3y+6)(3y-6)+(2y+1)^2$
   (*e*) $(3p+5)(2p-6)+(2p+7)^2$
   (*f*) $(x+5)^2-(x-2)^2$

12. Expand.
   (*a*) $3(x+4)(x+6)$
   (*b*) $-2(a-3)(a+5)$
   (*c*) $5(3a-4b)^2$
   (*d*) $-2(5y-7)^2$

5(a) x^2+5x+6
(d) $15+8a+a^2$
(e) $24-10b+b^2$

6(a) $y^2+4y-32$
(e) $20-x-x^2$

7(a) y^2+4y+4
(d) $25-10h+h^2$

8(a) $x^2-2x-15$
(c) $-24+10p-p^2$
(g) $-y^2+6yz+40z^2$

9(a) $10x^2+21x+8$
(c) $10x^2-22x+4$
(e) $8+34p+21p^2$

10(a) $56x^2+19x-10$
(c) $20h^2-36h-8$
(e) $100-49p^2$

11(a) $2x^2+12x+20$
(c) $b^2+4b-18$
(e) $10p^2+20p+19$

12(a) $3x^2+30x+72$
(c) $45a^2-120ab+80b^2$

7.2 Factors of Variable Expressions

A Difference of Squares

1. Expand.
   - (a) $(x+4)(x-4)$
   - (b) $(y+3)(y-3)$
   - (c) $(2b-7)(2b+7)$
   - (d) $(3-8y)(3+8y)$

What happens to the middle term?

EXPANDING →

$(x+4)(x-4) = x^2 - 16$

← FACTORING

Study your answers to each part of Question 1. What kind of terms appear in the products?

2. State the factors.
   - (a) $y^2 - 9$
   - (b) $t^2 - 16$
   - (c) $\overset{\text{Perfect squares!}}{\overbrace{4x^2 - 25}}$
   - (d) $x^2 - y^2$

Pattern for a Difference of Squares

$$(x+y)(x-y) = x^2 - y^2$$
$$x^2 - y^2 = (x+y)(x-y)$$

B Simple Trinomial

1. Expand using a short cut.
   - (a) $(a+2)(a+3)$
   - (b) $(t-4)(t-7)$
   - (c) $(y-5)(y-5)$
   - (d) $(p+5)(p+7)$

Here's help.

1(a) $(a+2)(a+3)$
$= a^2 + \underset{\substack{\swarrow\\ 2+3}}{5}\,a + \underset{\substack{\downarrow\\ 2\times 3}}{6}$

By factoring, $a^2 + 5a + 6 = (a+2)(a+3)$

2. Factor.
   - (a) $x^2 + 6x + 5$
   - (b) $y^2 + 7y + 10$
   - (c) $t^2 - 8t + 15$
   - (d) $h^2 - 10h + 21$

2(a) $\square + \triangle = 6$

$\square \times \triangle = 5$

Pattern for a Simple Trinomial

$$(a+3)(a+4) = a^2 + 7a + 12$$
$$a^2 + 7a + 12 = (a+3)(a+4)$$

C Here are some examples of special trinomials!

Example 1

Factor (*a*) $x^2 + 12x + 36$ (*b*) $a^2 - 10a + 25$

Factors	Sum
36 = 36 × 1	37
36 = 18 × 2	20
36 = 12 × 3	15
36 = 9 × 4	13
36 = 6 × 6	12

Solution

(*a*) By checking the factors of 36

$$\begin{aligned} x^2 + 12x + 36 &= (x+6)(x+6) \\ &= (x+6)^2 \end{aligned}$$

(*b*)

$$\begin{aligned} a^2 - 10a + 25 &= (a-5)(a-5) \\ &= (a-5)^2 \end{aligned}$$

Notice that we can express the answer as the square of a binomial.

Example 2

Factor (*a*) $a^2 + 3a - 28$ (*b*) $b^2 - 5b - 6$

Solution

(*a*) -28 is the product of a positive and a negative number whose sum is $+3$. By trial, we find we want 7 and -4.

$\therefore a^2 + 3a - 28 = (a+7)(a-4)$

(*b*) $b^2 - 5b - 6 = (b-6)(b+1)$

What do you notice about the factors when the third term is negative?

Checking is always possible when factoring and is an essential step. Using FOIL we can check (*b*).

$$\begin{aligned} (b-6)(b+1) &= b^2 + b - 6b - 6 \\ &= b^2 - 5b - 6 \end{aligned}$$

Example 3

(*a*) Expand $(x - 2y)(x + 5y)$.
(*b*) Factor $r^2 + 7rs + 10s^2$.

Solution

(*a*)

$$\begin{aligned} (x-2y)(x+5y) &= x^2 + 5xy - 2xy - 10y^2 \\ &= x^2 + 3xy - 10y^2 \end{aligned}$$

(*b*) $r^2 + 7rs + 10s^2 = (r+2s)(r+5s)$

Example 4

Factor (*a*) $3c^2 - 12c - 15$ (*b*) $18x^2 - 50y^2$

Solution

(*a*) $3c^2 - 12c - 15$
$= 3(c^2 - 4c - 5)$
$= 3(c - 5)(c + 1)$

(*b*) $18x^2 - 50y^2$
$= 2(9x^2 - 25y^2)$
$= 2(3x - 5y)(3x + 5y)$

Always look for a common factor first!

Exercise 7.2

Discussion

1. Expand.
   (*a*) $(y + 3)(y - 3)$
   (*b*) $(4h + 8)(4h - 8)$
   (*c*) $(t - 7)(t + 7)$
   (*d*) $(6 - 3x)(6 + 3x)$

   What is similar about your answers to each part?

2. Factor.
   (*a*) $k^2 - 25$
   (*b*) $z^2 - 9$
   (*c*) $36t^2 - 1$
   (*d*) $64m^2 - 49$
   (*e*) $x^2 - y^2$
   (*f*) $x^4 - 1$

3. Select from the following list terms that are perfect squares.
   x^2, $9a^2$, y^3, $15b^2$, 25, 1, $100a^2$, -1, t^4,
   $80z^2$, 81, x^6

4. State whether each of the following is true or false.
   (*a*) $a^2 - b^2 = (a + b)(a - b)$
   (*b*) $x^2 + y^2 = (x + y)^2$
   (*c*) $a^6 - 1 = (a^3 - 1)(a^3 + 1)$
   (*d*) $t^2 - 12 = (t + 4)(t - 3)$
   (*e*) $(x + y)^2 = x^2 + 2xy + y^2$
   (*f*) $x^2 + 4 = (x + 2)(x + 2)$

5. Expand.
   (*a*) $(y + 6)(y + 4)$
   (*b*) $(a - 2b)(a - 7b)$
   (*c*) $(c - 3)(c + 5)$
   (*d*) $(x + 4y)(x - 7y)$

6. Factor.
   (*a*) $x^2 + 5x + 4$
   (*b*) $p^2 + 6pq + 8q^2$
   (*c*) $s^2 - 6s + 9$
   (*d*) $b^2 - 9bc + 20c^2$
   (*e*) $y^2 - y - 30$
   (*f*) $z^2 + 2z - 24$

Written Solutions

7. Factor and check by expanding.
   - (*a*) $x^2 - 16$
   - (*b*) $z^2 - 4$
   - (*c*) $9 - a^2$
   - (*d*) $m^2 - 64$
   - (*e*) $x^2 - 4y^2$
   - (*f*) $36 - n^2$

8. Factor and check.
   - (*a*) $81x^2 - 16y^2$
   - (*b*) $49a^2 - 1$
   - (*c*) $100 - 9z^2$
   - (*d*) $x^4 - 1$
   - (*e*) $y^6 - 1$
   - (*f*) $a^8 - 1$

9. Factor and check.
   - (*a*) $a^2 + 3a + 2$
   - (*b*) $p^2 + 9p + 14$
   - (*c*) $k^2 + 13k + 22$
   - (*d*) $t^2 - 12th + 32h^2$
   - (*e*) $r^2 - 14r + 45$
   - (*f*) $s^2 - 16st + 63t^2$

10. Factor and check.
    - (*a*) $x^2 - 10x + 25$
    - (*b*) $y^2 + 6yz - 16z^2$
    - (*c*) $t^2 + 8t + 16$
    - (*d*) $h^2 + 2h - 35$
    - (*e*) $g^2 - 3gh - 54h^2$
    - (*f*) $k^2 + 4k - 21$

11. Factor and check.
    - (*a*) $w^2 - 7w + 6$
    - (*b*) $z^2 - 7z - 8$
    - (*c*) $r^2 + 2r - 80$
    - (*d*) $t^2 - 2t - 80$
    - (*e*) $h^2 + 3h + 2$
    - (*f*) $k^2 + 3k - 4$
    - (*g*) $c^2 + 5c - 24$
    - (*h*) $c^2 - 5c - 24$

12. Factor and check.
    - (*a*) $t^2 + 8t + 12$
    - (*b*) $r^2 - 7r + 12$
    - (*c*) $s^2 + 13s + 12$
    - (*d*) $q^2 - q - 12$
    - (*e*) $x^2 - 21x + 20$
    - (*f*) $y^2 + 9y + 20$
    - (*g*) $z^2 - 12z + 20$
    - (*h*) $w^2 - 19w - 20$

13. Factor and check.
    - (*a*) $x^2 - 16$
    - (*b*) $y^2 - 16y + 48$
    - (*c*) $y^2 - 9$
    - (*d*) $z^2 - 9z + 18$
    - (*e*) $b^2 - 36$
    - (*f*) $c^2 - 12c + 36$
    - (*g*) $h^2 - 49k^2$
    - (*h*) $h^2 + 14hk + 49k^2$

14. Factor into a product of three factors.
    - (*a*) $5x^2 - 15x + 10$
    - (*b*) $12a^2 - 27b^2$
    - (*c*) $x^3 - 7x^2 + 12x$
    - (*d*) $c^3 - b^2c$
    - (*e*) $2x^2 - 4x - 70$
    - (*f*) $2xy^2 - 8xy - 10x$

In (d) and (f) don't stop after the first step!

7(a) $(x-4)(x+4)$
(f) $(6-n)(6+n)$

8(d) $(x^2+1)(x+1)(x-1)$
(f) $(a^4+1)(a^2+1)(a+1)$
$(a-1)$

9(b) $(p+2)(p+7)$
(e) $(r-9)(r-5)$

10(a) $(x-5)^2$
(e) $(g+6h)(g-9h)$

11(a) $(w-6)(w-1)$
(c) $(r+10)(r-8)$
(e) $(h+2)(h+1)$
(g) $(c-3)(c+8)$

12(a) $(t+2)(t+6)$
(c) $(s+12)(s+1)$
(e) $(x-20)(x-1)$
(g) $(z-2)(z-10)$

13(a) $(x-4)(x+4)$
(c) $(y-3)(y+3)$
(e) $(b-6)(b+6)$
(g) $(h+7k)(h-7k)$

14(a) $5(x-2)(x-1)$
(c) $x(x-3)(x-4)$
(e) $2(x-7)(x+5)$

7.3 Trinomial Factoring

A 1. State the product.

(*a*) $(x+3)(x+5)$ (*c*) $(z+6)(z+6)$
(*b*) $(y+5)(y-5)$ (*d*) $(t-3)(t-4)$

2. State the product.

(*a*) $(4x+1)(x+5)$ (*c*) $(6y+3)(2y-3)$
(*b*) $(5x-3)(2x+1)$ (*d*) $(4r-2)(5r+1)$

Why is $4x^2$ the first term in 2(a), while the first term in 1(a) is x^2?

To factor expressions such as $4x^2+21x+5$ it is important for you to see that factors such as $(x+\square)(x+\square)$ are not correct. Two possible forms are $(2x+\square)(2x+\square)$ and $(x+\square)(4x+\square)$.

3. State the first terms possible in the factors of the following trinomials.

(*a*) $2x^2+7x+3$ (*d*) $12b^2-20b+3$
(*b*) $5x^2-7x+2$ (*e*) $15c^2-12c-3$
(*c*) $10y^2-y-3$ (*f*) $8h^2-2h-15$

(12b)(b)
(6b)(2b)
(4b)(3b)

To complete the factoring, the last term of the trinomial is used. However several answers are possible. For 3(*a*) possible solutions are

$$\begin{aligned}&2x^2+7x+3 \\ &=(x+1)(2x+3)\end{aligned} \qquad \text{or} \qquad \begin{aligned}&2x^2+7x+3 \\ &=(2x+1)(x+3)\end{aligned}$$

Which is correct? How do you know?

B It appears that the only way to be sure is to *check the middle term.* When you do this,

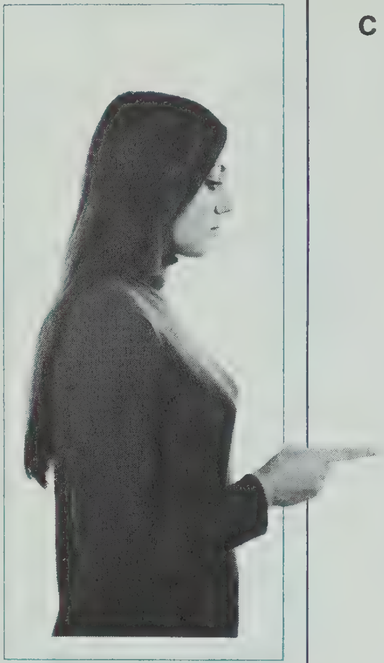
$$\begin{aligned}&(2x+1)(x+3) \\ &=2x^2+6x+x+3 \\ &=2x^2+7x+3\end{aligned} \qquad \text{while} \qquad \begin{aligned}&(x+1)(2x+3) \\ &=2x^2+3x+2x+3 \\ &=2x^2+5x+3\end{aligned}$$

$$\therefore 2x^2+7x+3=(2x+1)(x+3) \text{ is correct.}$$

FOIL!

In 3(*b*),
$$\begin{aligned}&5x^2-7x+2 \\ &=(5x-1)(x-2)\end{aligned} \qquad \text{or} \qquad \begin{aligned}&5x^2-7x+2 \\ &=(x-1)(5x-2)\end{aligned}$$

Which is correct?



Can you see why only factors with "–" signs need to be checked?

C Often there are far more than just two possibilities. Study the following examples.

Example 1

Factor $12b^2 - 20b + 3$.

Solution

$12b^2 - 20b + 3$	*Check on the second term*	
$= (12b - 3)(b - 1)$	$-12b - 3b = -15b$	
or $(b - 3)(12b - 1)$	$-b - 36b = -37b$	
or $(6b - 3)(2b - 1)$	$-6b - 6b = -12b$	
or $(2b - 3)(6b - 1)$	$-2b - 18b = -20b$	← Aha!
or $(4b - 3)(3b - 1)$	$-4b - 9b = -13b$	
or $(3b - 3)(4b - 1)$	$-3b - 12b = -15b$	

$\therefore$ The correct solution is $12b^2 - 20b + 3 = (2b - 3)(6b - 1)$.

Example 2

Factor $9y^2 - 12y + 4$.

Solution

$9y^2 - 12y + 4$	*Check on the second term*	
$= (9y - 4)(y - 1)$	$-9y - 4y = -13y$	
or $(y - 4)(9y - 1)$	$-y - 36y = -37y$	
or $(3y - 4)(3y - 1)$	$-3y - 12y = -15y$	
or $(9y - 2)(y - 2)$	$-18y - 2y = -20y$	
or $(3y - 2)(3y - 2)$	$-6y - 6y = -12y$	← Yes!

$$\therefore\ 9y^2 - 12y + 4 = (3y - 2)(3y - 2) = (3y - 2)^2$$

Example 3

Factor (*a*) $4q^2 - 6q - 10$ (*b*) $3x^2 - 8xy + 5y^2$

Solution

(*a*) $4q^2 - 6q - 10$
$= 2(2q^2 - 3q - 5)$
$= 2(2q - 5)(q + 1)$

(*b*) $3x^2 - 8xy + 5y^2$
$= (3x - 5y)(x - y)$

(a) $2(2q + 5)(q - 1)$
$2(2q + 1)(q - 5)$
$2(2q - 5)(q + 1)$
$2(2q - 1)(q + 5)$

(b) $(3x - 5y)(x - y)$
$(3x - y)(x - 5y)$

Exercise 7.3

Discussion

1. Expand using FOIL.
   - (*a*) $(2x+5)(x+4)$
   - (*b*) $(3y-7)(4y-2)$
   - (*c*) $(5b-1)(3b+7)$
   - (*d*) $(4c+6)(2c-5)$

2. List possible pairs of factors for the following trinomials.
   - (*a*) $2x^2+5x+3$
   - (*b*) $3y^2-10y+7$
   - (*c*) $6h^2-5h+1$
   - (*d*) $8t^2-17t+2$
   - (*e*) $10s^2+14s+4$
   - (*f*) $20r^2+52r+5$
   - (*g*) $3x^2+7x-6$
   - (*h*) $5z^2-6z-8$

Let's call this type of expression a "trial" trinomial.

3. Is there any way of telling what signs should be used in the factors of a trinomial? Use Question 2 for examples.

4. $9x^2+12x+4$ is an example of *a square of a binomial*. Select from the following list other possible squares.
   - (*a*) x^2+6x+9
   - (*b*) $5y^2+9y+16$
   - (*c*) $4z^2-20z+25$
   - (*d*) $49b^2-50b+1$
   - (*e*) $4x^2-18x+8$
   - (*f*) $25y^2-30y-9$

   What further test is necessary before concluding that these trinomials are squares of binomials?

Notice that $9x^2$ and 4 are perfect squares!

Written Solutions

5. Expand.
   - (*a*) $(2x-7)(x+4)$
   - (*b*) $(3y-10)^2$
   - (*c*) $(5a-6)(3a-5)$
   - (*d*) $(3b+6)(8b+2)$
   - (*e*) $(12y+5)(3y-2)$
   - (*f*) $(4a+9)(10a-7)$

6. Simplify.
   - (*a*) $(4b-2)(8b+1)+(3b-5)(2b-1)$
   - (*b*) $(3y-7)(4-y)+(2y-3)^2$
   - (*c*) $2(3y-7)^2-3y(y-5)$
   - (*d*) $(2h-5)^2-2(h+3)^2$

7. Factor and check.
   - (*a*) $2x^2+11x+5$
   - (*b*) $2y^2+7y+5$
   - (*c*) $4a^2-4a+1$
   - (*d*) $4b^2-5b+1$
   - (*e*) $3h^2-22h+7$
   - (*f*) $3h^2-10h+7$
   - (*g*) $3h^2-4h-7$
   - (*h*) $3h^2+4h-7$

5(a) $2x^2+x-28$
(c) $15a^2-43a+30$
(e) $36y^2-9y-10$

6(a) $38b^2-25b+3$
(b) $y^2+7y-19$
(c) $15y^2-69y+98$
(d) $2h^2-32h+7$

Strategy of Factoring

1. Check for a common factor.
2. Two terms $\longrightarrow$ difference of squares.
3. Three terms
   - $\longrightarrow$ simple trinomial
   - $\longrightarrow$ square of a binomial
   - $\longrightarrow$ "trial" trinomial

8(a) $2(5a-3)(a-1)$
(d) $2(5d-1)(d-3)$
(f) $2(5f-3)(f+1)$
(i) $2(5i+1)(i-3)$

14(b) $3(k-2)(k+2)$
(g) $4(x+3)(x-7)$
(h) $3(2t+1)(3t-4)$

8. Factor and check.
(a) $10a^2 - 16a + 6$
(b) $10b^2 - 61b + 6$
(c) $10c^2 - 17c + 6$
(d) $10d^2 - 32d + 6$
(e) $10e^2 - 23e + 6$
(f) $10f^2 + 4f - 6$
(g) $10g^2 - 59g - 6$
(h) $10h^2 + 17h - 6$
(i) $10i^2 - 28i - 6$
(j) $10j^2 - 7j - 6$

9. Factor and check.
(a) $5y^2 + 16y + 3$
(b) $7y^2 - 78y + 11$
(c) $2r^2 - 15r + 13$
(d) $5t^2 + 13t + 6$
(e) $3p^2 - 8p + 4$
(f) $7q^2 - 30q + 8$

10. Factor and check.
(a) $10x^2 + 17x + 3$
(b) $18y^2 - 21y + 5$
(c) $21z^2 + 13z + 2$
(d) $8m^2 - 18m + 9$
(e) $14h^2 + 19h + 6$
(f) $4s^2 - 32s + 15$

11. Factor and check.
(a) $2s^2 + 5s - 3$
(b) $5r^2 - 3r - 2$
(c) $2a^2 - 5a - 3$
(d) $5h^2 + 3h - 2$
(e) $7h^2 + 2h - 5$
(f) $2m^2 + 3m - 9$
(g) $7t^2 - 2t - 5$
(h) $2k^2 - 3k - 9$

12. Factor and check.
(a) $5t^2 - 7t - 6$
(b) $8h^2 + 2h - 3$
(c) $5x^2 - 23x - 10$
(d) $4y^2 - 16y - 9$
(e) $5s^2 + 4s - 12$
(f) $14z^2 - 11z - 15$

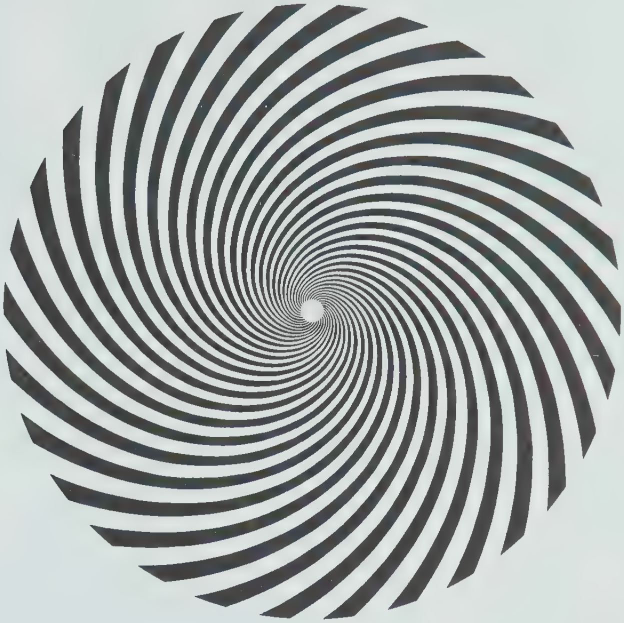
13. Factor and check.
(a) $s^2 - 10s + 21$
(b) $15m^2 - 8m + 1$
(c) $6t^2 - t - 5$
(d) $8y^2 + 22y + 9$
(e) $3a^2 - 11a - 20$
(f) $16x^2 - 40x + 25$
(g) $4r^2 + 2r - 30$
(h) $8 - 13h - 6h^2$
(i) $12p^2 - 32p + 5$
(j) $49z^2 - 84z + 36$

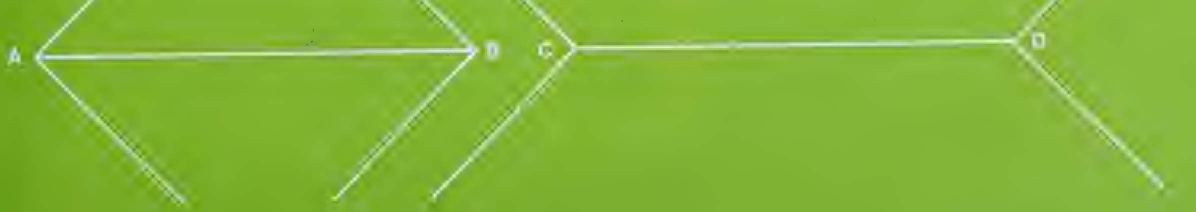
14. Factor.
(a) $x^2 - xy$
(b) $3k^2 - 12$
(c) $25x^2 + 10x + 1$
(d) $12x^2 - 40x - 4$
(e) $15m^2 + 29m - 14$
(f) $a^2 - 14a - 72$
(g) $4x^2 - 16x - 84$
(h) $18t^2 - 15t - 12$

15. Factor completely.

$$5x^4 - 65x^2 + 180$$

Illusions





Measure AB and CD.

Unit 8
Further Operations with Expressions

We know how to add, subtract, multiply, and divide with numbers. Now we will perform these same operations with variable expressions.

Dividend

$$\frac{6}{3} = 2 \leftarrow \text{Quotient}$$

Divisor

Recall

$$a^5 \div a^4 = \frac{a \times a \times a \times a \times a}{a \times a \times a \times a}$$

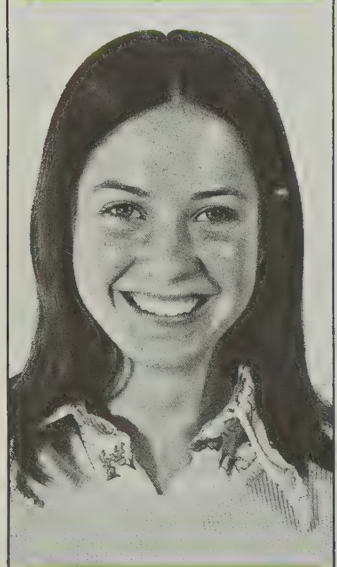
$$= a^1$$

$$= a$$

$$\therefore a^5 \div a^4 = \frac{a^5}{a^4}$$

$$= a^{5-4}$$

$$= a$$



8.1 Dividing Monomials

1. State the quotient in each case.
   - (*a*) 26 ÷ 2
   - (*b*) 15 ÷ (−3)
   - (*c*) −32 ÷ 8
   - (*d*) 35 ÷ (−7)
   - (*e*) −56 ÷ (−8)
   - (*f*) −42 ÷ 6
   - (*g*) 3 ÷ 5
   - (*h*) 14 ÷ (−6)
   - (*i*) −37 ÷ 5
   - (*j*) 51 ÷ (−11)

2. Divide.
   - (*a*) $\frac{x^5}{x^2}$
   - (*b*) $\frac{y^7}{y^3}$
   - (*c*) $a^5 \div a^4$
   - (*d*) $5^4 \div 5^3$
   - (*e*) $s^{15} \div s^6$
   - (*f*) $k^{10} \div k^{10}$
   - (*g*) $x^2 \div x^5$
   - (*h*) $t^7 \div t^8$

Compare the following examples.

Example 1

Find $-24xy^2 \div 14y$.

Solution

$$\frac{-24xy^2}{14y}$$

$$= -\frac{{}^{12}\cancel{24}}{\cancel{14}_{7}} \times \frac{x\cancel{y^2}\,{}^{y}}{\cancel{y}\,{}_{1}}$$

$$= -\frac{12}{7} \times xy$$

$$= -\frac{12}{7}xy$$

Example 2

Find $\frac{35a^2b}{-15a}$

Solution

$$\frac{35a^2b}{-15a}$$

$$= -\frac{\cancel{5a} \times 7ab}{\cancel{5a} \times 3}$$

$$= -\frac{7ab}{3}$$

In Example 1 the numerical and variable parts of the expression were separated, the divisions completed, and the parts put together again. In Example 2 factoring was used.

Check Example 1 by using factoring. Try to do Example 2 by the method shown in Example 1.

3. Find the quotient.
   - (*a*) $6ab \div (-3a)$
   - (*b*) $-25x^2 \div (5x)$

Example 3

Divide $-42r^5s^3t$ by $28r^2s^3t^7$

Factoring could be used here as well.

Solution

$$\frac{-42r^5s^3t}{28r^2s^3t^7}$$

$$= -\frac{42}{28} \times \frac{r^5}{r^2} \times \frac{s^3}{s^3} \times \frac{t}{t^7}$$

$$= -\frac{3}{2} \times r^3 \times 1 \times \frac{1}{t^6} = \frac{-3r^3}{2t^6} \longleftarrow$$

This could also be written as $-\frac{3}{2}r^3t^{-6}$, since $\frac{t}{t^7} = t^{1-7} = t^{-6}$

Exercise 8.1

Discussion

1. Divide

(a) $-\frac{15}{3}$

(b) $\frac{-28}{4}$

(c) $\frac{52}{-13}$

(d) $\frac{-30}{-10}$

(e) $\frac{72}{-6}$

(f) $\frac{t^5}{t^2}$

(g) $\frac{-x^3}{x^2}$

(h) $\frac{y^7}{-y^4}$

(i) $\frac{-m^{15}}{-m^{12}}$

2. Divide.

(a) $\frac{6a}{2}$

(b) $\frac{-28x}{4}$

(c) $\frac{45a}{-a}$

(d) $\frac{15x^2}{x}$

(e) $\frac{21t^3}{t^2}$

(f) $\frac{-6x}{4}$

(g) $\frac{12ab}{-a}$

(h) $\frac{18h}{-9h}$

(i) $\frac{25xy}{-5y}$

Recall that $3x$, 16, $5yz$, 2, $-7z^2$ are monomials (One term)

3. What number may not be used as a divisor? In 2(*c*) what *restriction* must be placed on the value of a?

4. State the HCF of the divisor and dividend.

(a) $\frac{24a}{8a}$

(b) $\frac{15b^2}{10b}$

(c) $\frac{30rs}{12r}$

(d) $\frac{-40a^3}{4a^2}$

(e) $\frac{28y^2z}{-21y}$

(f) $\frac{-39xy^2}{-26x^2y}$

HCF

means "Highest Common Factor"

Careful in (f)!

5. (*a*) A journey of 480 km was completed in 6 h. State the average speed.
   (*b*) 15 dozen eggs cost x¢. State the cost per dozen.
   (*c*) The area of a rectangle is $24xy$ m² and the width is $3y$ metres. State the length.

Written Solutions

6. Simplify.

(*a*) $\frac{-12}{10}$ (*d*) $\frac{xy}{x}$ (*g*) $\frac{t^7}{t^3}$

(*b*) $\frac{-56}{-21}$ (*e*) $\frac{ab}{b}$ (*h*) $\frac{-xy^2}{xy}$

(*c*) $\frac{32t}{-16}$ (*f*) $\frac{-6t^2}{3}$ (*i*) $\frac{3a^2}{6a}$

7. Simplify.

(*a*) $\frac{5r \times 3s}{5r}$ (*c*) $\frac{3rs \times 5}{3rs \times 5}$ (*e*) $\frac{0}{abc}$

(*b*) $\frac{4t \times (-3y)}{4t \times 5}$ (*d*) $\frac{(-6x) \times 3y}{(-6x) \times 4}$ (*f*) $\frac{xyz}{0}$

8. Simplify.

(*a*) $\frac{12a^2}{8a}$ (*d*) $\frac{18klm}{-9lm}$ (*g*) $\frac{-63rs}{9r^2}$

(*b*) $\frac{20xy}{-5x}$ (*e*) $\frac{-30xy}{-20y}$ (*h*) $\frac{75ab}{-15b^3}$

(*c*) $\frac{-14h^2}{-7h}$ (*f*) $\frac{49x^2y^5}{-14xy^3}$ (*i*) $\frac{27xyz}{81xz}$

9. 300 kg of apples cost $25$x$. Find the cost per kilogram and the cost of 60 kg.

10. A person walks $35x$ kilometres in $5x$ hours. Find the average rate of walking and the time required to walk $42y$ kilometres.

11. Zogs sell at $3x$ zogs for $21y$¢. Find the cost of $5x$ zogs.

12. In the game of Zip-Zap one zip $= \frac{rs}{2}$ zaps. Convert r^2s^3 zaps to zips.

13. Solve the following equations for x.

(*a*) $mx = m^2n$ (*b*) $3h^2x = 9hr^2$

6(a) $-\frac{6}{5}$ **(e)** a **(i)** $\frac{a}{2}$

7(a) $3s$ **(f) Impossible**

8(a) $\frac{3a}{2}$ **(f)** $-\frac{7}{2}xy^2$

(e) $\frac{3x}{2}$ **(h)** $-\frac{5a}{b^2}$

9. $\$\frac{x}{12}$**/kg,** $\$5x$

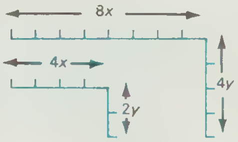
10. 7 km/h, $6y$ **h**

11. $35y$**¢**

12. $2rs^2$ **zips**

13(a) mn **(b)** $\frac{3r^2}{h}$

8.2 Dividing Polynomials by Monomials



Example 1

Divide $8x + 4y$ by 2.

Solution

$$\frac{8x + 4y}{2} = \frac{8x}{2} + \frac{4y}{2} = 4x + 2y$$

← or →

$$\frac{8x + 4y}{2} = \frac{2(4x + 2y)}{2} \text{ by factoring} = 4x + 2y$$

Another way of writing the first solution is

$$\frac{8x+4y}{2} = \frac{1}{2}(8x) + \frac{1}{2}(2y) = 4x + 2y$$

1. State the quotient.
   (*a*) $\dfrac{5a + 10b}{5}$ (*b*) $\dfrac{12r - 8s}{4}$ (*c*) $\dfrac{x^2 - 2x}{x}$
2. Factor the dividend and simplify.
   (*a*) $\dfrac{3x + 6}{3}$ (*b*) $\dfrac{14a - 35b}{7}$ (*c*) $\dfrac{9a + 15b}{6}$

Example 2

Find the quotient.

(*a*) $\dfrac{8x^3 + 4x^2 - 6x}{2x}$ (*b*) $\dfrac{6a^2b^2 - 12ab^2 + 3ab}{3ab}$

Solution

(*a*)
$$\frac{8x^3 + 4x^2 - 6x}{2x} = \frac{8x^3}{2x} + \frac{4x^2}{2x} - \frac{6x}{2x} = 4x^2 + 2x - 3$$

(*b*)
$$\frac{6a^2b^2 - 12ab^2 + 3ab}{3ab} = \frac{6a^2b^2}{3ab} - \frac{12ab^2}{3ab} + \frac{3ab}{3ab} = 2ab - 4b + 1$$

By factoring,

$$\frac{8x^3 + 4x^2 - 6x}{2x} = \frac{2x(4x^2 + 2x - 3)}{2x} = 4x^2 + 2x - 3$$

Try (*b*) by factoring!

5(a) $x + 5$

(c) $\frac{(r - 3s)}{2}$

(f) $\frac{5x^2 + 9x - 16}{4}$

Exercise 8.2

Discussion

1. Factor.
   (a) $3x^2 + 6x$
   (b) $5a - 10b$
   (c) $6y^2 - 8y$
   (d) $15xy - 18y$
   (e) $4x^3 + 6x^2 + 8x$
   (f) $6a^3 - 8a^2 + 2a$

2. State the quotient.
   (a) $\frac{2x + 4}{2}$
   (b) $\frac{5x - 10}{5}$
   (c) $\frac{6a^2 + 5a}{a}$
   (d) $\frac{10t^2 - 7t}{t}$
   (e) $\frac{6z^2 - 3z}{3z}$
   (f) $\frac{ab - a}{a}$

3. Divide.
   (a) $2a + 4b + 6c - 8d$ by 2
   (b) $x^3 + x^2 - x$ by x

Compare the number of terms in the quotient with the number in the dividend. What do you notice?

4. Which is correct?

   (a) $\frac{x + 1}{x} = 1$ or (b) $\frac{\overset{1}{\cancel{x}} + 1}{\cancel{x}^1} = \frac{2}{1} = 2$ or (c) $\frac{x + 1}{x} = \frac{x}{x} + \frac{1}{x} = 1 + \frac{1}{x}$

   Check by substituting $x = 2$.

Written Solutions

5. Find the quotient.
   (a) $(2x + 10) \div 2$
   (b) $\frac{5x - 15}{-5}$
   (c) $(7r - 21s) \div 14$
   (d) $\frac{15y + 24}{9}$
   (e) $(18a - 24b + 36) \div (-6)$
   (f) $\frac{25x^2 + 45x - 80}{20}$

6. Simplify.

(*a*) $\dfrac{3r^2 + 6r}{3r}$ (*d*) $\dfrac{32ab - 40b^2}{8b}$

(*b*) $\dfrac{2xy - 4y^2}{2y}$ (*e*) $\dfrac{10r^3 + 15r^2 - 20r}{-5r}$

(*c*) $\dfrac{16x^3 - 8x^2 + 12x}{-4x}$ (*f*) $\dfrac{18t^3 - 15t}{9t}$

7. Find the average rate if a distance of $(10ax + 15ay)$ kilometres is covered in $5a$ hours.

8. The perimeter of an equilateral triangle is $(3x^2 - 6x + 9)$ centimetres. Find the length of one side. Check using $x = 1$.

9. The area of a rectangle is $(40xyz + 80x^2)$ m^2 and the length is $40x$ metres. Find the width.

10. The total surface area of a cube is $(6x^2 - 36x + 54)$ cm^2. Find an expression for the length of each side of the cube.

8.3 Dividing a Polynomial by a Binomial

A *Example 1*

Divide $x^2 + 5x + 6$ by $x + 2$.

Solution

$$\frac{x^2 + 5x + 6}{x + 2} = \frac{\cancel{(x + 2)}(x + 3)}{\cancel{x + 2}} = x + 3$$

1. Use factoring to divide.

(*a*) $a^2 + 7a + 12$ by $a + 4$ (*c*) $y^2 + 6y + 9$ by $y + 3$

(*b*) $x^2 - 9$ by $x - 3$ (*d*) $b^2 + 3b + 2$ by $b + 4$

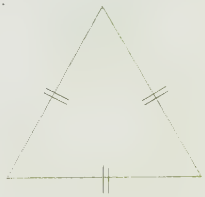
What difficulty arises in 1(*d*)? Does this mean that division in this case is impossible?

In working with numbers,

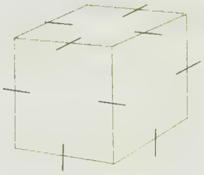
$$\frac{63}{7} = \frac{\cancel{7} \times 9}{\cancel{7}} = 9 \qquad \text{and} \qquad \frac{43}{7} = 6 + \frac{1}{7} = 6\tfrac{1}{7}$$

6(a) $r + 2$
(c) $-4x^2 + 2x - 3$
(e) $-2r^2 - 3r + 4$
7. $(2x + 3y)$ km/h
8. $(x^2 - 2x + 3)$ cm
9. $(yz + 2x)$ m
10. $(x - 3)$ cm

8.



10.



Does factoring work in every case with numbers?

B Study the following example noting the steps in the familiar procedure of "long division."

Example 2

Divide 987 by 52.

Solution

Step 1

$$\begin{array}{r} 1 \\ 52\overline{)987} \\ 52 \\ \hline 46 \end{array}$$

Step 2

$$\begin{array}{r} 18 \\ 52\overline{)987} \\ 52 \\ \hline 467 \\ 416 \\ \hline 51 \end{array}$$

Division statement

$$\therefore \frac{987}{52} = 18 + \frac{51}{52}$$

1. Find the quotient and conclude with a division statement.
   (*a*) $7241 \div 35$ (*b*) $28.74 \div 2.1$ (*c*) $358.2 \div 57$

Example 3

If one number is a factor of another number, it divides into that number exactly! (no remainder)

Test whether $b + 4$ is a factor of $b^2 + 3b + 2$. (To answer refer back to the pattern for long division.)

Solution

$$\begin{array}{r} b\phantom{{}+3b+2} \\ b+4\,\overline{)b^2+3b+2} \end{array}$$

What terms were divided to give b as the first term of the quotient?

$$\begin{array}{r} b\phantom{{}+3b+2} \\ b+4\,\overline{)b^2+3b+2} \\ b^2+4b\phantom{{}+2} \\ \hline -b\phantom{{}+2} \end{array}$$

Explain.

Recall the rule for subtraction.

$$\begin{array}{r} b-1\phantom{{}+3b+2} \\ b+4\,\overline{)b^2+3b+2} \\ b^2+4b\phantom{{}+2} \\ \hline -b+2 \\ -b-4 \\ \hline +6 \end{array}$$

$\because$ There is a remainder,
$\therefore$ $b + 4$ is not a factor.

Check:

$(b-1)(b+4)+6$
$= b^2+3b-4+6$
$= b^2+3b+2$

Example 4

Divide $x^3 - 5x + 3x^2 + 1$ by $x - 1$.

Solution

$$\begin{array}{r l}
 & x^2 + 4x - 1 \\
x - 1 & \overline{)x^3 + 3x^2 - 5x + 1} \\
 & \underline{x^3 - x^2} \\
 & \phantom{)x^3 + {}}4x^2 - 5x \\
 & \phantom{)x^3 + {}}\underline{4x^2 - 4x} \\
 & \phantom{)x^3 + 3x^2 {}}-x + 1 \\
 & \phantom{)x^3 + 3x^2 {}}\underline{-x + 1} \\
 & \phantom{)x^3 + 3x^2 - x + {}}0
\end{array}$$

$\therefore (x^3 + 3x^2 - 5x + 1) \div (x - 1) = x^2 + 4x - 1$

How could this quotient be checked? *

Note that we must change the order of the given terms. (Both divisor and dividend must be in the same order – usually descending.)

*

$(x-1)(x^2+4x+1) = ?$

Exercise 8.3

Discussion

1. Subtract.

(*a*) $\begin{array}{r} 3x \\ \underline{5x} \end{array}$ (*b*) $\begin{array}{r} -6 \\ \underline{+2} \end{array}$ (*c*) $\begin{array}{r} 3y^2 \\ \underline{-y^2} \end{array}$ (*d*) $\begin{array}{r} -4 \\ \underline{-1} \end{array}$ (*e*) $\begin{array}{r} -2x \\ \underline{3x} \end{array}$

2. Multiply.

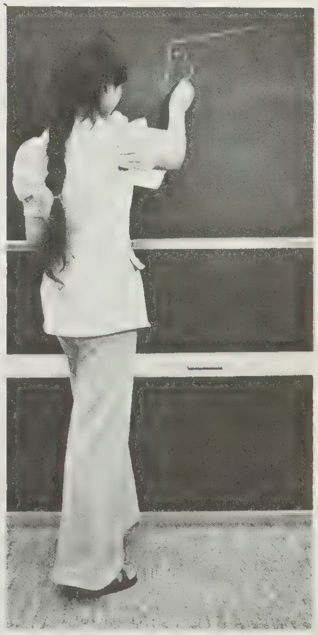
(*a*) $x + 3$ by $2x^2$
(*b*) $y - 4$ by $-3y$
(*c*) $1 + x$ by $-5x$
(*d*) $b - 2$ by $4b$
(*e*) $r + 6$ by -1
(*f*) $2x - 3$ by $-3x^2$

3. Divide.

(*a*) $4x^2$ by x
(*b*) $-5x$ by x
(*c*) $6y^2$ by $2y$
(*d*) x^3 by x
(*e*) $7y$ by y
(*f*) $-10x$ by $2x$

4. (*a*) When $x^2 - 5x + 6$ is divided by $x - 3$ the quotient is $x - 2$ and the remainder is 0. What is the division statement for this example?

(*b*) When $3x^2 - 10x + 12$ is divided by $x - 2$ the quotient is $3x - 4$ and the remainder is 4. Give the division statement.



Put both divisor and dividend in order of descending powers of the variable.

5. How would you rewrite the following before dividing?
(*a*) $(x + x^2 - 2) \div (x - 2)$
(*b*) $(a^3 - 2a + 3a^2 + 1) \div (a - 1)$
(*c*) $(x^2 - 1) \div (x + 1)$
(*d*) $(x^3 + x - 2) \div (x - 1)$

$$\frac{x^2 - 1}{x - 1} = \frac{(x - 1)(x + 1)}{x - 1}$$

6. Are the following correct? Explain.
(*a*) $\frac{x^2 - 1}{x - 1} = x - 1$ (*b*) $\frac{a + 4}{4} = a$ (*c*) $\frac{x + 2}{x} = 2$

Written Solutions

7. Factor.
(*a*) $x^2 - 9$
(*b*) $y^2 - 4$
(*c*) $a^2 - 6a + 8$
(*d*) $y^2 + 6y + 9$
(*e*) $b^2 + 3b - 28$
(*f*) $t^2 - 5t - 24$
(*g*) $4a^2 - 25$
(*h*) $m^2 - 12m + 36$

8. Divide by factoring. Check the quotient by multiplying.
(*a*) $(3x + 6) \div (x + 2)$
(*b*) $(5y - 15) \div (y - 3)$
(*c*) $(x^2 - 4) \div (x + 2)$
(*d*) $(a^2 - 7a + 10) \div (a - 5)$
(*e*) $(b^2 + b - 20) \div (b - 4)$
(*f*) $\frac{h^2 + 6h + 9}{h + 3}$
(*g*) $\frac{a^2 + 4a - 12}{a - 2}$
(*h*) $\frac{b^2 + 8b + 15}{2b + 6}$

9. Divide using the long division procedure.
(*a*) $\frac{x^2 + 5x + 4}{x - 1}$
(*b*) $\frac{a^2 + 8a + 12}{a + 6}$
(*c*) $\frac{x^2 + 4x - 21}{x - 3}$
(*d*) $\frac{y^3 + 3y^2 - y - 3}{y - 1}$

10. Divide and write the resulting division statement.
(*a*) $\frac{y^2 + 5y + 2}{y + 3}$
(*b*) $\frac{y^3 + 2y^2 - y + 3}{y - 2}$
(*c*) $\frac{a^3 - 6a^2 + 5a - 1}{a + 3}$
(*d*) $\frac{x^2 - 3x - 7}{x - 5}$

11. Rewrite and divide.
(*a*) $(a^3 + 2 - 9a - 3a^2) \div (a + 2)$
(*b*) $(3x + x^3 - 2x^2 + 4) \div (x - 3)$
(*c*) $(x^2 - 1) \div (x - 1)$
(*d*) $(a^3 - 2a - 1) \div (a + 1)$

8(a) 3
(c) $x - 2$ **(g)** $a + 6$
9(a) $x + 4$ **(d)** $y^2 + 4y + 3$
10(a) $y + 2 - \frac{4}{y + 3}$
(c) $a^2 - 9a + 32 - \frac{97}{a + 3}$
11(a) $a^2 - 5a + 1$
(b) $x^2 + x + 6 + \frac{22}{x - 3}$
(d) $a^2 - a - 1$

8.4 Adding and Subtracting Algebraic Fractions

Numerical fractions

$$\frac{1}{4}, \frac{-5}{6}, \frac{7}{2}, \frac{19}{3}$$

Algebraic fractions

$$\frac{5x}{m}, \frac{3}{a}, \frac{-a}{b}, \frac{5x^2y}{3z}$$

Algebraic fractions behave very much like numerical fractions. Keeping this in mind, study the following examples carefully.

To add or subtract fractions you must have a common denominator!!!

Example 1

$$\begin{aligned}\frac{5}{8}+\frac{1}{8} &= \frac{5+1}{8}\\ &= \frac{6}{8} \leftarrow \\ &= \frac{3}{4}\end{aligned}$$

Simplify by dividing out common factors.

Example 2

$$\begin{aligned}\frac{5}{m}+\frac{1}{m} &= \frac{5+1}{m}\\ &= \frac{6}{m}\end{aligned}$$

Example 3

$$\begin{aligned}\frac{5x+1}{x+2}+\frac{x+11}{x+2} &= \overset{*}{\frac{(5x+1)+(x+11)}{x+2}}\\ &= \frac{6x+12}{x+2}\\ &= \frac{6\cancel{(x+2)}}{\cancel{x+2}}\\ &= 6\end{aligned}$$

* Here we already have a common denominator, so we just add the numerators.

Example 4

$$\begin{aligned}\frac{a+4}{b}-\frac{a-2}{b} &= \frac{(a+4)\overset{*}{-}(a-2)}{b}\,*\\ &= \frac{a+4-a+2}{b}\\ &= \frac{6}{b}\end{aligned}$$

* Can you see why this sign changes? (Careful - this is a "GOTCHA"!)

Example 5

$$\begin{aligned}\frac{2}{3}+\frac{3}{4} &= \frac{2\times 4}{3\times 4}+\frac{3\times 3}{4\times 3}\\ &= \frac{8}{12}+\frac{9}{12}\\ &= \frac{17}{12}\end{aligned}$$

Recall equivalent fractions!

Example 6

$$\begin{aligned}\frac{3x}{x^2-4}-\frac{x}{x-2} &= \frac{3x}{(x-2)(x+2)}-\frac{x\,(x+2)}{(x-2)\,(x+2)}\\ &= \frac{3x-x(x+2)}{(x-2)(x+2)}\\ &= \frac{3x-x^2-2x}{(x-2)(x+2)}\\ &= \frac{-x^2+x}{(x-2)(x+2)}\end{aligned}$$

Factoring gives $\frac{x(-x+1)}{(x-2)(x+2)}$, but we cannot simplify further!

Exercise 8.4

Discussion

1. Before combining numerators when adding or subtracting fractions, what must you do to the denominators?

2. Add or subtract as indicated.

(*a*) $\frac{5}{7}-\frac{3}{7}$ (*d*) $\frac{4}{m}+\frac{7}{m}$ (*g*) $\frac{a}{x+1}-\frac{b}{x+1}$

(*b*) $\frac{5}{2}-\frac{1}{2}$ (*e*) $\frac{4k}{9}-\frac{k}{9}$ (*h*) $\frac{3m}{x-y}-\frac{2k}{x-y}$

(*c*) $\frac{3y}{8}+\frac{2y}{8}$ (*f*) $\frac{2a}{b}+\frac{c}{b}$ (*i*) $\frac{2a}{a+b}+\frac{2b}{a+b}$

Remember to simplify your answer by dividing out common factors.

Written Solutions

3. Add or subtract as indicated.

(*a*) $\frac{7}{x-y}+\frac{5}{x-y}$ * (*c*) $\frac{y-2}{y+2}-\frac{y-3}{y+2}$

(*b*) $\frac{2x}{a+b}+\frac{5x}{a+b}$ (*d*) $\frac{b^2}{b-5}-\frac{25}{b-5}$

Careful! *
(c) is a "GOTCHA"!

4. Add or subtract as indicated.

(*a*) $\frac{3m}{5}+\frac{m}{2}$ (*d*) $\frac{6}{a+2}+\frac{3}{a+3}$ (*g*) $\frac{y-2}{y-4}-\frac{5}{3y}$

(*b*) $\frac{4}{x}-\frac{5}{y}$ (*e*) $\frac{8}{p-6}-\frac{2}{p+6}$ (*h*) $\frac{9}{k+3}-\frac{4}{k-3}$

(*c*) $\frac{3x}{5y}+\frac{2y}{3x}$ (*f*) $\frac{x}{x-4}+\frac{x+1}{x-5}$ (*i*) $\frac{3}{a-1}-\frac{4}{a}$

Watch those "−" signs!

5. Add or subtract as indicated.

(*a*) $\frac{6}{5(x-1)}-\frac{4}{3(x-1)}$

(*b*) $\frac{3x}{4(x+3)}-\frac{2x}{5(x-2)}$

(*c*) $\frac{1}{a+b}-\frac{1}{a-b}+\frac{2a}{a^2-b^2}$

(*d*) $\frac{1}{2(k-m)}-\frac{1}{2(k+m)}-\frac{m}{k^2-m^2}$

(*e*) $\frac{x}{x^2+5x+6}+\frac{15}{x^2+9x+14}-\frac{12}{x^2+10x+21}$

3(a) $\frac{12}{x-y}$ (c) $\frac{1}{y+2}$

4(a) $\frac{11m}{10}$

(c) $\frac{9x^2+10y^2}{15xy}$

(e) $\frac{6p+60}{(p-6)(p+6)}$

(f) $\frac{2x^2-8x-4}{(x-4)(x-5)}$

(h) $\frac{5k-39}{(k+3)(k-3)}$

(j) $\frac{5}{(y+2)(y+3)}$

5(a) $\frac{-2}{15(x-1)}$

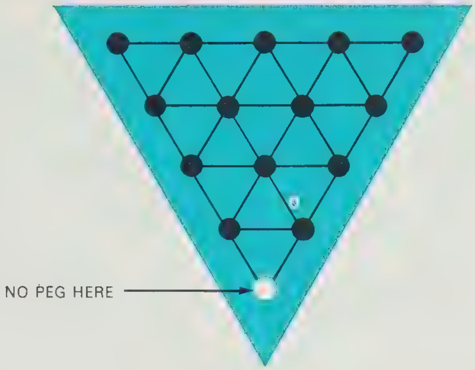
(b) $\frac{7x^2-54x}{20(x+3)(x-2)}$

(c) $\frac{2}{a+b}$

(d) 0

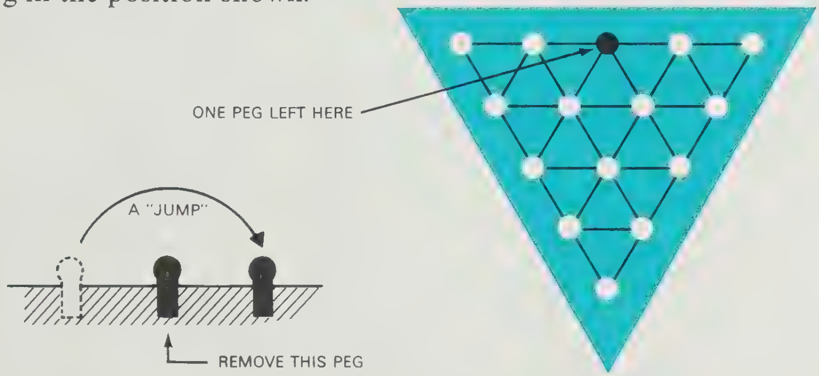
The Tricky Triangle

Make a pegboard in this shape. Set up the board with 14 pegs as shown.

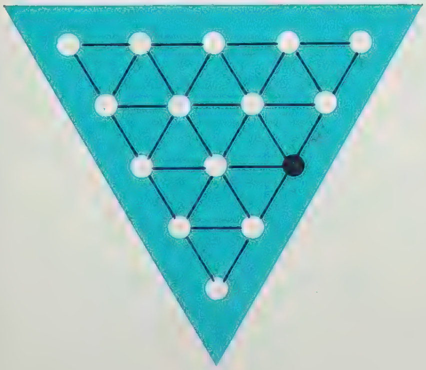


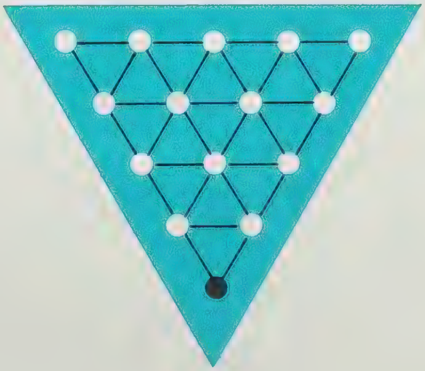
You can use nails for pegs!

The game: By jumping a peg, and removing it, end up with one peg in the position shown.


ONE PEG LEFT HERE
A "JUMP"
REMOVE THIS PEG

For a variation, start with the original board and attempt to arrive at one of the following situations.





These are much tougher!

Maintaining Basic Skills

Exercise 2

1. State as rational or irrational numbers.
   (*a*) 6.35 (*c*) 0.42 42 42. . . (*e*) −4.875 000 0. . .
   (*b*) $4.7\dot{1}\dot{3}$ (*d*) 17 (*f*) 4.03 003 0003 00003. . .

2. Express as fractions in lowest terms.
   (*a*) 0.35 (*c*) 0.125 (*e*) 4.3333 . . .
   (*b*) 1.5 (*d*) 1.25 (*f*) $7.\dot{2}\dot{1}$

3. Evaluate.
   (*a*) $5.23 - 1.87$ (*d*) $(0.2)^2$ (*g*) $4.1 + 3 \times 0.5$
   (*b*) 7.2×0.8 (*e*) $\frac{1}{4} - \frac{3}{8}$ (*h*) $0.375 - \frac{3}{8}$
   (*c*) $\frac{3}{7} \times \frac{21}{9}$ (*f*) $1\frac{1}{2} + 4\frac{1}{4}$ (*i*) $\frac{4}{5} \div \frac{2}{15}$

4. Evaluate.
   (*a*) $\frac{2}{3} + \frac{5}{6} - \frac{1}{4}$ (*c*) $\frac{4}{5} \div 1\frac{1}{3} + 1\frac{2}{5}$
   (*b*) $4\frac{1}{2} - 1\frac{2}{5}$ (*d*) $\frac{1}{2}$ of $1\frac{3}{4} \div \frac{5}{8}$

5. Copy and complete the table.

Fraction	*Decimal*	*Percent*
$\frac{2}{5}$	🟩	🟩
🟩	0.7	🟩
🟩	🟩	15
$\frac{5}{6}$	🟩	🟩
🟩	$1.\dot{6}$	🟩
🟩	🟩	16.25

6. Express as entire radicals.
   (*a*) $5\sqrt{2}$ (*c*) $7\sqrt{2}$ (*e*) $x\sqrt{5}$
   (*b*) $2\sqrt{3}$ (*d*) $4\sqrt{5}$ (*f*) $a\sqrt{b}$

7. Convert to mixed radicals.
   (*a*) $\sqrt{12}$ (*c*) $\sqrt{80}$ (*e*) $\sqrt{72}$
   (*b*) $\sqrt{48}$ (*d*) $\sqrt{18}$ (*f*) $\sqrt{288}$

8. Simplify.
   (*a*) $4\sqrt{2} - 2\sqrt{2} + 7\sqrt{2}$ (*c*) $2\sqrt{18} - 3\sqrt{50}$
   (*b*) $\sqrt{48} - \sqrt{12} + \sqrt{27}$ (*d*) $\sqrt{180} - 7\sqrt{20}$

9. Evaluate correct to 3 digits, using the Formal Method.
   (*a*) $\sqrt{18.3564}$ (*c*) $\sqrt{0.412\ 73}$ (*e*) $\sqrt{12\ 500}$
   (*b*) $\sqrt{295.5}$ (*d*) $\sqrt{3}$ (*f*) $\sqrt{7}$

1(a) rational	**(c) rational**
2(a) $\frac{7}{20}$	**(c)** $\frac{1}{8}$
3(b) 5.76	**(e)** $-\frac{1}{8}$
4(a) $1\frac{1}{4}$	**(c) 2**
6(c) $\sqrt{98}$	**(d)** $\sqrt{80}$
7(a) $2\sqrt{3}$	**(b)** $4\sqrt{3}$
8(a) $9\sqrt{2}$	**(c)** $-9\sqrt{2}$
9(b) 17.2	**(d) 1.73**

10. Collect like terms.

(*a*) $3a + 2a + 4b - b$
(*b*) $6x - 7x - 2y + y$
(*c*) $4m - 2 + 6 - 3m$
(*d*) $x^2 + 2x + 3x^2 + 4x$
(*e*) $3h^2 - 2t^2 - 6 + 4t^2 + 2$
(*f*) $-5x + 1 - 3x^2 + 10x - 4$

11. Find the product or quotient.

(*a*) $(5x)(3x)$
(*b*) $-12y^2 \div (3y)$
(*c*) $6a^2 \times (-2a)$
(*d*) $\frac{3}{4}a \div \frac{3}{5}b$
(*e*) $-5t^2 \times (-2t)$
(*f*) $-15m^3 \div \frac{1}{5}m^2$

12. Multiply or divide.

(*a*) $2(a + b)$
(*b*) $(6a - 9b) \div 3$
(*c*) $4(x^2 + x - 1)$
(*d*) $(6x^2 - 4x) \div 2x$
(*e*) $-2a(3a^2 - 4a + 5)$
(*f*) $5ab(a^2 - b^2)$

13. Simplify.

(*a*) $3(a - b) - 2(a + b)$
(*b*) $x(x + y) - y(x - y)$
(*c*) $2(x^2 - 2x) - 3(x^2 + x - 4)$
(*d*) $3t(2t - 1) + 4t(3t + 2)$

14. Expand.

(*a*) $(2a - b)(2a + b)$
(*b*) $(5x + 4y)(2x - 3y)$
(*c*) $(5n + 7p)^2$
(*d*) $(3y - 4z)(5y + z)$
(*e*) $(5s - 3t)(5s + 3t)$
(*f*) $(2x - 8y)^2$

15. Simplify.

(*a*) $(x + 2)(x + 3) + (x + 4)^2$
(*b*) $(h - 3)^2 - 2(h + 2)^2$
(*c*) $2(3b - 4c)(5b - c) + (b - c)^2$
(*d*) $(3m + 5n)(m - 2n) - (m + n)^2$

16. Factor.

(*a*) $5a + 15b - 10c$
(*b*) $x^2 - 2xy + 5x$
(*c*) $16a^2 - 24ab$
(*d*) $(x - 2)x + (x - 2)5$

17. Factor.

(*a*) $9x^2 - 4y^2$
(*b*) $2t^2 - 72$
(*c*) $4h^2 + 28h + 49$
(*d*) $16b^2 - 48bc + 36c^2$

18. Divide.

(*a*) $\dfrac{a^2 + 6a + 5}{a^2 + 2a + 1}$
(*b*) $(x^3 - 5x^2 + x - 1) \div (x - 1)$

19. Simplify.

(*a*) $\dfrac{5a}{2b} + \dfrac{2b}{5a}$
(*b*) $\dfrac{m}{m - 3} - \dfrac{m}{m + 2}$

10(b) $-x - y$ **(e)** $3h^2 + 2t^2 - 4$
11(a) $15x^2$ **(c)** $-12a^3$
12(d) $3x - 2$
(e) $-6a^3 + 8a^2 - 10a$
13(b) $x^2 - y^2$
14(b) $10x^2 - 7xy - 12y^2$
(e) $25s^2 - 9t^2$
15(a) $2x^2 + 13x + 22$
(c) $31b^2 - 48bc + 9c^2$
16(b) $x(x - 2y + 5)$
(d) $(x - 2)(x + 5)$
17(a) $(3x - 2y)(3x + 2y)$
(b) $2(t - 6)(t + 6)$
18(a) $\dfrac{a + 5}{a + 1}$
19(a) $\dfrac{25a^2 + 4b^2}{10ab}$
(b) $\dfrac{5m}{(m - 3)(m + 2)}$

Summary of Units 1 to 8

	Do you understand . . . ?	*Can you . . . ?*
Unit 1 **Computation**		
1.1 Accuracy	the difference between exact numbers and approximations significant digits	add, subtract, multiply, and divide rational numbers *accurately* round off a number to a specified accuracy
1.2 Estimating Answers	estimating answers	find a close approximation to a product or quotient locate the decimal correctly in a given product or quotient
Unit 2 **Operating with Power Terms**		
2.1 Multiplying and Dividing Powers	base, exponent, power	multiply powers divide powers
2.2 Powers of Products and Quotients	the power of a power, a product, or a quotient	expand a power term simplify a power of a product or quotient
2.3 Powers with Zero and Negative Exponents	the meaning of a zero exponent and a negative exponent	simplify expressions containing terms with zero and negative exponents
2.4 Numbers in Standard Form	the standard form of a number	express numbers in standard form estimate the value of expressions using standard form
2.5 Calculating with Numbers in Standard Form		calculate using numbers in standard form

	Do you understand . . . ?	*Can you . . . ?*
Unit 3 **Statistics**		
3.1 Representing Data	an array of data frequency distribution range of a set of data vertical and horizontal bar graphs	find the range of a set of data organize data into a frequency distribution and represent it by a graph
3.2 Mode, Median, and Mean	mode, median, and arithmetic mean of a set of numbers weighted average	calculate the mode, median, and mean of a set of data
3.3 Statistics Illustrated by Graphs	comparison bar graphs	construct bar graphs and comparison bar graphs from data interpret a graph
3.4 Standard Deviation	range of a set of data average deviation from the mean standard deviation	determine the range, average deviation, and standard deviation of a set of data
Unit 4 **The Number System**		
4.1 Review of the Rational Numbers	the number system	classify numbers according to their sets
4.2 Fractional Numbers and Periodic Decimals	period and length of period abbreviated form of a periodic decimal fractional equivalent of a decimal, in lowest terms non-periodic decimal	convert a fractional number to a periodic decimal and state the period and the length convert a periodic decimal to its fractional equivalent
4.3 Working with Rational Numbers	common denominator reciprocal substitution BODMAS	add, subtract, multiply, and divide rational numbers evaluate by substitution
4.4 Percent	percent equivalent ratios	convert fractions and decimals to percents convert percents to fractions and decimals

	Do you understand . . . ?	*Can you . . . ?*
Unit 5 **Operating with Square Root**		
5.1 Multiplying and Dividing Radicals	perfect square, square root, radical sign, radical	list perfect squares to 625 multiply and divide radicals
5.2 Adding and Subtracting Radicals	mixed and entire radicals like radicals	change a mixed radical to an entire radical change an entire radical to a mixed radical add and subtract like radicals
5.3 Approximating a Square Root	table of square roots	use the table of square roots, for both squaring and finding square root
5.4 Other Methods of Finding Square Root	Newton's Method for finding square root trial divisor Formal Method of finding square root	find the square root of a number using either Newton's Method or the Formal Method
5.5 Square Root Problems		
Unit 6 **Introduction to Variable Expressions**		
6.1 Review of Operations with Variable Expressions	terms, like terms, monomial, binomial, trinomial, polynomial, coefficients	add, subtract, multiply, and divide terms
6.2 Multiplying and Factoring	factors of an expression expanding highest common factor (HCF)	multiply an expression by a term find the HCF of an expression

	Do you understand . . . ?	*Can you . . . ?*
Unit 7 **Multiplying Expressions**		
7.1 Multiplying Binomials	FOIL	use FOIL to multiply 2 binomials
7.2 Factors of Variable Expressions	Common Factor Difference of Squares Simple Trinomial	recognize a term that is a perfect square identify the type of factoring that is required
7.3 Trinomial Factoring	Square of a Binomial Trial Trinomial	write all the possible factors of a number in a trial trinomial check factoring by multiplying
Unit 8 **Further Operations with Expressions**		
8.1 Dividing Monomials	dividend, divisor, quotient	divide monomials
8.2 Dividing Polynomials by Monomials		use factoring to divide expressions
8.3 Dividing a Polynomial by a Binomial	remainder division statement	do “long division” put the terms of the dividend in order divide a polynomial by a binomial determine whether a binomial is a factor of a polynomial
8.4 Adding and Subtracting Algebraic Fractions	algebraic fractions	find a common denominator for algebraic fractions add and subtract algebraic fractions

Review Exercise, Units 1 to 8

1. Perform the operations indicated.
   - (*a*) $(1\frac{1}{2}+\frac{3}{4})\div\frac{3}{8}$
   - (*b*) $5.73-8.94$
   - (*c*) $643.1+11.2+191.13$
   - (*d*) 68% of 93
   - (*e*) 123.2×16.1
   - (*f*) $122.22\div 6.3$
   - (*g*) $4+9\div 3-6\times 2$
   - (*h*) $12\frac{1}{2}-2\frac{3}{4}+3\frac{5}{8}$
   - (*i*) $\dfrac{6.4\times 5.2}{0.26\times 19.2}$

2. If $a=1$, $b=-2$, and $c=3$, evaluate.
   - (*a*) $2a-3b+c$
   - (*b*) a^2-b^2
   - (*c*) $(a-b)\div c$
   - (*d*) b^{ac}

3. Each of the measured lengths that follow has been rounded off. For each length, state the largest and smallest possible values.
   - (*a*) 17.3 cm
   - (*b*) 42 km
   - (*c*) 0.64 cm
   - (*d*) 0.002 m

4. Express in standard form and estimate an answer. Check by evaluating to 3 significant digits.
   - (*a*) 7730×452
   - (*b*) $\dfrac{28.6\times 73\,200}{5280}$
   - (*c*) $38\,600\div 0.008\,92$
   - (*d*) $\dfrac{598\times 0.683}{22\,500\times 0.0731}$

5. What is the length of the longest straight line that can be drawn on a rectangular sheet of newspaper measuring 59 cm by 84 cm? (Answer to 1 decimal place.)

6. How long is the side of a square card table whose area is 2020 cm² (nearest whole number)?

7. Find the total cost.
   - 5 kg nails at \$2.65/kg
   - 4 hinges at \$1.49 each
   - 2 handles at \$2.29 each
   - 4 m metal trim at \$1.35/m
   - 6 m² insulation at \$1.42/m²

8. How many kilolitres of water are required to fill a trough 8.3 m long, 2.1 m high, and 4.6 m wide?

9. Calculate the area (to 1 decimal place) of a record 30 cm in diameter. (Use $\pi \doteq 3.14$.)

1(a) 6 (g) −5
(c) 845.43 (i) $6.\dot{6}$
(e) 1983.52

2(a) 11 (c) 1

3(a) 17.34 cm and 17.25 cm

4(a) 3.49×10^6
(c) 4.33×10^6

5. 102.6 cm

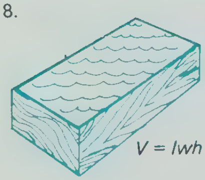
6. 45 cm

7. \$37.71

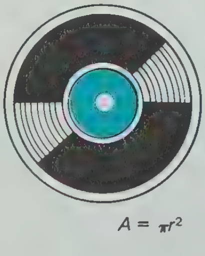
8. 80.178 kl

9. 706.5 cm²

8.



9.



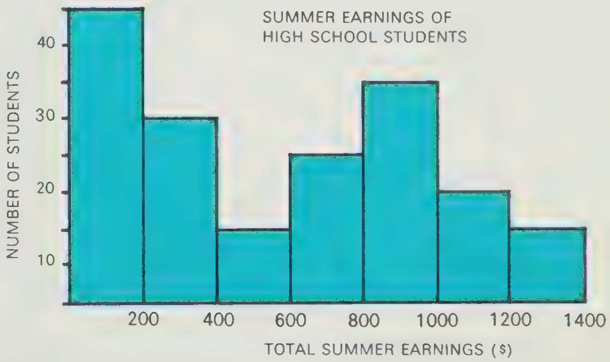
10. Determine (to the nearest whole number) the surface area of a soccer ball 28 cm in diameter. ($A = 4\pi r^2$ and $\pi \doteq 3.14$.)

11. Simplify the following expressions.
    (*a*) 3^0 (*c*) $\left(\frac{2}{3}\right)^2$ (*e*) $\left(\frac{1}{4}\right)^{-2}$
    (*b*) 5^{-1} (*d*) $(-3)^3$ (*f*) $\left(-\frac{3}{5}\right)^{-1}$

12. Simplify by removing brackets.
    (*a*) $(xy)^3$ (*d*) $-16xy \div (8x^2)$
    (*b*) $(3x^2)(-5x)$ (*e*) $15s^2t^3 \div (-5st^2)$
    (*c*) $\left(\frac{a}{b}\right)^2$ (*f*) $(x^2yz^3)^5$

13. Find the range, median, mean and average deviation of the following sets of data.
    (*a*) 110 kg, 92 kg, 108 kg, 120 kg, 97 kg, 130 kg, 103 kg, 117 kg, 102 kg, 109 kg
    (*b*) 15.2 years, 16.1 years, 16.4 years, 15.1 years, 15.8 years, 17.0 years, 15.9 years, 16.2 years

14. Using the graph, answer the following questions.
    (*a*) How many students earned between \$600 and \$800? \$200 or less?
    (*b*) Find the number of students in the survey.
    (*c*) Using the midpoint of each interval as the average for that group, find the weighted average of the summer earnings.



15. Construct a horizontal bar graph to illustrate the following data.

Height (cm)	*Men*	*Women*
160	0	15
165	2	30
170	40	90
175	100	125
180	150	80
185	120	20
190	75	5
195	10	0

10. 2462 cm^2
12(b) $-15x^3$ (e) $-3st$
13(a) 92–130 kg, 108.5 kg, 108.8 kg, 8.4 kg

16. Express as fractions in lowest terms.
(*a*) 0.07
(*b*) $1.\dot{3}$
(*c*) 3.15
(*d*) $0.\dot{2}$
(*e*) 5.45
(*f*) $0.\dot{3}\dot{1}$

17. Find the decimal equivalent of each of the following fractional numbers and state the period and the length.
(*a*) $\frac{1}{2}$
(*b*) $\frac{3}{4}$
(*c*) $\frac{4}{9}$
(*d*) $\frac{3}{7}$
(*e*) $\frac{7}{11}$
(*f*) $\frac{2}{13}$

18. How many skirts, each requiring 1.2 m of material, can be made from a bolt of cloth 11 m long? How long is the remnant of cloth left over?

19. A roll of wallpaper is 7 m long and its pattern repeats every 40 cm. How many "repeats" of the pattern occur in the roll? Why is it necessary to allow more for waste with this paper than one in which the repeat occurs at 6 cm intervals?

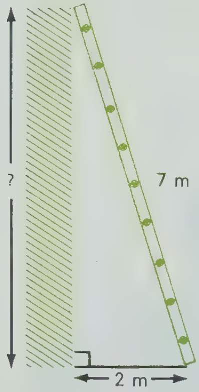
20. After 6 weeks on a muscle-building program, Sam Strain's biceps measurement had increased from 30 cm to 33 cm. State this as a percent increase.

21. Simplify.
(*a*) $\sqrt{7} \times \sqrt{5}$
(*b*) $\sqrt{18} \div \sqrt{2}$
(*c*) $2\sqrt{5} + 7\sqrt{5} - \sqrt{5}$
(*d*) $\sqrt{3} \times \sqrt{27}$
(*e*) $2\sqrt{7} + \sqrt{63} - 3\sqrt{28}$
(*f*) $\frac{1}{2}\sqrt{80} + \frac{2}{5}\sqrt{125}$

22. Evaluate, using square root tables.
(*a*) $\sqrt{74}$ (*b*) $\sqrt{260\,000}$ (*c*) $\sqrt{0.39}$ (*d*) $\sqrt{0.0013}$

23. How far up a vertical wall will a 7 m ladder reach if the foot of the ladder is 2 m from the wall? (Correct to 2 decimal places.)

23.



24. An object is accelerated from an initial velocity (u) to a final velocity (v) in a given distance (d) by an acceleration (a) according to the formula $v = \sqrt{u^2 + 2ad}$. Determine the final velocity (v) if $u = 20$ m/s, $a = 9.8$ m/s^2, and $d = 90$ m.

25. Evaluate by the Formal Method, to the accuracy required.
(*a*) $\sqrt{5.3}$ (1 decimal)
(*b*) $\sqrt{275}$ (1 decimal)
(*c*) $\sqrt{0.0052}$ (3 decimals)
(*d*) $\sqrt{942.7}$ (1 decimal)

16(b) $\frac{4}{3}$ **(e)** $\frac{109}{20}$

17(a) **0.5** **(e)** $0.\dot{6}\dot{3}$
(c) $0.\dot{4}$

18. 9 skirts
Remnant = 0.2 m

20. 10% increase

21(b) 3 **(e)** $-\sqrt{7}$

22(a) 8.6 **(c) 0.624**

23. 6.71 m

24. $v = 46.5$ **m/s**

25(b) 16.6 **(d) 30.7**

26. A telephone cable is to be strung from cottage A to cottage C, as shown. Using the diagram, calculate the length AC, correct to 1 decimal place.

27. Simplify.

(*a*) $3x + 5y - 2x - 2y$ (*c*) $5x^2 - 3x - x^2 + 4x$
(*b*) $2(x + 3) - 3(x - 2)$ (*d*) $a(a - 2) + 3(a + 4)$

28. Expand.

(*a*) $(x + 2)(x + 3)$ (*d*) $(2m - 3n + p)(-2mnp)$
(*b*) $3a(a^2 - 2a - 4)$ (*e*) $(5x - 3y)^2$
(*c*) $(5b - 1)(4b + 3)$ (*f*) $(2t - 3)(t^2 + t - 5)$

29. Find the quotient.

(*a*) $(4x + 8y - 12) \div (-4)$ (*d*) $(a^2 + 5a + 6) \div (a + 2)$
(*b*) $(x + 4)(x - 3) \div (x - 3)$ (*e*) $(b^2 - 2b - 1) \div (b - 1)$
(*c*) $15xyz \div (-3y)$ (*f*) $(r^3 + 3r^2 - 5r + 1) \div (r - 1)$

30. Factor each expression.

(*a*) $y^2 - 8y + 12$ (*c*) $9g^2 - h^2$ (*e*) $t^2 - 3t - 18$
(*b*) $2a + 4b - 6c$ (*d*) $x^2 + 4x + 4$ (*f*) $x^2 + xy - 5x$

31. Use factoring to simplify.

(*a*) $\dfrac{x^2 - 7x + 10}{x^2 - 4}$ (*c*) $\dfrac{x^2 - 9y^2}{x^2 + xy - 12y^2}$
(*b*) $\dfrac{x^2 - 8x + 16}{x^2 - 4x}$ (*d*) $\dfrac{9x^2 - 12x + 4}{3x^2 - 2x}$

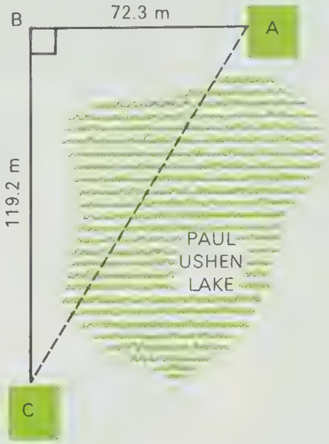
32. Simplify by expanding and adding like terms.

(*a*) $(5a + b)^2 + (3a - b)^2$ (*c*) $(3x - 2)(5x + 1) + (2x + 1)$
(*b*) $-2(3y + 4)(5y - 2)$ (*d*) $12(3x - y)^2 - 3(x + y)^2$

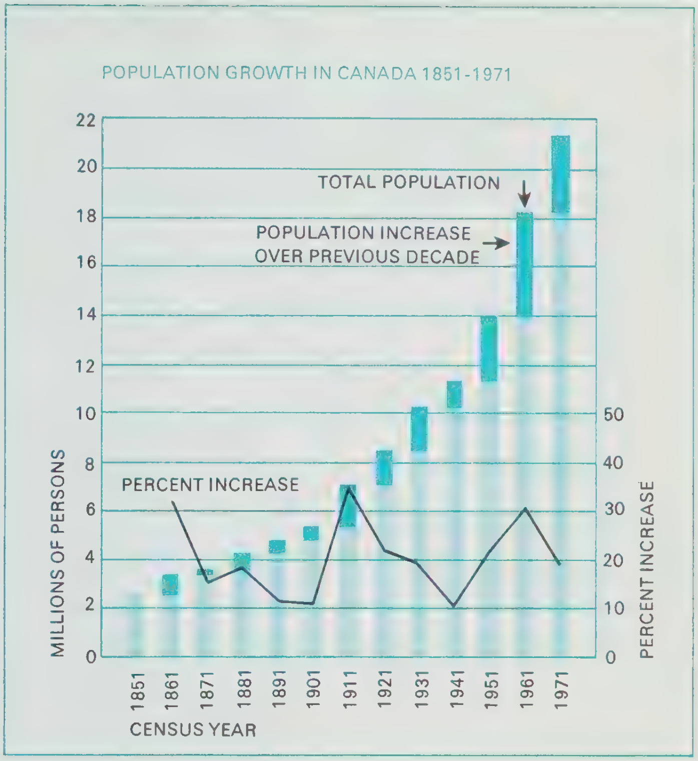
33. Simplify.

(*a*) $\dfrac{3}{x} + \dfrac{5}{x}$ (*d*) $\dfrac{1}{2x} - \dfrac{1}{3y}$
(*b*) $\dfrac{2a}{3b} + \dfrac{7a}{3b}$ (*e*) $\dfrac{3y + 2}{y + 2} - \dfrac{y - 2}{y + 2}$
(*c*) $\dfrac{5}{m} + \dfrac{2}{k}$ (*f*) $\dfrac{2a}{a + 1} + \dfrac{5a}{a + 3}$

26.



26. 139.4 m
27(b) $12 - x$ **(d)** $a^2 + a + 12$
28(c) $20b^2 + 11b - 3$
(f) $2t^3 - t^2 - 13t + 15$
29(a) $-x - 2y + 3$ **(d)** $a + 3$
30(c) $(3g - h)(3g + h)$
(f) $x(x + y - 5)$
31(a) $\dfrac{x - 5}{x + 2}$ **(c)** $\dfrac{x + 3y}{x + 4y}$
32(a) $34a^2 + 4ab + 2b^2$
(c) $15x^2 - 5x - 1$
33(a) $\dfrac{8}{x}$ **(e)** 2
(c) $\dfrac{5k + 2m}{km}$



Unit 9
Relations and Their Graphs

Today the price of an article often depends on its supply: the scarcer a commodity is, the more it is likely to cost. A good example of this relationship is money obtained through a mortgage or a loan. If money is scarce, the interest rate (the cost of borrowing) rises. There is a direct relationship between supply and demand.

There is also a direct relationship between the number of litres consumed per 100 km in driving a car, and the average speed; between wages, and experience and qualifications; between the incidence of traffic accidents in a city, and the number of street intersections; and so on.

Many practical problems can be solved by studying these relationships. Sometimes we picture the relationship in the form of a graph and sometimes we list it using a table of values. Frequently in our everyday conversation we state a relationship in a general way using a sentence. In this unit you will study the various ways of looking at relationships.

9.1 Graphing Ordered Pairs

A Here are 2 examples of relationships expressed using tables of values.

Time (h)	*Weekly Wages*
30	$120
35	140
40	160
45	270
50	300

Height (cm)	*Mass* (kg)
145	44
150	46
155	48
160	50
165	52

Notice that the wages increase more between 40 h and 50 h than between 30 h and 40 h. Why?

The table shows that 30 h of work produces earnings of $120. To earn $160, a 40 h week is required. The numbers go together in pairs (30, 120), (35, 140), . . . , (50, 300). These are *ordered* pairs of the form (time, weekly wages); the time is called the *first component* and weekly wages is the *second component*. In the example, the ordered pair (40, 160) means something quite different from (160, 40) since we are writing the time component first.

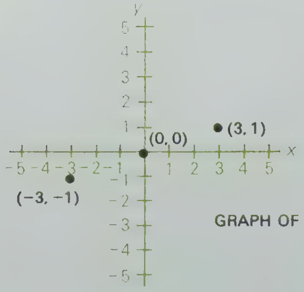
(Time, wages)

(40, 160) means that $160 wages are paid for a 40 h week. (160, 40) means that...

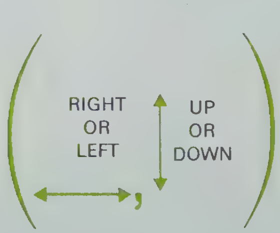
Referring to the *Height–Mass* table, answer the following questions.

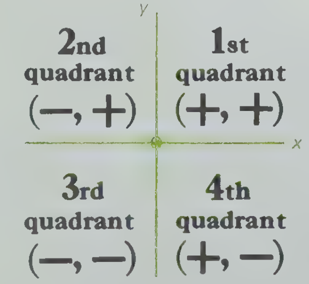
1. Write all the ordered pairs.
2. List the set of (*a*) first components, (*b*) second components.
3. What does (155, 48) mean? What would (48, 155) mean?

B Recall from your previous work in mathematics that ordered pairs can be graphed using 2 number lines or axes.



GRAPH OF {(−3, −1), (0, 0), (3, 1)}

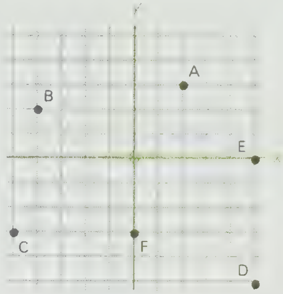




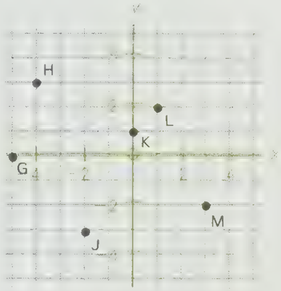
The 2 number lines are drawn in vertical and horizontal position, as shown. These lines intersect at right angles.

1. The axes meet at the *orgin.* State the coordinates of the origin.
2. How far is (3, 1) from the y axis? From the x axis?
3. On which side of the y axis would $(-2, 5)$ lie?
4. Where would $(4, -2)$ lie with respect to the x axis?
5. State the coordinates of the points A, B, C, D, E, F in the diagram.
6. Name the points in the diagram whose coordinates are $(1, 2)$, $(-4, 3)$, $(3, -2)$, $(0, 1)$, $(-2, -3)$, $(-5, 0)$.

5.



6.

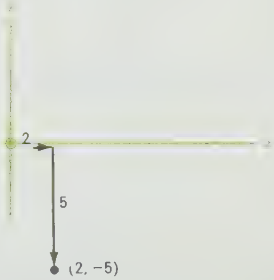


Exercise 9.1

Discussion

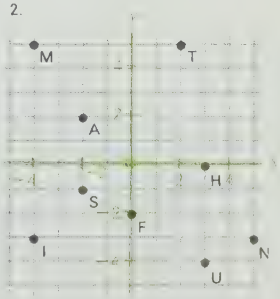
1. Starting from the origin, $(2, -5)$ is 2 to the right and 5 down. In the same way, locate $(3, 8)$, $(-5, 4)$, $(0, 5)$, $(-2, 0)$, $(-3, -5)$, $(-5, 10)$, naming the quadrant where possible.

1.



2. State the coordinates of the points in the diagram.

2.



3. State the coordinates of the origin.
4. (*a*) What kind of integer is the x coordinate of every point in the 1st and 4th quadrants?
   (*b*) In which quadrants are the y coordinates of all points negative?
5. If a point has one coordinate that is 0, what can you tell about the position of that point?
6. List three ordered pairs satisfying each condition.
   (*a*) The second component is twice the first component.
   (*b*) The second component is 5 greater than the first.
   (*c*) The second component is the square of the first.

Written Solutions

7. Graph each set of points on a separate set of axes.
   (*a*) $\{(5, 0), (0, 4), (-3, 0), (0, -1)\}$
   (*b*) $\{(-4, -2), (-3, -1), (-2, 0), (-1, 1), (0, 2), (1, 3)\}$
   (*c*) $\{(-2, 9), (-1, 7), (0, 5), (1, 3), (2, 1), (3, -1), (4, -3)\}$
   (*d*) $\{(4, 5), (4, -3), (-2, 5), (-2, -3)\}$
   (*e*) $\{(4, 6), (2, -1), (-1, 2), (-4, -6)\}$

8. Write a set of ordered pairs for each table of values and then graph the ordered pairs of the set.

(*a*)

Distance (km)	*Time* (h)
60	1
90	$1\frac{1}{2}$
120	2
150	$2\frac{1}{2}$
180	3
210	$3\frac{1}{2}$

(*c*)

Number	*Square of the Number*
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9

(*b*)

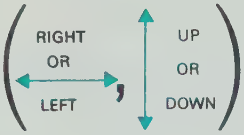
Number of Books	*Total Cost*
5	$20
10	40
15	55
20	70
25	85
30	100

(*d*)

x	$y = 2x + 1$
−3	−5
−2	−3
−1	−1
0	1
1	3
2	5

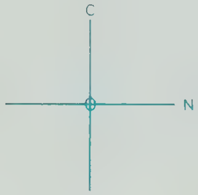
9. List a set of six ordered pairs in which the second component is twice the first component. Graph the set and note the pattern.

10. List a set of six ordered pairs in which the first component is the opposite (or the negative) of the second component. Graph the set and note the pattern.



8(a)

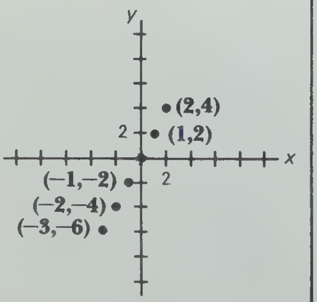
8(b)



8(c)

8(d)

9. (−3, −6), (−2, −4), (−1, −2), (0, 0), (1, 2), (2, 4)

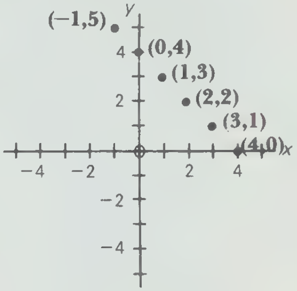


Pattern is a straight line.

11. List a set of six ordered pairs that justify each of the following conditions. Graph each set.
   (*a*) The sum of the components of each ordered pair is 4.
   (*b*) The sum of the components of each ordered pair is less than 3.

12. Determine a sequence of ordered pairs that will spell your name on graph paper, using straight line segments.

11(a) **(0, 4), (1, 3), (2, 2), (3, 1), (4, 0), (−1, 5)**



9.2 Graphing a Relation

A A relation in mathematics can be described in *words*: for example,

The diameter of a circle is twice its radius.

Statement!

This relation can be expressed by a *formula* or *defining sentence*,

$$d = 2r$$

Formula!

A relation can be written as a *set of ordered pairs*,

$$\{(1, 2), (2, 4), (3, 6), \ldots\}$$

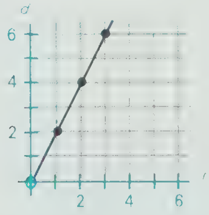
Ordered Pairs!

or as a table of values,

r	1	2	3	....
d	2	4	6	....

Table of Values!

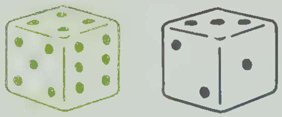
and these may be pictured in a *graph*.



Graph!

"a picture is worth a thousand words."

The nature of the problem involving the relation determines which form of it you work with. You often work with *formulas* in school, but an insurance agent will use a *table of values* to quote a premium on a policy, not a formula. Scientists frequently record results of an experiment on a *graph*. There is a visual advantage to a graph not possessed by the other ways of expressing a relation.



B An ordinary pair of dice can be used to illustrate the basic ideas about relations.

Example 1

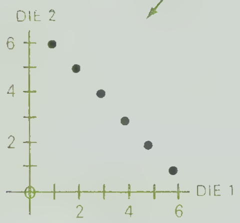
List the set of ordered pairs of the relation T satisfying the condition that the sum shown on a pair of dice is 7.

Solution

In our ordered pairs each component represents the number shown by one of the two dice.

$$\therefore\ T = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$$

Why is it meaningless to join these points?

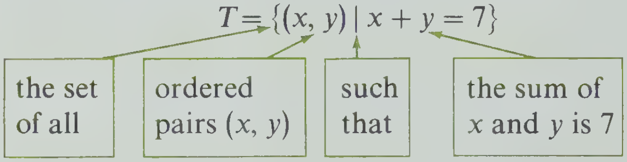


GRAPH OF T

If x represents the number on one die and y the number on the other, then T can be defined by the sentence "the sum is 7" or $x + y = 7$.

Sometimes you will write T as follows.

$$T = \{(x, y) \mid x + y = 7\}$$



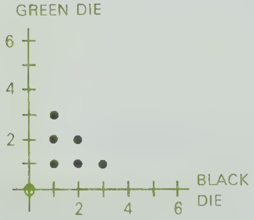
Example 2

A black die and a green die are rolled. List and graph the relation B in which the sum showing on the 2 dice is less than 5. State the defining sentence for B.

Solution

$$B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)\}$$

$\therefore$ The defining sentence is $x + y < 5$.



GRAPH OF B

1. Read the following.
   (*a*) $\{(x, y) \mid y = 2x, x \in I\}$
   (*b*) $\{(x, y) \mid x + y = 3, \underbrace{x, y \in I}_{*}\}$

*

$x, y \in I$ means that x and y are integers.

2. State the relation in words, and give a possible defining sentence for each of the following sets.
   (*a*) $\{(1, 1), (2, 2), (3, 3), \ldots\}$
   (*b*) $\{(1, 2), (-1, -2), (2, 4), (-2, -4), (0, 0), \ldots\}$

Exercise 9.2

Discussion

1. State five ways in which you can describe a relation.

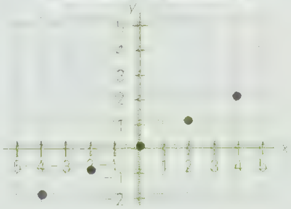
2. If x represents the first component and y the second component, state in words the relation defined by
   (*a*) $x = y$
   (*b*) $y = 2x$
   (*c*) $y = x - 3$
   (*d*) $x - y = 1$
   (*e*) $y = 2x + 5$
   (*f*) $x + y > 2$

3. (*a*) State the ordered pair for each point on the graph.
   (*b*) In words, what is the relationship between y (the second component) and x (the first component)?
   (*c*) State this relationship as a formula.

4. Read the following relations. x, y are integers.
   (*a*) $\{(x, y) \mid x + y = 5\}$
   (*b*) $\{(x, y) \mid y = 5x\}$
   (*c*) $\{(x, y) \mid x - y = 6\}$
   (*d*) $\{(x, y) \mid y = x + 2\}$

5. For each set of ordered pairs state in words how the second number is obtained from the first. Express the relation by a formula.
   (*a*) $\{(1, 3), (2, 6), (3, 9), \ldots\}$
   (*b*) $\{(10, 5), (6, 3), (4, 2), (2, 1), (0, 0), (-2, -1), \ldots\}$
   (*c*) $\{(-1, 0), (0, 1), (1, 2), (2, 3), \ldots\}$
   (*d*) $\{(0, 5), (1, 4), (2, 3), (-1, 6), (-2, 7), \ldots\}$

3.

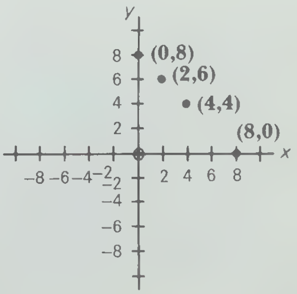


Written Solutions

6. Each of the following is a relation stated in words. Express each as a formula in x and y.
   (*a*) The second number is 5 times the first number.
   (*b*) The sum of the two components is 8.
   (*c*) The second number is 3 greater than the first number.
   (*d*) The second number is 1 less than the first number.

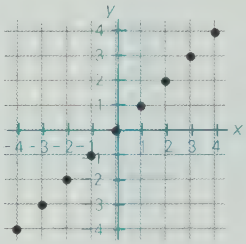
7. For each of the relations in Question 6 find four ordered pairs and show these on a graph.

6(a) $y = 5x$ **(c)** $y = x + 3$
7(b) (0, 8), (2, 6), (4, 4), (8, 0)

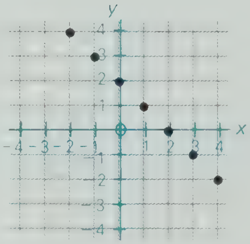


8. Each of the graphs below defines a relation. For each,
   (*i*) list the set of ordered pairs for the points of the graph,
   (*ii*) state in words a relationship between the x and y coordinates that is satisfied by each point, and
   (*iii*) write a formula in x and y that represents this relationship.

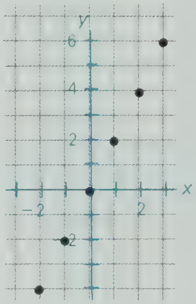
(*a*)



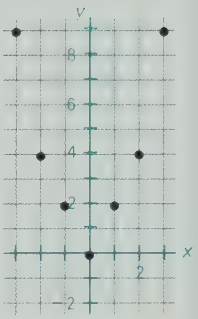
(*b*)



(*c*)



(*d*)

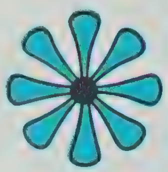


9. A red die and a green die are rolled.
   (*a*) Using ordered pairs list all possible outcomes.*
   (*b*) Graph the ordered pairs in (*a*).
   (*c*) Circle on the graph all points representing outcomes where
   (*i*) the sum is 3, (*ii*) the sum is 7, (*iii*) the sum is 9.
   Do you notice any pattern?

10. Draw the same graph as used in Question 9 to represent all the outcomes possible in throwing two dice.
   (*a*) Circle the points representing outcomes where the red die has the number 3 on it. Repeat for the number 5.
   (*b*) Circle the points representing outcomes in which the green die has a 2. Repeat for the number 4.

   What patterns do you notice?

11. Draw the 36-point graph for rolling a pair of dice.
   (*a*) Circle the points representing a sum of 6.
   (*b*) Mark with a cross all points representing a sum > 6.
   (*c*) Mark with a heavy dot all points representing a sum < 6.
   (*d*) State in words the position of the crosses; the position of the dots.



*Did you get all 36 of them?

8(a)

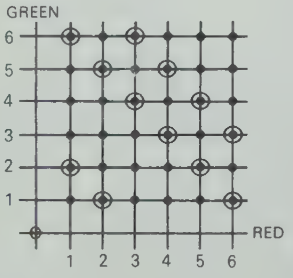
(i) $\{(-4, -4), (-3, -3), (-2, -2), (-1, -1), (0, 0), (1, 1), (2, 2), (3, 3), (4, 4)\}$

(ii) **The** x **and** y **coordinates are equal.**

(iii) $x = y$

9(a) $\{(1, 1), (1, 2), \ldots, (1, 6),$
$(2, 1), (2, 2), \ldots, (2, 6),$
.
.
.
$(6, 1), (6, 2), \ldots, (6, 6)\}$

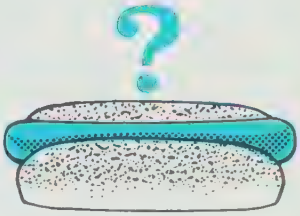
(b), (c)



9.3 The Possible Values of a Relation

A The speed of a car is limited by its mass, the power produced by the engine, and many other factors. The temperature of a sick person is restricted to values that normally range from 36°C to 41°C, and the reading depends on the nature of the illness.

1. State the range of masses that *you* are able to lift.
2. What is the range of permissible speeds for a car on the streets of a town or city?
3. List the prices paid in your community for hamburgers and hot dogs.
4. The power of a light bulb is quoted in watts. State the possible values for the power of bulbs normally in use in the home.
5. Select from Questions 1 to 4 the cases where
   (*a*) any numerical value in the range is possible, and
   (*b*) where only some specific values are possible.



27.4¢ for a hot dog?

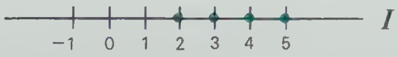
B The stage lights in an auditorium are usually controlled by a rheostat. What freedom does this give the stage crew? Compare this to the effect of a standard light switch in your home.

On a stove compare the effect of an "infinite heat" burner control to a switch that has the readings LOW, MEDIUM, HIGH on it.

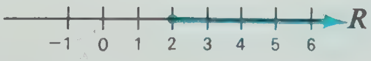
Can you think of other examples where on one hand there is a *continuous* set of values, while on the other a few specific or *discrete values* can be used?

You have met this situation in previous work in mathematics. Compare the two graphs below. Why is one graph a set of dots and the other a solid line?

Graph of $\{x \mid x \geq 2,\ x \in I\}$



Graph of $\{x \mid x \geq 2,\ x \in R\}$



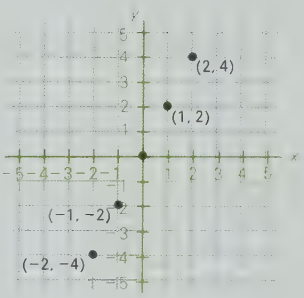
For $x \geq 2$, $x \in R$, we want all the real numbers, (that is, all possible decimal numbers) greater than or equal to 2.

In a similar way graphs using two axes can consist either of separate points or of solid lines or curves. Study Example 1 and do the questions following.

x	y
-2	-4
-1	-2
0	0
1	2
2	4

Example 1

Graph $B = \{(x, y) \mid y = 2x,\ x \in I\}$



Solution

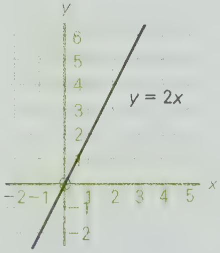
If $x = 1$, $\therefore\ y = 2x$
$= 2(1)$
$= 2 \qquad \therefore\ (1, 2) \in B$

If $x = 2$, $\therefore\ y = 2(2)$
$= 4 \qquad \therefore\ (2, 4) \in B$

Hence $B = \{(0, 0), (1, 2), (-1, -2), (2, 4), (-2, -4), \ldots\}$

If you allowed the values of x to include any real number, then x could be replaced by numbers like 1.5, 2.75, -3.3.

1. Draw the graph of Example 1.
2. Substitute values such as 1.5, -3.25 for x and find more points on the graph between the points already graphed.
3. If all real values of x—say between 1 and 2—are used, how would the graph appear between the points (1, 2) and (2, 4)?
4. Draw the graph of $\{(x, y) \mid y = 2x,\ x \in R\}$.



Example 2

Draw the graph of each relation. $x, y \in R$.

(*a*) $\{(x, y) \mid y = x + 1\}$
(*b*) $\{(x, y) \mid x = 4\}$
(*c*) $\{(x, y) \mid y = 2\}$

Solution

(*a*) $y = x + 1$

x	y
1	2
-1	0
0	1
1.3	2.3
$\frac{1}{2}$	$1\frac{1}{2}$

If $x = 1$, $\therefore y = x + 1$
$= 1 + 1$
$= 2$

$\therefore$ (1, 2) is in the graph. Similarly, we can determine other ordered pairs for our table of values.

(*b*) $x = 4$

x	y
4	-2
4	0
4	1
4	3.7
4	-1.8

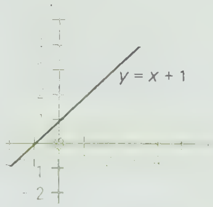
If $x = 4$ and y can be any real number, then $(4, -2)$, $(4, 0)$, $(4, 1)$, $(4, 3.7)$, $(4, -1.8)$, . . . are in the graph.

(*c*) $y = 2$

x	y
-5	2
0	2
$\frac{1}{4}$	2
-3.1	2
2	2

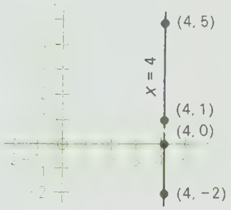
If $y = 2$ and x is any real number, then $(-5, 2)$, $(0, 2)$, $(\frac{1}{4}, 2)$, $(-3.1, 2)$, $(2, 2)$, . . . are in the graph.

(a)



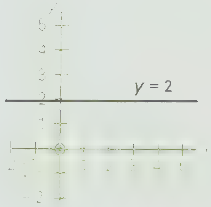
GRAPH OF $\{(x, y) \mid y = x + 1\}$

(b)



GRAPH OF $\{(x, y) \mid x = 4\}$

(c)



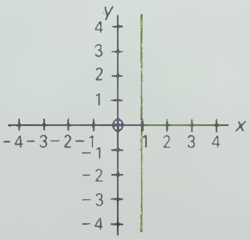
GRAPH OF $\{(x, y) \mid y = 2\}$

Exercise 9.3

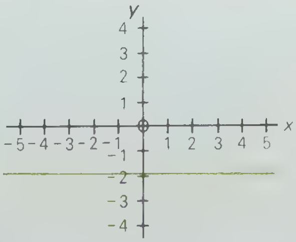
Discussion

1. State three ordered pairs from each of the following relations. $x, y \in R$.
   - (*a*) $\{(x, y) \mid y = 3x\}$
   - (*b*) $\{(x, y) \mid x + y = 5\}$
   - (*c*) $\{(x, y) \mid y = -1\}$
   - (*d*) $\{(x, y) \mid y = x + 2\}$
   - (*e*) $\{(x, y) \mid x - 3 = 0\}$
   - (*f*) $\{(x, y) \mid x - y = 2\}$
2. State the formula for each of the relations graphed below.

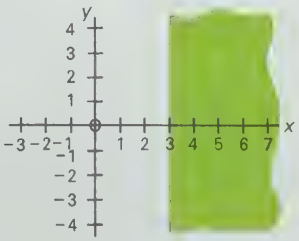
(*a*)



(*b*)



(*c*)



Written Solutions

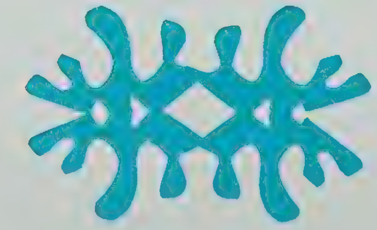
3. Find 6 ordered pairs for each of the following relations. $x, y \in R$.
   (a) $\{(x, y) \mid y = x\}$
   (b) $\{(x, y) \mid y = x + 3\}$
   (c) $\{(x, y) \mid x + y = 6\}$
   (d) $\{(x, y) \mid 2y = 3x\}$
   (e) $\{(x, y) \mid y = 2x - 3\}$
   (f) $\{(x, y) \mid x - y = 1\}$

4. (a) Graph the relation $A = \{(x, y) \mid y = 3x - 4,\ x, y \in I\}$
   (b) Graph $B = \{(x, y) \mid y = 3x - 4,\ x, y \in R\}$

5. Graph the relation defined by each formula. Use at least 5 ordered pairs in each case. $x, y \in R$.
   (a) $y = 2x$
   (b) $y = x + 3$
   (c) $y = x - 2$
   (d) $y = 3x + 1$
   (e) $y = 5 - 2x$
   (f) $x = 2y + 1$

6. The guarantee offered by the Heatright Waterheater Co. on its waterheaters gives 100% of selling price on a replacement during the first year, 90% during the second year, 80% during the third year, and so on until the guarantee runs out.
   (a) Construct a graph that shows the amount payable by the company on a waterheater that sold for \$180 on January 1, 1975 until the guarantee is worthless.
   (b) During 1981, if the waterheater must be replaced and the cost at that time is \$240 for a new heater, how much money must the owner pay?
   (c) When does the guarantee run out?

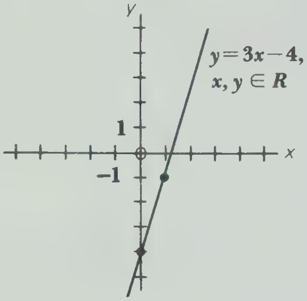
7. Write computer programs that will produce ordered pairs for the following relations.
   (a) $y = 3x - 1$ (b) $x - y = 7$ (c) $2x - 4y = 5$.



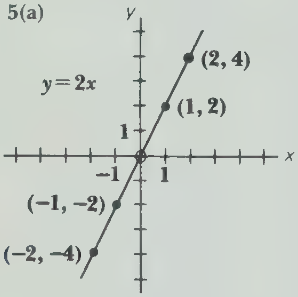
3(a) $\{(-1, -1), (-2, -2), (0, 0), (1, 1), (2, 2), (3, 3)\}$

(c) $\{(-3, 9), (-2, 8), (-1, 7), (0, 6), (1, 5), (2, 4)\}$

4(b)



5(a)



6(b) \$168
(c) December 31, 1984

Graphing Fun

1. On a single sheet of graph paper, center the origin to allow for $-16 \leqslant x \leqslant 16$ and $-18 \leqslant y \leqslant 18$. Now join consecutive points as follows:

   $(-5, -3)$, $(-2, -2)$, $(-2, 0)$, $(-3, 1)$, $(-3, 3)$, $(-2, 4)$,
   $(2, 4)$, $(3, 3)$, $(0, 3)$, $(2, 2)$, $(2, 3)$, $(5, 3)$, $(6, 2)$, $(6, 0)$, $(5, 1)$,
   $(3, 1)$, $(2, 0)$, $(2, -2)$, $(5, -3)$, $(7, -3)$, $(10, 0)$, $(7, 0)$, $(7, -2)$,
   $(10, -2)$, $(13, 1)$, $(6, 1)$, $(6, -3)$, $(11, -3)$, $(16, 2)$, $(6, 2)$,
   $(5, 3)$, $(5, 7)$, $(4, 8)$, $(3, 8)$, $(2, 7)$, $(2, 6)$, $(4, 6)$, $(3, 5)$, $(3, 3)$,
   $(5, 3)$, $(5, 13)$, $(4, 11)$, $(2, 9)$, $(1, 7)$, $(-1, 7)$, $(-2, 9)$, $(-4, 11)$,
   $(-5, 13)$, $(-4, 10)$, $(-1, 7)$, $(-1, 6)$, $(1, 6)$, $(1, 7)$, $(4, 10)$,
   $(5, 13)$, $(5, 15)$, $(4, 14)$, $(2, 13)$, $(1, 11)$, $(1, 9)$, $(-1, 9)$, $(-1, 11)$,
   $(-2, 13)$, $(-4, 14)$, $(-5, 15)$, $(-4, 14)$, $(-3, 12)$, $(-1, 11)$,
   $(-2, 10)$, $(0, 8)$, $(2, 10)$, $(1, 11)$, $(3, 12)$, $(4, 14)$, $(5, 15)$, $(4, 17)$,
   $(3, 17)$, $(2, 15)$, $(0, 13)$, $(-2, 15)$, $(0, 14)$, $(2, 15)$, $(-2, 15)$,
   $(-3, 17)$, $(-4, 17)$, $(-5, 15)$, $(-5, 3)$, $(-3, 3)$, $(-3, 5)$,
   $(-4, 6)$, $(-2, 6)$, $(-2, 7)$, $(-3, 8)$, $(-4, 8)$, $(-5, 7)$, $(-5, 3)$,
   $(-6, 2)$, $(-16, 2)$, $(-11, -3)$, $(-6, -3)$, $(-6, 1)$, $(-13, 1)$,
   $(-10, -2)$, $(-7, -2)$, $(-7, 0)$, $(-10, 0)$, $(-7, -3)$, $(-5, -3)$,
   $(-5, -10)$, $(-1, -10)$, $(-1, -4)$, $(-4, -4)$, $(-4, -9)$,
   $(-2, -9)$, $(-2, -5)$, $(-3, -5)$, $(-3, -8)$, $(-5, -10)$,
   $(-5, -12)$, $(-3, -14)$, $(-6, -14)$, $(-6, -18)$, $(-5, -17)$,
   $(-5, -15)$, $(-4, -15)$, $(-4, -18)$, $(-3, -17)$, $(-3, -15)$,
   $(-2, -15)$, $(-2, -18)$, $(-1, -17)$, $(-1, -15)$, $(1, -15)$,
   $(1, -17)$, $(2, -18)$, $(2, -15)$, $(3, -15)$, $(3, -17)$, $(4, -18)$,
   $(4, -15)$, $(5, -15)$, $(5, -17)$, $(6, -18)$, $(6, -14)$, $(3, -14)$,
   $(5, -12)$, $(5, -10)$, $(3, -8)$, $(3, -5)$, $(2, -5)$, $(2, -9)$,
   $(4, -9)$, $(4, -4)$, $(1, -4)$, $(1, -10)$, $(5, -10)$, $(5, -3)$.

Patience and perseverance, now!

2. On a single sheet of graph paper, locate the origin to allow for $-10 \leqslant x \leqslant 20$ and $-6 \leqslant y \leqslant 18$. Now graph each of the following line segments.

   (a) $y = -x + 15$ *
   $5 \leq x \leq 18$

   (b) $y = -2$
   $-8 \leq x \leq 17$

   (c) $y = \frac{1}{3}x + \frac{46}{3}$
   $2 \leq x \leq 5$

   (d) $y = -3$
   $-9 \leq x \leq 20$

   (e) $y = \frac{17}{14}x + \frac{111}{14}$
   $-9 \leq x \leq 5$

   (f) $x = 5$
   $-3 \leq y \leq 17$

   (g) $y = -6$
   $-6 \leq x \leq 10$

   (h) $y = -\frac{1}{3}x + \frac{50}{3}$
   $2 \leq x \leq 5$

   (i) $y = -x - 12$
   $-9 \leq x \leq -6$

   (j) $y = \frac{3}{8}x - \frac{78}{8}$
   $10 \leq x \leq 18$

*

Like this!

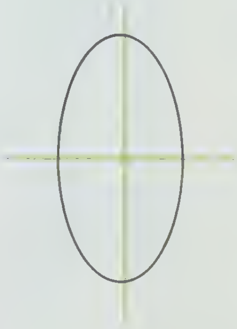
If $x = 5$
$\therefore\ y = -(5) + 15$
$= 10$

$\therefore$ $(5,10)$ is one end of the line segment.

If $x = 18$
$\therefore\ y = -(18) + 15$
$= -3$
$\therefore$ $(18,-3)$ is the other end of the line segment.

– Easy, eh?!

10



Unit 10
Graphing Lines and Curves

Not all relations result in straight line graphs. In this unit, we will review the straight line graph and then look at graphs of some common curves — the circle, ellipse, and parabola.

All variables in this unit represent real numbers.

10.1 Graphing Straight Lines

1. Can you draw a single line through any two fixed points? Any three fixed points?

How many nails fix a board in place?

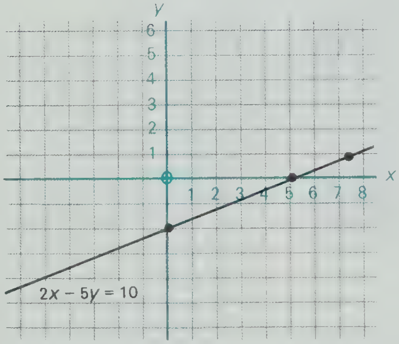
2. How many points fix a line in position?
3. What is the least number of ordered pairs you could use to draw the graph of a linear equation? We shall agree to use three, the third being a *check point*.
4. Draw the graphs of the following equations.

   (*a*) $y = 2x - 3$ (*b*) $y = \dfrac{3x - 12}{4}$ (*c*) $y = -2x$

Not all equations are given in the convenient $y = ▢$ form. Equations such as $3x + 4y = 6$ and $2x - 5y = 10$ can be easily graphed as follows.

An accurate graph results from well-spaced points.

Example Graph the equation $2x - 5y = 10$.



Solution

	Point on Graph
If $x = 0$, $\therefore\ 2(0) - 5y = 10$	
$-5y = 10$	
$\therefore\ y = -2$	(0, −2)
If $y = 0$, $\therefore\ 2x - 5(0) = 10$	
$2x = 10$	
$\therefore\ x = 5$	(5, 0)
Check	
If $y = 1$, $\therefore\ 2x - 5(1) = 10$	
$2x = 15$	
$\therefore\ x = 7.5$	(7.5, 1)

The line intersects the x axis at (5,0) and the y axis at (0,−2).

Note that 5 is called the *x*-intercept of the line and −2 is called the *y*-intercept of the line.

To find the *x*-intercept, put $y = 0$ and find x.

To find the *y*-intercept, put $x = 0$ and find y.

1. Find the *x*-intercept and the *y*-intercept of the graphs of each of the following lines.

   (*a*) $3y + 4x = 12$ (*b*) $2x - 5y = 8$ (*c*) $3x - y + 4 = 0$

Exercise 10.1

Discussion

1. State the y-intercept of each of the following lines.
   (*a*) $y = 2x + 1$ (*b*) $y = \frac{1}{2}x - 3$ (*c*) $y = 3x + 4$
   At what point would each line cross the y axis?

2. From the following list select equations of lines that pass through the origin.*
   (*a*) $y = 3x - 2$ (*c*) $y = 2x$ (*e*) $x + 2y + 1 = 0$
   (*b*) $y = 2x$ (*d*) $4x + 3y = 0$ (*f*) $y = -x$

Written Solutions

3. Graph the following equations by finding where the lines cross each axis. Use a third point as a check.
   (*a*) $2x - 3y = 6$ (*d*) $-2x + 6y = 18$
   (*b*) $4x + 5y = 10$ (*e*) $\frac{1}{2}x - \frac{1}{3}y = \frac{1}{4}$
   (*c*) $y - 3x = 6$ (*f*) $0.5x + 0.7y = 3.5$

4. By graphing, find the point where each pair of lines intersect.

5. Check by substitution to see if the point $(3, -1)$ lies on the lines $x + y = 2$ and $2x + 4y = 2$. What can you conclude?

6. Discuss the nature of the intersection of the lines $x + y = 2$ and $2x + 2y = 7$.

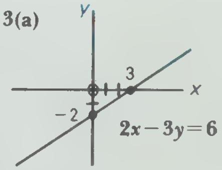
7. (*a*) On the *same set of axes*, construct the graphs of $y = x$, $y = 2x$, $y = 3x$, $y = \frac{x}{2}$. State any pattern you notice.
   (*b*) Repeat part (*a*) using the equations $y = -x$, $y = -2x$, $y = -3x$, $y = -\frac{x}{2}$.

8. On the same set of axes, construct the graphs of $y = x + 1$, $y = x + 4$, $y = x - 3$, $y = x - 5$. What do these lines have in common?

9. Draw the graphs of $y = 2x$, $y = 2x + 3$, $y = 2x - 4$ on the same diagram. What pattern do you notice?

*
Hint: Does (0,0) satisfy the equation?

3(a)



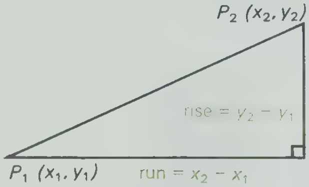
4(a) (2, 4)
(b) (5, 3)
(c) (2, −1)

10.2 Graphing Lines Using Slope

A The term "*slope*" is often used in everyday living. We talk of the "slope of a road," "a sloping roof," or "skiing on the slopes."

In mathematics, the *slope* of a line describes its "steepness"; the steeper the line, the greater the slope. Slope is usually represented by m and is defined as

$$\boxed{m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}}$$

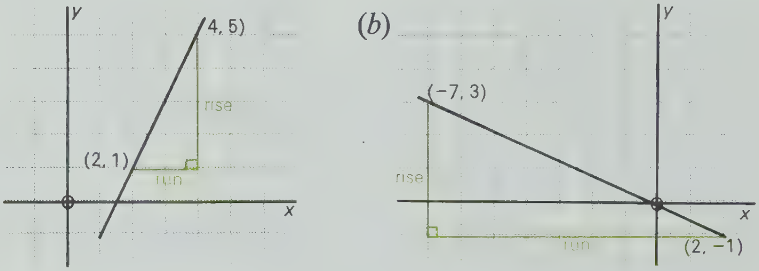


Example 1

Find the slope of the line passing through the points
(*a*) (2, 1) and (4, 5) (*b*) (2, −1) and (−7, 3).

Solution

In (a),
$(4,5) = (x_2, y_2)$
and $(2,1) = (x_1, y_1)$



(*a*)

$$\text{slope } m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{5-1}{4-2} = \frac{4}{2} = 2$$

(*b*)

$$\text{slope } m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{3-(-1)}{-7-2} = \frac{4}{-9}$$

Note that slope may be either *positive* or *negative*.

Recall that division by zero is impossible!

1. Find the slope of the line segment joining each pair of points.
   (*a*) (0, 1), (5, 11)
   (*b*) (9, 1), (12, 14)
   (*c*) (−3, 4), (1, −6)
   (*d*) (−7, −2), (0, −5)
   (*e*) (5, −3), (−2, −3)
   (*f*) (4, 2), (4, 9)

2. Find the slope of the line segment joining A(4, −5) and B(−2, 1) using A(4, −5) as (x_1, y_1) and B(−2, 1) as (x_2, y_2). Repeat using A(4, −5) as (x_2, y_2) and B(−2, 1) as (x_1, y_1). What do you notice?

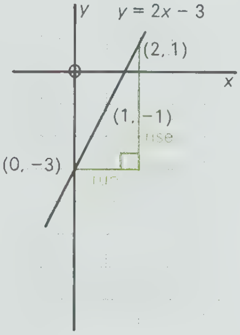
B If we are given the equation of a line, it is quite easy to find its slope by first graphing the line.

Example 2

Graph each line and determine its slope.
(*a*) $y = 2x - 3$ (*b*) $y = -3x + 5$

Solution

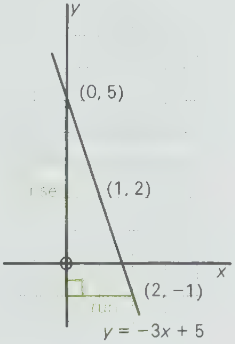
(*a*)



x	y
0	-3
1	-1
2	1

$$m = \frac{1-(-3)}{2-0} = \frac{4}{2} = 2$$

(*b*)

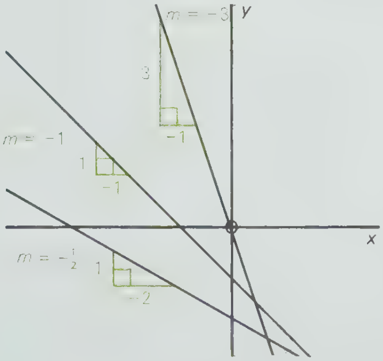


x	y
0	5
1	2
2	-1

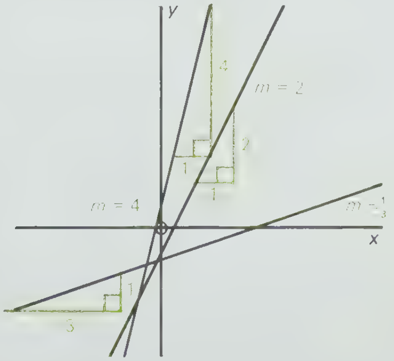
$$m = \frac{5-(-1)}{0-2} = \frac{6}{-2} = -3$$

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

Since slope may be either positive or negative, the following idea may help you to remember which is which.

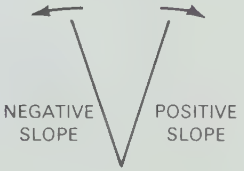


Lines "falling" left.



Lines "falling" right.

"falling" left
↓
$m = -3$
↑
"steepness" is 3



1. Graph each line and determine its slope. State whether the line is "falling" right or left.
(*a*) $y = 2x + 1$ (*c*) $y = -x + 1$ (*e*) $y = -4x - 1$
(*b*) $y = 5x - 2$ (*d*) $y = -3x + 4$ (*f*) $y = -5$

*
If $y = 4x - 3$, then $m = ?$

To find the y-intercept put $x = 0$ and solve for y.

$$y = \underset{\uparrow\ y\text{-INTERCEPT}}{\overset{\text{SLOPE}\ \downarrow}{m}x + b}$$

C Is it possible to determine the slope of a line from its equation,* without first graphing the line? Let's find out!

1. Graph each of the following lines, and find its slope and y-intercept.
   (*a*) $y = 2x + 3$ (*c*) $y = -3x - 2$
   (*b*) $y = -x + 1$ (*d*) $y = x - 5$

2. Each equation in Question 1 is of the form

$$y = ■\, x + ▲$$

Compare this number with the slope of the line (■)

Compare this number with the y-intercept of the line (▲)

What do you notice?

> In general, the line $y = mx + b$ has slope m and y-intercept b.

3. State the slope and y-intercept of each line.
   (*a*) $y = 3x + 5$ (*c*) $y = -\frac{5}{4}x + 7$ (*e*) $y = -14$
   (*b*) $y = x - 2$ (*d*) $y = x$ (*f*) $y = 7.5x + 9.3$

D The idea of slope helps in graphing lines.

Example 3

Graph (*a*) $y = 3x + 2$ (*b*) $3x + 2y = 5$

Solution

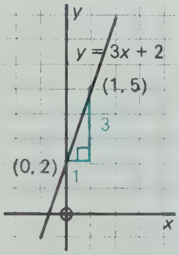
(*a*)

$$\text{slope } m = 3 = \frac{3}{1} = \frac{\text{rise}}{\text{run}}$$

$$y = \boxed{3}x + \boxed{2}$$

y-intercept is 2

The graph of $y = 3x + 2$ is shown in the margin.



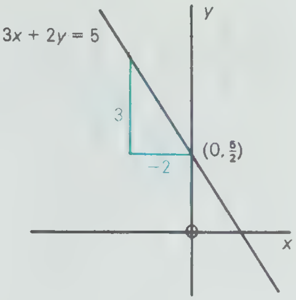
Note the steps:

1. Use the y-intercept to locate (0, 2).
2. Use the slope to locate a second point (1, 5).
3. Draw a line through (0, 2) and (1, 5).

(*b*) To graph $3x + 2y = 5$ using slope, we first express the equation in the form $y = mx + b$.

If $3x + 2y = 5$,
$\therefore\ 2y = -3x + 5$
$\therefore\ y = -\frac{3}{2}x + \frac{5}{2}$
$y = mx + b$
$\therefore$ The slope is $-\frac{3}{2}$ and
the y-intercept is $\frac{5}{2}$.

$$\text{slope } m = \frac{\text{rise}}{\text{run}} = -\frac{3}{2}$$



Exercise 10.2

Discussion

1. State the slope and y-intercept of each line and which way each line is "falling."
   (*a*) $y = 4x - 2$ (*c*) $y = -3x + \frac{1}{7}$ (*e*) $y = 3$
   (*b*) $y = \frac{1}{2}x + 5$ (*d*) $y = 9x$ (*f*) $x = -2$

2. How do the slopes of parallel lines compare?

3. Which lines are parallel to the line $y = 3x - 1$?
   (*a*) $y = 2x - 1$ (*b*) $y = 3x + 11$ (*c*) $2y = 6x + 1$

Written Solutions

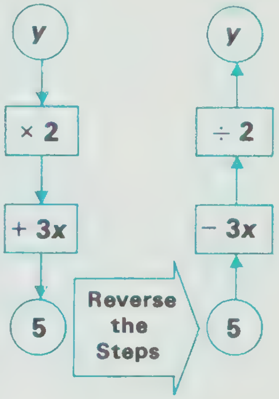
4. Determine the slope of the line through each pair of points.
   (*a*) (1, 3), (4, 9) (*d*) (−9, 2), (−3, −1)
   (*b*) (−2, 5), (3, 0) (*e*) (6, −3), (6, 41)
   (*c*) (−2, 5), (7, 5) (*f*) (5, −1), (0, 6)

5. Express each equation in the form $y = mx + b$. State the slope and y-intercept of the line.
   (*a*) $x + y = 2$ (*c*) $x - y + 3 = 0$ (*e*) $4x - 3 = 3y$
   (*b*) $2y = x + 4$ (*d*) $2x = y + 1$ (*f*) $5x - 4y = 3$

6. Graph each line given in Question 5.

7. Given 2 lines, $2x - 3y = 5$ and $5x - 7y + 1 = 0$, find which line is "steeper."

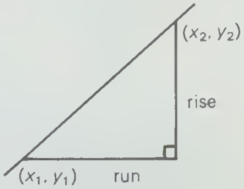
8. Using slopes, graph and find the intersection of the lines $x + 2y = 0$ and $3x - y = 7$.



$\therefore\ \frac{5-3x}{2} = y$

or $y = -\frac{3}{2}x + \frac{5}{2}$

SLOPE $m = \frac{y_2 - y_1}{x_2 - x_1}$



SLOPE
$y = mx + b$
y-INTERCEPT

4(a) 2 (e) Undefined
(c) 0

5(a) −1, 2
(c) 1, 3

8. (2, −1)

10.3 Graphing the Circle, Ellipse, and Parabola

A Equations may contain many different types of terms. We will consider only equations in one or two variables (usually x and y) and use exponents no greater than 2.

$3x$ is a first degree term
$5y^2$ is a second degree term
$-2x^3$ is a third degree term.

1. $3x + y = 5$ is an equation of the *first* degree in x and y. Make up five similar first degree equations.
2. $y = x^2 + 4x$ is an equation of the *second* degree in x and the first degree in y. Make up five others similar to it.
3. $x^2 + y^2 = 9$ is an equation of the second degree in both x and y. Make up four similar second degree equations.
4. An equation is first degree if *both* x and y are of the first degree. If either x or y are of the second degree, the equation is a second-degree equation. Classify as first- or second-degree equations.
   (*a*) $2x^2 + y^2 = 3$ (*c*) $3x = y^2$
   (*b*) $x - 5y = 7$ (*d*) $2x + 9 = 4y$

$3x + y = 5$
$x - 5y = 7$
$2x + 9 = 4y$
...straight lines!

As you have seen, the graphs of first-degree equations in x and y are straight lines. Such equations are called *linear equations*. The graphs of second-degree equations, however, are curves.

$y = x^2 + 4x$
$x^2 + y^2 = 9$
$2x^2 + y^2 = 3$
$3x = y^2$
... curves!

B *The Circle.* A very common curve found in Nature is the circle.

Example 1

Construct the graph of $x^2 + y^2 = 25$.

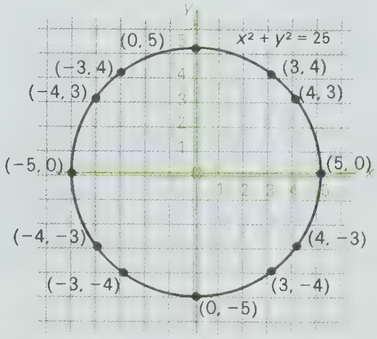
Solution *

Points on Graph

If $x = 0$, $0^2 + y^2 = 25$
$\therefore\ y = 5 \text{ or } -5$ — (0, 5), (0, −5)
If $y = 0$, $x^2 + 0 = 25$
$\therefore\ x = 5 \text{ or } -5$ — (5, 0), (−5, 0)
If $x = 3$, $3^2 + y^2 = 25$
and $9 + y^2 = 25$
$\therefore\ y^2 = 16$
$\therefore\ y = \pm 4$ — (3, 4), (3, −4)
Similarly if $y = 3$, then $x = \pm 4$ — (4, 3), (−4, 3)
If $x = -3$, $\therefore\ (-3)^2 + y^2 = 25$
and $9 + y^2 = 25$
$\therefore\ y^2 = 16$
$\therefore\ y = \pm 4$ — (−3, 4) (−3, −4)
Similarly if $y = -3$, then $x = \pm 4$ — (4, −3) (−4, −3)

*
Remember intercepts!

$y = 4$ or $y = -4$
may also be written
$y = \pm 4$
plus or minus



Notice that this graph is a circle with center at the origin and radius 5.

1. Graph each circle and record its radius.
   (*a*) $x^2 + y^2 = 9$ *
   (*b*) $x^2 + y^2 = 36$
   (*c*) $x^2 + y^2 = 16$
   (*d*) $x^2 + y^2 = 13$

In general, $x^2 + y^2 = r^2$ represents a circle with radius r and center at the origin.

*
$x^2 + y^2 = 9 = (3)^2$
and radius $r = 3$

C *The Ellipse.* An ellipse is an oval shape that looks like a "squashed" circle. Its equation is similar to that of a circle, too!

Example 2

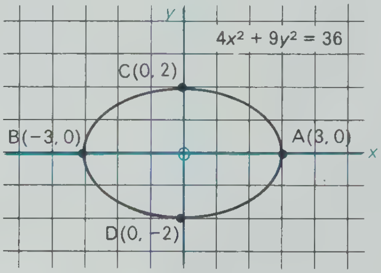
Construct the graph of $4x^2 + 9y^2 = 36$.

Solution

	Points on Graph
If $y = 0$, $\therefore 4x^2 + 0 = 36$	
or $x^2 = 9$	
$\therefore x = \pm 3$	(3, 0), (−3, 0)
Similarly, if $x = 0$, then $y = \pm 2$	(0, 2), (0, −2)

The idea of intercepts helps us here!

Other points may be found by choosing other values for x and y, as shown in the table.



x	± 1	± 2
y	± 1.9	± 1.5

AB is the *major* axis
CD is the *minor* axis

If $x = 2$,
$\therefore 4(2)^2 + 9y = 36$
or $16 + 9y^2 = 36$
then $9y^2 = 20$
$\therefore y^2 = \frac{20}{9}$
$\therefore y = +\sqrt{\frac{20}{9}}$ or $-\sqrt{\frac{20}{9}}$
$= +\frac{2\sqrt{5}}{3}$ or $-\frac{2\sqrt{5}}{3}$
$= +1.5$ or -1.5
$\therefore (2, 1.5), (2, -1.5)$ are on the graph.

now, if $x = -2$,
⋮

1. State the lengths of the major and minor axes of the ellipse in the example above.

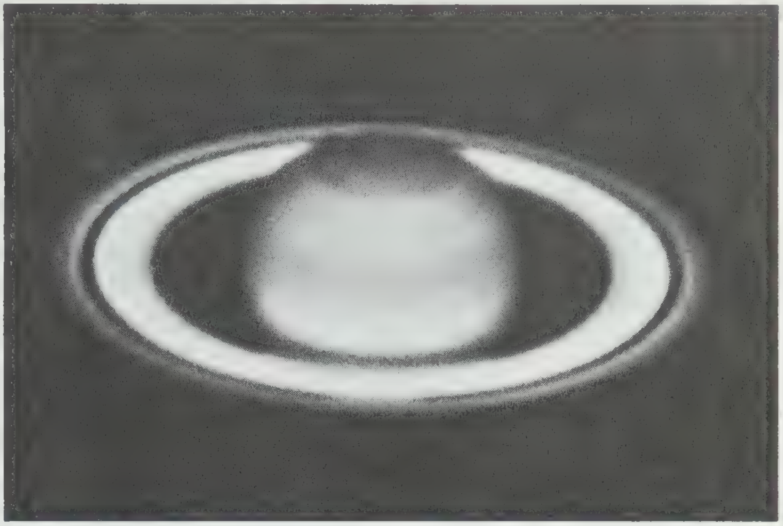
2. By first making a table of values, graph each ellipse.
   (*a*) $4x^2 + 25y^2 = 100$
   (*b*) $9x^2 + 16y^2 = 144$
   (*c*) $9x^2 + 4y^2 = 36$
   (*d*) $4x^2 + y^2 = 4$

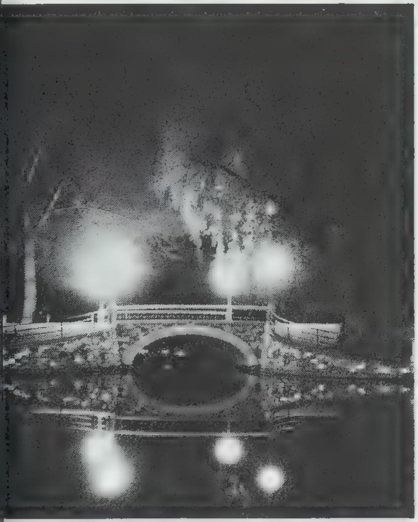
3. Classify each of the following equations according to those representing circles or those representing ellipses.
   (*a*) $x^2 + y^2 = 49$
   (*b*) $3x^2 + 6y^2 = 18$
   (*c*) $x^2 + 4y^2 = 4$
   (*d*) $5x^2 + y^2 = 10$
   (*e*) $y^2 + x^2 = 25$
   (*f*) $2x^2 + 2y^2 = 8$

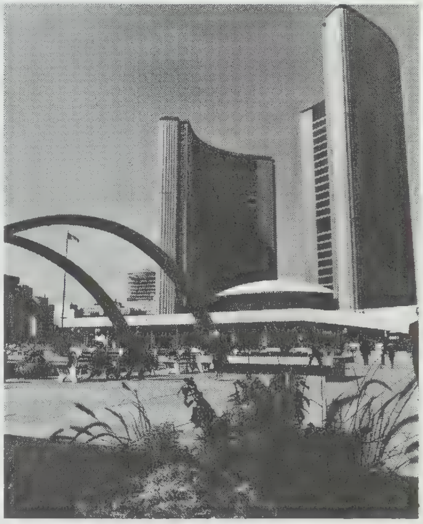
4. State the radius of each circle given in Question 3.

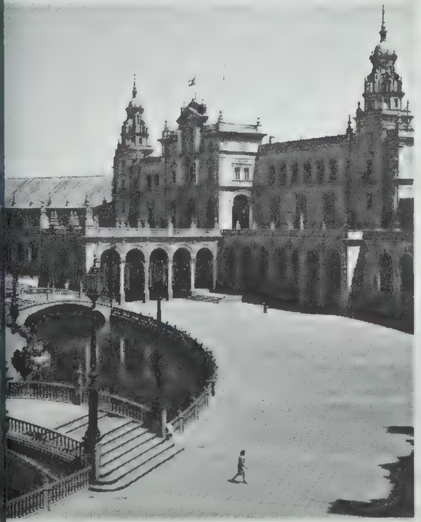
5. Determine the lengths of the major and minor axes of each ellipse in Question 3.

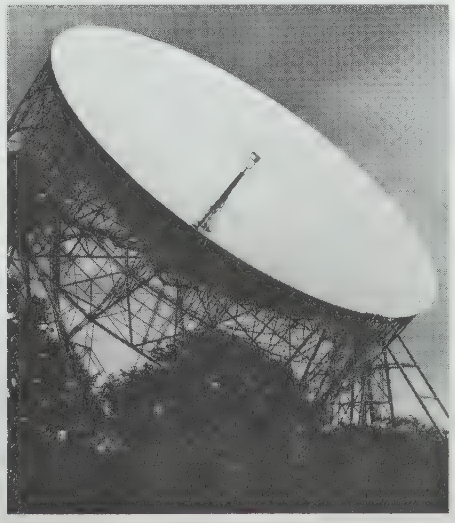
The ellipse has many important applications in industry and science. Some gears are made using this shape, and the paths of the planets around the sun are ellipses. Have you ever seen an elliptical pool table?

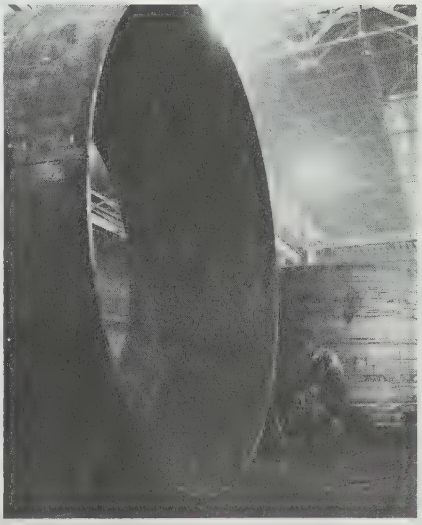




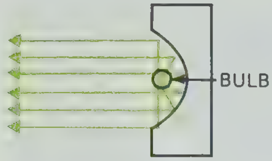








D *The Parabola.* Unlike the equations representing circles or ellipses, the equation of a parabola contains only *one* squared term.



Flashlight and auto headlight reflectors are parabolic reflectors.

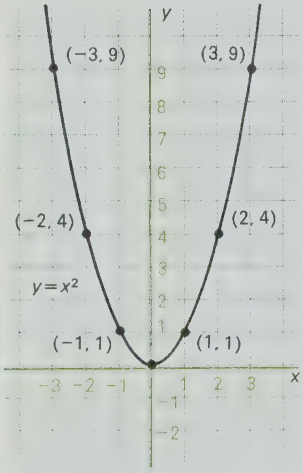
Example 3

Construct the graph of $y = x^2$.

Solution

		Points on Graph
If $x = 0$,	$\therefore y = (0)^2$	
	$\therefore y = 0$	(0, 0)
If $x = \pm 1$,	$\therefore y = (\pm 1)^2$	
	$\therefore y = 1$	(1, 1), (−1, 1)
If $x = \pm 2$,	$\therefore y = (\pm 2)^2$	
	$\therefore y = 4$	(2, 4), (−2, 4)
If $x = \pm 3$,	$\therefore y = (\pm 3)^2$	
	$\therefore y = 9$	(3, 9), (−3, 9)

The graph is shown in the margin.



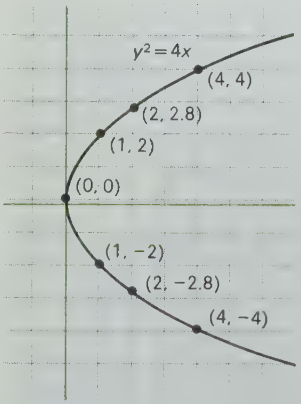
Example 4

Construct the graph of $y^2 = 4x$.

Solution

		Points on Graph
If $x = 0$,	$\therefore y^2 = 4\,(0)$	
	$= 0$	
	$\therefore y = 0$	(0, 0)
If $x = 1$,	$\therefore y^2 = 4\,(1)$	
	$= 4$	
	$\therefore y = \pm 2$	(1, 2), (1, −2)
If $x = -1$,	$\therefore y^2 = 4\,(-1)$	
	$= -4$	
No real number satisfies!		No points!
If $x = 2$,	$\therefore y^2 = 4\,(2)$	
	$= 8$	
	$\therefore y = \pm\sqrt{8}$	
	$\doteq \pm 2.8$	(2, 2.8), (2, −2.8)
If $x = 4$,	$\therefore y^2 = 4\,(4)$	
	$= 16$	
	$\therefore y = \pm 4$	(4, 4), (4, −4)

The graph is shown in the margin.



Exercise 10.3

Discussion

1. State whether each equation represents a straight line, circle, ellipse, or parabola.
   (*a*) $x^2 + y^2 = 1$
   (*b*) $3x - 4y = 12$
   (*c*) $25x^2 + 16y^2 = 400$
   (*d*) $y = \frac{1}{3}x^2$
   (*e*) $x^2 + 9y^2 = 9$
   (*f*) $x = -y^2$
   (*g*) $4x^2 + 4y^2 = 16$
   (*h*) $3x = 2y$

Can you state the radius of each circle?

Written Solutions

2. Graph.
   (*a*) $y = 2x - 3$ (*b*) $2y = 4$ (*c*) $3x - y = 6$

Remember intercepts!

3. Graph each circle and state its radius.
   (*a*) $x^2 + y^2 = 49$ (*b*) $x^2 + y^2 = 4$ (*c*) $x^2 + y^2 = 36$

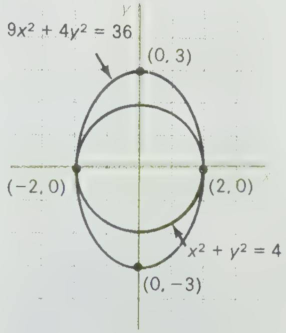
4. Using intercepts, sketch the graph of each ellipse.
   (*a*) $16x^2 + 9y^2 = 144$
   (*b*) $9x^2 + 16y^2 = 144$
   (*c*) $25x^2 + 9y^2 = 225$
   (*d*) $9x^2 + 25y^2 = 225$

5. Graph each parabola by obtaining at least 5 points for each graph.
   (*a*) $y = \frac{1}{2}x^2$ (*b*) $y = -\frac{1}{2}x^2$ (*c*) $x = y^2$

6. Draw the graph represented by each equation in Question 1.

7. Graph each pair of equations on the same set of axes to find their points of intersection.
   (*a*) $x^2 + 4y^2 = 16$
   $x - 2y = 4$
   (*b*) $x^2 + y^2 = 25$
   $4x = 3y$
   (*c*) $x^2 + y^2 = 4$
   $9x^2 + 4y^2 = 36$

7 (c)

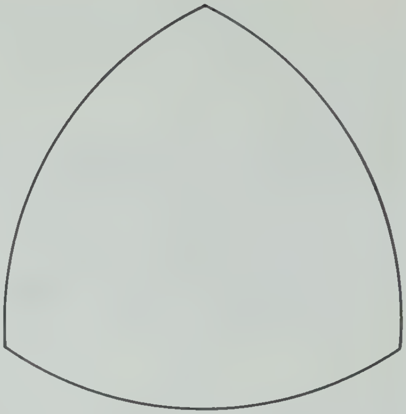


8. Graph $y = x^2 - 2x - 3$. ← This looks like a toughie!

9. Write a computer program that will produce a table of values in x and y for the equation $4x^2 + y^2 = 16$.

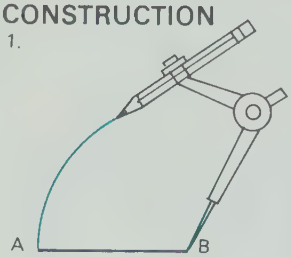
Reuleaux Triangles

The triangular shape shown on the right is called a Reuleaux triangle. Its construction is shown by the series of diagrams in the margin.



CONSTRUCTION

1.



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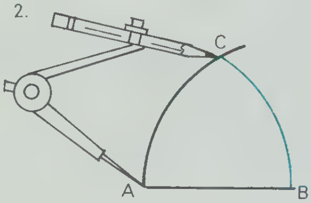

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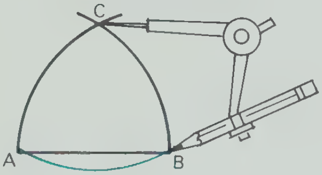


Project 1

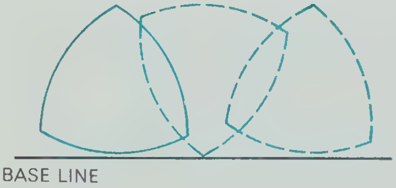
1. Make a tracing of the large Reuleaux triangle above, and carefully cut out the triangle you have traced.

2. Place the triangle on a sheet of lined notepaper, as shown in the margin, using one line as a base line.

3. Trace around the triangle and then move the triangle into several new positions, tracing around it each time.

4. Draw a line that just touches the top of each traced figure. What do you notice?

PROJECT 1



Project 2

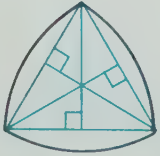
1. Make 4 Reuleaux triangles of identical shape out of light wood. Locate the center point of each.

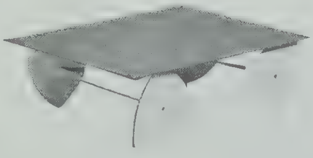
2. Fasten pairs of triangles together as shown in the margin, to form 2 "wheel-and-axle" arrangements.

3. Place the "wheels-and-axles" on a smooth surface, and put a sheet of light wood on top.

4. Lightly push on the end of the board.

PROJECT 2





Curves such as Reuleaux triangles are called *curves of constant width*. Can you think of a common geometric shape that is also a curve of constant width?

Geodesic Domes

One of the most recent innovations in architecture is the *geodesic dome*, pioneered by R. Buckminster Fuller in the United States.

Project

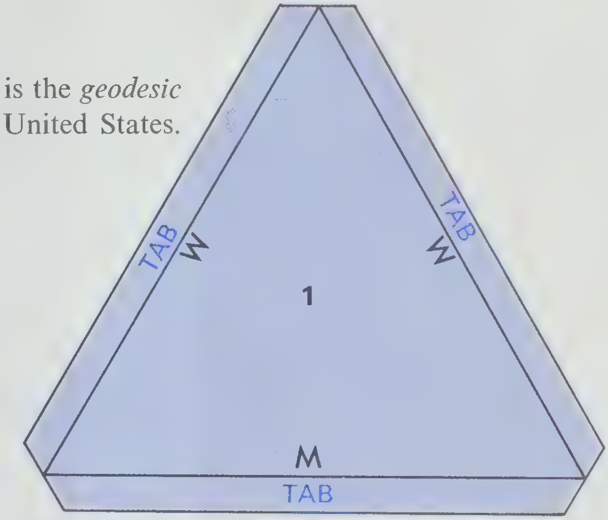
The patterns at the right may be used to build a model of a geodesic dome.

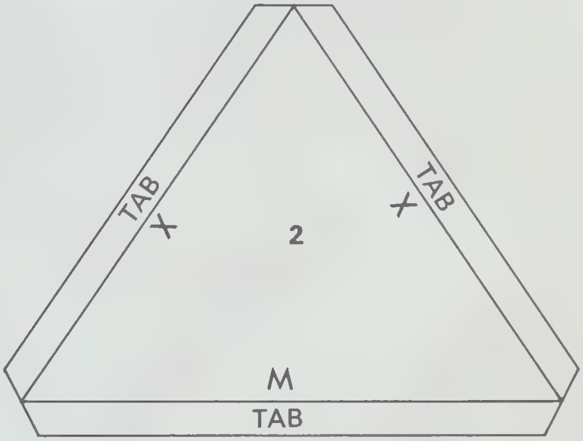
1. On paper or light cardboard, trace and cut out 10 triangles (1) of one color, and 30 triangles (2) of another color, and fold back the tabs.
2. Use 5 triangles (2) to form the top of the dome, as shown, by stapling or gluing the tabs together on the back.
3. Use the remaining triangles to form the middle and bottom sections, as shown, fastening the tabs in back as before.
4. Form the bottom and middle rows into a circle, fastening the free ends together.
5. Put the top in place and complete the dome by fastening all the tabs.

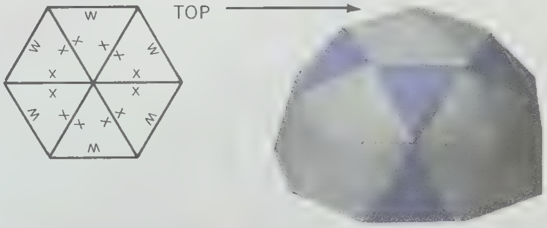
Discussion

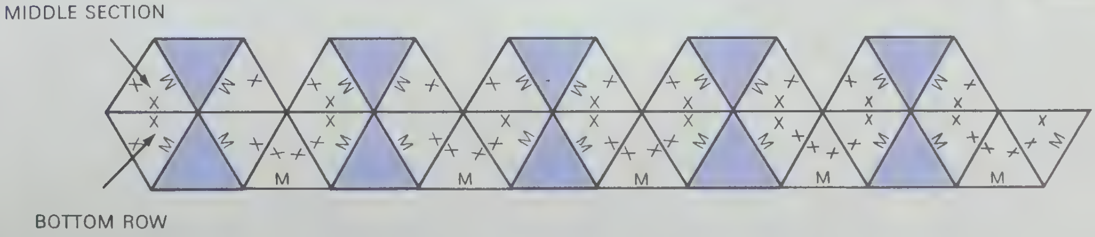
The geodesic dome uses less material to enclose a given space than does a "rectangular" building. In fact, the more closely the dome resembles a sphere, the less material is required to construct it.

Discuss the advantages and disadvantages of a building shaped like a geodesic dome. Why are geodesic domes used in the Arctic?









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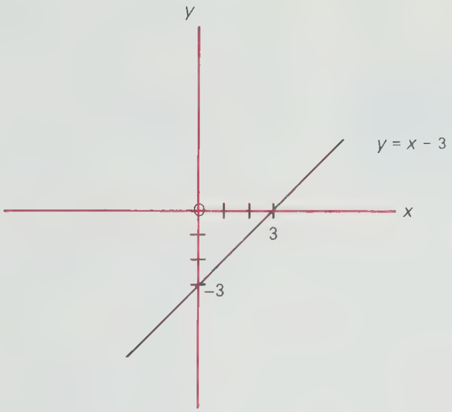
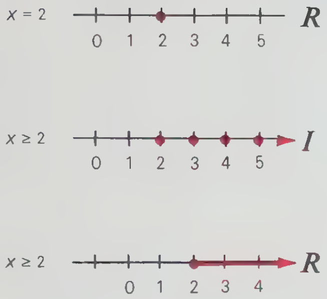
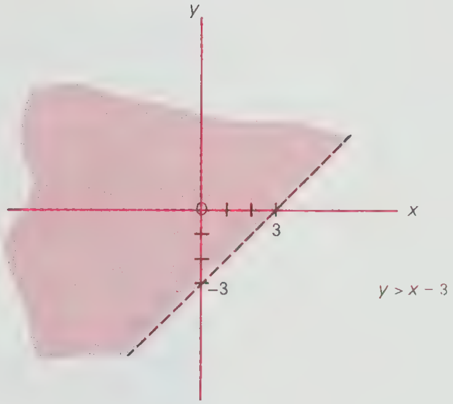



x = 2
0 1 2 3 4 5
R
x ≥ 2
0 1 2 3 4 5
I
x ≥ 2
0 1 2 3 4
R



Unit 11

Graphing Regions

The graph of an equation in x and y is a line or curve. What kind of graph is defined by $y \geq x - 3$? To help answer this question we will recall some of the ideas involved in graphing an inequation in one variable such as $x \geq 2$, $x \in R$.

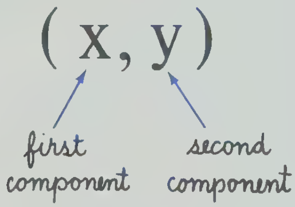
11.1 Graphing Simple Regions

Watch the domain of x!

Recall that R is the set of all decimal numbers.

A 1. Graph the solution set of each of the following on an R-line.
- (*a*) $x - 3 = 0, x \in R$
- (*b*) $x > 3, x \in I$
- (*c*) $x > 3, x \in R$

2. On the same R-line graph the solution set for
- (*a*) $x < 1, x \in R$
- (*b*) $x = 1, x \in R$
- (*c*) $x > 1, x \in R$



B 1. From the list, select the ordered pairs that satisfy the condition that the first component is greater than 1.

$(4, 3), (-2, 0), (1, 5), (0, -1), (3, 6), (10, 5)$

2. Each of the following conditions refers to the ordered pairs of a relation. Express each by an equation or an inequation.
- (*a*) The first number of each ordered pair is 0.
- (*b*) The second number of each ordered pair is greater than or equal to 2.
- (*c*) The first number of each ordered pair is less than 5.
- (*d*) The second number of each ordered pair equals 5.

3. List 3 ordered pairs satisfying each of the following conditions.
- (*a*) first component equals 4
- (*b*) second component is greater than 2
- (*c*) $x \leq -2$
- (*d*) $y = 1$

4. From the following list select ordered pairs that satisfy
- (*a*) $x \geq 1$
- (*b*) $y < -2$
- (*c*) $x = 4$

$(1, 3), (4, -4), (-2, 5), (0, 2), (-3, 0), (6, -5), (4, 3)$

Example 1

Construct the graph of $\{(x, y) \mid x = 2, x, y \in R\}$.

Solution

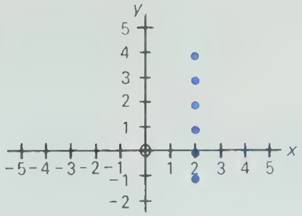
We could try to list this set, making sure that the first component is 2.

$(2, -1), (2, 0), (2, 1), (2, 2), (2, 3), \ldots$

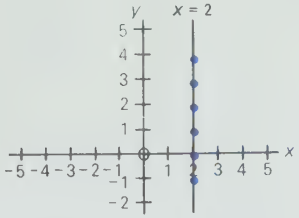
all satisfy the condition.

?

Can the listing ever be completed?



PARTIAL GRAPH OF $\{(x, y) \mid x = 2, x, y \in R\}$



COMPLETE GRAPH OF $\{(x, y) \mid x = 2, x, y \in R\}$

Example 2

Construct the graph of

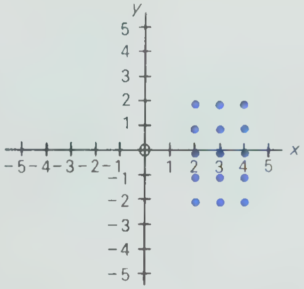
(*a*) $\{(x, y) \mid x \geq 2, \ x, y \in R\}$ (*b*) $\{(x, y) \mid x < 2, \ x, y \in R\}$

Solution

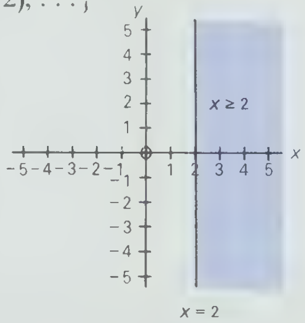
(*a*) Here again we can try to list the solution set.

$$\{(x, y) \mid x \geq 2, x, y \in R\} = \{\ldots, (2, -2), \ (2, -1), \ (2, 0), \ (2, 1), \ldots$$
$$\ldots, (3, -2), (3, -1), (3, 0), (3, 1), \ldots,$$
$$\ldots, (4, -2), (4, -1), (4, 0), (4, 1), \ldots,$$
$$\ldots, (5, -2), \ldots\}$$

In words, "the first component is greater than or equal to 2".



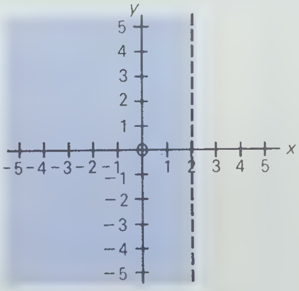
PARTIAL GRAPH OF $x \geq 2$



COMPLETE GRAPH OF $x \geq 2$

Why can the graph be shaded in completely?

(*b*) The graph for (*a*) includes all points satisfying $x \geq 2$. The remaining points of the plane satisfy the condition that $x < 2$. The boundary is *dotted* since the points that form it do not satisfy the condition.



GRAPH OF $x < 2$

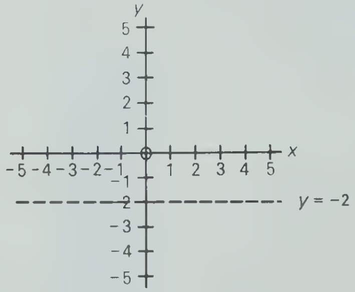
C

1. List 5 ordered pairs in which the second component is 4.
2. Express the condition of Question 1 using the variable y in an equation.
3. List 5 ordered pairs in which the second component is less than -2. Express this condition using the variable y in an inequation.
4. Construct the graph of
   - (*a*) $\{(x, y) \mid y = 1,\ x,\ y \in I\}$
   - (*b*) $\{(x, y) \mid y = 1,\ x,\ y \in R\}$

Example 3

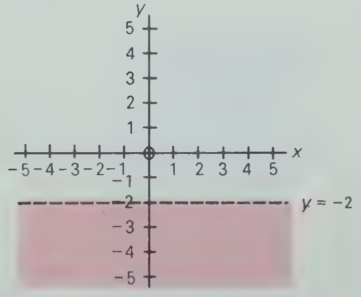
Construct the graph of $\{(x, y) \mid y < -2,\ x, y \in R\}$.

Solution



GRAPH OF $y = -2$

Step 1
Construct the boundary of the region by drawing the graph of $y = -2$. All points whose y coordinate is -2 lie on this graph.



GRAPH OF $y < -2$

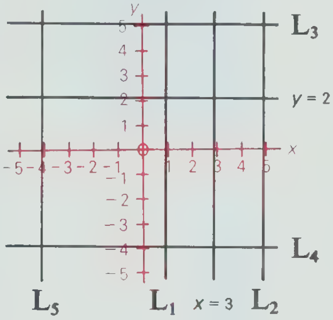
Step 2
All points whose y coordinates are less than -2 are below the line $y = -2$. The graph is as shown here.

Why is a dotted line used for the boundary?

Exercise 11.1

Discussion

1. $x = 3$ is a vertical line and $y = 2$ is a horizontal line. State the equations of L_1, L_2, L_3, L_4, and L_5.



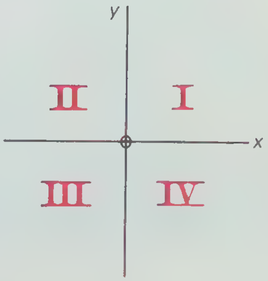
2. The graph of $x = 3$ is a vertical line 3 units to the right of the y axis. In a similar way describe the graphs of each of the following equations.
   (*a*) $x = 7$ (*b*) $x = -2$ (*c*) $y = 1$ (*d*) $y + 1 = 0$

3. Each of the following relations has a graph that is a region bound by a straight line. State the equation of the boundary.
   (*a*) $\{(x, y) \mid x > 1,\ x,\ y \in R\}$
   (*b*) $\{(x, y) \mid y \geq -3,\ x,\ y \in R\}$
   (*c*) $\{(x, y) \mid x \leq -3,\ x,\ y \in R\}$
   (*d*) $\{(x, y) \mid y \leq 5,\ x,\ y \in R\}$

   In which cases would the boundary be part of the graph? How do you show that the boundary is included in the required graph?

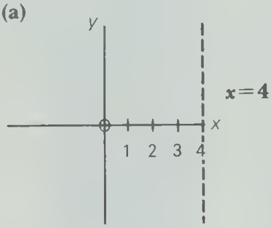
4. (*a*) What is the condition on the x or y coordinate satisfied by each point of the x axis?
   (*b*) State the equation of the x axis.
   (*c*) State the equation of the y axis.

5. (*a*) State the condition satisfied by all points to the right of the y axis; above the x axis. Express each condition by an inequation using x or y.
   (*b*) What condition is satisfied by each point of the 2nd quadrant? Repeat for each quadrant and express as an inequation.

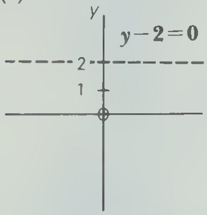


6(a) $x > 2$ **(e)** $y > 1$
(b) $y \leq 1$

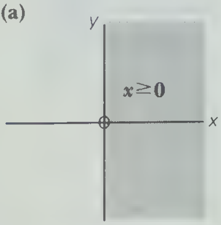
7(a)



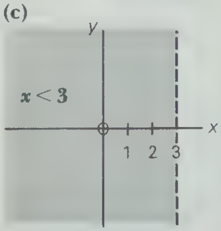
7(c)



8(a)



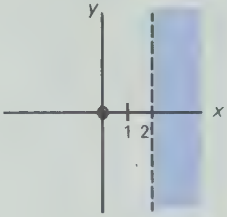
8(c)



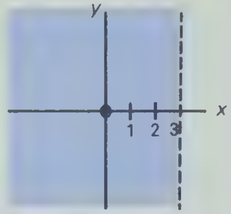
Written Solutions

6. State the inequation for each graph.

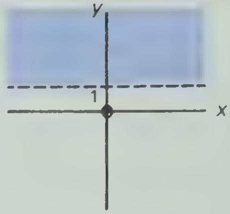
(*a*)



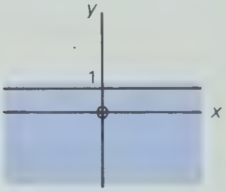
(*c*)



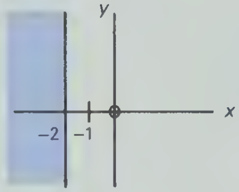
(*e*)



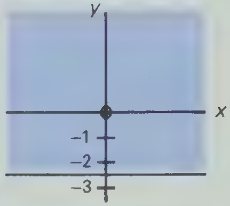
(*b*)



(*d*)



(*f*)



7. Construct the graph of each relation. $x, y \in R$.
(*a*) $\{(x, y) \mid x = 4\}$
(*b*) $\{(x, y) \mid y = -3\}$
(*c*) $\{(x, y) \mid y - 2 = 0\}$
(*d*) $\{(x, y) \mid x + 6 = 0\}$
(*e*) $\{(x, y) \mid x - 5 = 0\}$
(*f*) $\{(x, y) \mid y + 6 = 0\}$

8. Graph the set of all ordered pairs (x, y) that satisfy each condition.
(*a*) $x \geq 0$
(*b*) $y \leq 0$
(*c*) $x < 3$
(*d*) $x > -1$
(*e*) $y > 2$
(*f*) $y \leq 1$
(*g*) $x < -4$
(*h*) $y \geq -2$
(*i*) $x \geq 3$

9. Graph the relations $x \geq 3$ and $y \geq 2$ on the same set of axes. What does the "double-shaded" area represent?

11.2 Graphing Linear Inequations

A How do you test whether the points (3, 9), (2, −1), and (−3, 3) lie on the line $y = 2x + 3$? Make the test for each point. By substitution you can test whether ordered pairs satisfy a given equation. Read the example and then do Question 1.

Example 1

Does the ordered pair (3, 5) satisfy $y \geq -2x + 1$?

Substitute $x = 3, y = 5$ into $y = -2x + 1$ and compare L S and R S.

Solution

LS	RS
y	$-2x + 1$
$= 5$	$= -2(3) + 1$
	$= -5$

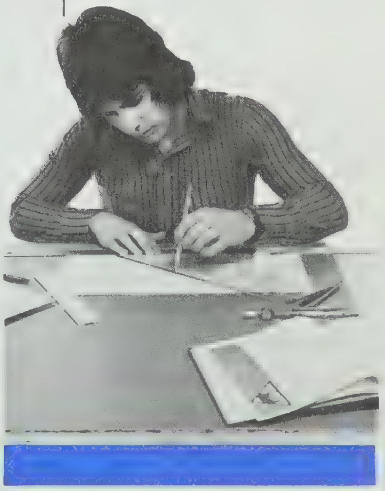
$\because\ 5 \geq -5$

$\therefore$ (3, 5) satisfies the inequation $y \geq -2x + 1$.

1. Test whether the ordered pair satisfies the inequation.
   (*a*) $y \leq 2x + 3$; (4, 1)
   (*b*) $y > x - 1$; (5, 7)
   (c) $y \geq 2x - 5$; (−2, −9)
   (*d*) $x + y < 2$; (1, 3)
   (*e*) $3x - 2y + 1 < 0$; (−2, −6)
   (*f*) $x + 6y \geq -10$; (0, 0)

B

1. Each of the following ordered pairs satisfies one of the inequations $y > x$ or $y < x$. List the points that satisfy each inequation.
   (1, 0), (1, 2), (3, 0), (1, 3), (2, 1), (2, 4), (1, 5), (−1, 4), (0, 1), (2, 0), (2, 3), (4, 0), (3, 4), (5, 3), (3, −2), (−2, −1), (4, 3), (−1, 0), (0, −1), (−1, −2), (1, −2), (4, 2), (−2, 2), (3, 2)
2. Plot in ink the ordered pairs satisfying $y > x$, and plot in pencil those that satisfy $y < x$. What pattern do you notice?
3. On the same graph you made for Question 2, construct the line $y = x$.
4. Using your graph of Question 3, how would you describe the position of all points that satisfy the inequation $y > x$? Where would all points lie that satisfy the condition $y < x$?



C

1. Construct the graph of $y = -2x + 1$.
2. Plot the points (0, 5), (4, 0), (−1, 1), (1, −3), (−2, −1), (4, 3) on the diagram of Question 1.
3. Which of the points from Question 2 would you expect to satisfy the inequation $y > -2x + 1$? Test your guess! Test the remaining points using the inequation $y < -2x + 1$.

D More complicated inequations can be graphed as before by using the corresponding equation to obtain a boundary line.

Example 2

Draw the graph of the inequation $y \geq 2x - 3$.

Solution

Step 1

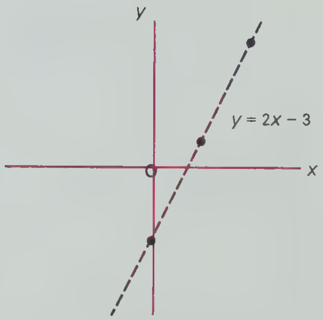
Graph the related equation $y = 2x - 3$.

x	y
0	−3
2	1
4	5

	Points on Graph
If $x = 0$, then $y = -3$	$(0, -3)$
If $x = 2$, then $y = 2(2) - 3 = 1$	$(2, 1)$
If $x = 4$, then $y = 2(4) - 3 = 5$	$(4, 5)$

Step 2

All the points on one side of this line satisfy $y > 2x - 3$, and those on the other side form the graph of $y < 2x - 3$. All we need is one test point.



Test Point

Does $(0, 0)$ satisfy $y \geq 2x - 3$?

LS	RS
y	$2x - 3$
$= 0$	$= 2(0) - 3$
	$= -3$

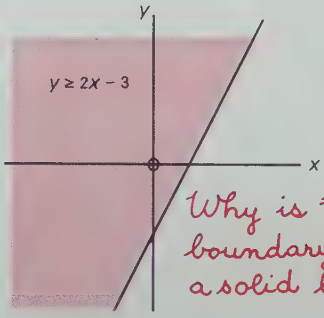
$\because 0 \geq -3$

$\therefore$ $(0, 0)$ satisfies the inequation $y \geq 2x - 3$.

$\therefore$ The required graph is the line $y = 2x - 3$ and the region above the line containing the test point $(0, 0)$.

Check by trying several points above the line!

If the test point satisfies the given inequation, the region to be graphed includes the test point.



GRAPH OF $y \geq 2x - 3$

Why is the boundary a solid line?

Example 3

Draw the graph of $3x - 2y - 6 > 0$.

Solution

Step 1

Graph the line $3x - 2y - 6 = 0$

Points on Graph

If $x = 0$, then $3(0) - 2y - 6 = 0$

or $-6 = 2y$

$\therefore$ $y = -3$ — $(0, -3)$

If $y = 0$, then $3x - 2(0) - 6 = 0$

or $3x = 6$

$\therefore$ $x = 2$ — $(2, 0)$

Remember intercepts!
– or use slope!
If $3x-2y-6=0$
then $2y=3x+6$
$\therefore$ $y=\frac{3}{2}x+3$
slope is $\frac{3}{2}$
y-intercept is 3

Step 2

Test $(0, 0)$ in $3x - 2y - 6 > 0$.

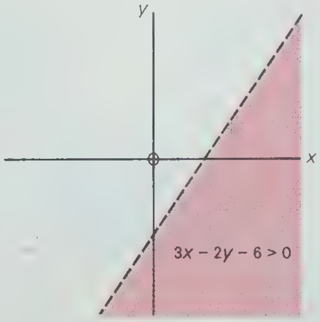
LS	RS
$3x - 2y - 6$ $= 3(0) - 2(0) - 6$ $= -6$	0

$\because$ $-6 < 0$

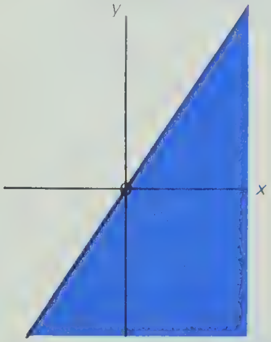
$\therefore$ $(0, 0)$ does *not* satisfy $3x - 2y - 6 > 0$.

The required graph is on the side of the line $3x - 2y - 6 = 0$ that does *not* contain the test point $(0, 0)$.

Note that we still require only one test point.



If the test point does not satisfy the given inequation, the region to be graphed will not include the test point.



Exercise 11.2

Discussion

1. Which of the following inequations are satisfied by (0, 0)?

(*a*) $y < x + 5$
(*b*) $y < x - 3$
(*c*) $x + y \geq 2$
(*d*) $x - y < 4$
(*e*) $y \geq x$
(*f*) $y < x + 1$

What can you conclude about the graph of each inequation?

2. State the equation of the boundary of the graph of each of the following inequations.

(*a*) $y < x + 2$
(*b*) $x + 2y \geq 3$
(*c*) $y \leq -2$
(*d*) $3x - 2y + 4 > 0$
(*e*) $y \leq 3x + 5$
(*f*) $y > 5 - x$

In which parts of this question would the boundary be part of the graph? How would you show this? How do you show that the boundary is excluded from the graph?

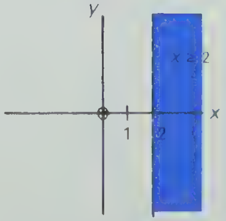
3. The origin, (0, 0) is usually the easiest test point to tell you which side of a boundary line to use for the graph. When can you not use the origin as a test point? What can you do in this special case?

4. Which of the following lines pass through the origin?

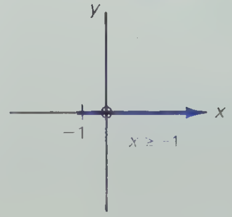
(*a*) $2x - y + 1 = 0$
(*b*) $3x - 2y = 0$
(*c*) $y = 2x$
(*d*) $y = 4x - 5$

5. Each graph contains at least one error. Identify the errors.

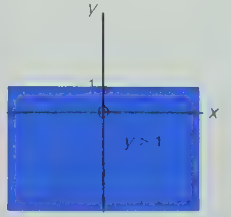
(*a*)



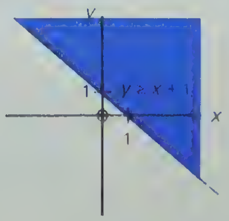
(*b*)



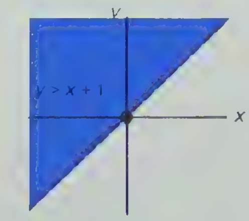
(*c*)



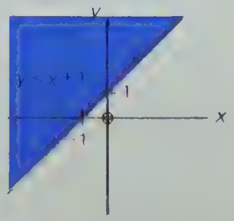
(*d*)



(*e*)



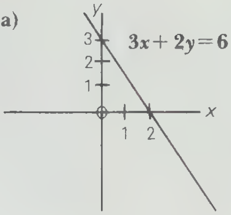
(*f*)



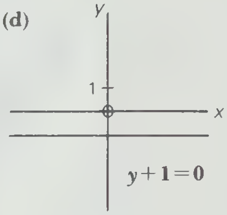
Written Solutions

6. Construct the graph of each of the following equations.
   - (*a*) $3x + 2y = 6$
   - (*b*) $y = 5x - 2$
   - (*c*) $x = 2$
   - (*d*) $y + 1 = 0$
   - (*e*) $4x - 3y + 12 = 0$
   - (*f*) $3x - y = 0$
7. Graph the boundary line of each region. Determine on which side of the boundary the region lies by using a test point such as $(0, 0)$ or $(1, 0)$.
   - (*a*) $y > 2x + 3$
   - (*b*) $y \leq 1 - x$
   - (*c*) $x + y \geq 0$
   - (*d*) $4x - 3y > 12$
   - (*e*) $y > 2x$
   - (*f*) $x \geq -2$
   - (*g*) $y \geq 2x - 3$
   - (*h*) $x + y - 4 > 0$
8. Draw the graph of each of the following inequations.
   - (*a*) $x + 3y > 6$
   - (*b*) $y < 3 - 2x$
   - (*c*) $x - 2 \geq 0$
   - (*d*) $y \geq 3x - 5$
   - (*e*) $2x - 3y + 4 \leq 0$
   - (*f*) $y + 3 > 0$
   - (*g*) $y > 1 - x$
   - (*h*) $x + y < 2$
9. Recall that *and* means *common to*. Draw the graph of the relation defined by each of the following conditions.
   - (*a*) $y < x + 1$ *and* $y < 1 - x$
   - (*b*) $2x - y < 0$ *and* $x \leq 3y - 2$
10. Using the origin as a test point, graph each inequation.
    - (*a*) $x^2 + y^2 \geq 9$
    - (*b*) $4x^2 + 9y^2 < 36$

6(a)

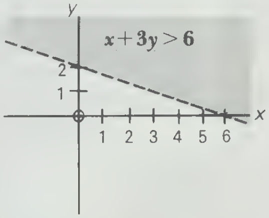


(d)

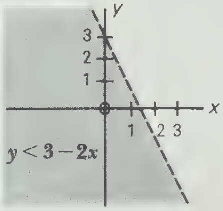


7(a) No **(e) No**
(c) Yes

8(a)



(b)



12

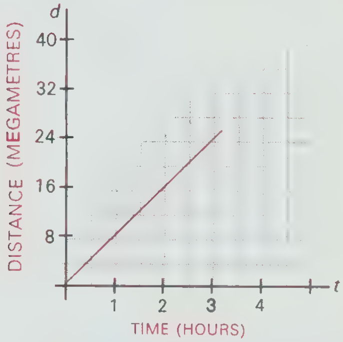
t	1	2
d	8000	16000

1 Mm = 1 000 000 m

If $d = 8000t$ and $t = 7$

$\therefore d = 8000(7)$

$= 56\,000$



Unit 12
Quadratic and Simultaneous Equations

Get-Set Exercise — Graphing, Factoring, and Equation Solving

Graphing

1. The Explorer II space probe sent to Mars travelled so that its distance from Earth (d kilometres) at any time (t hours) was given by the equation $d = 8000t$.
   (*a*) Make a table of values for $t = 1, 2, 3, \ldots, 12$.
   (*b*) List the set of ordered pairs (t, d) from your table of values in part (*a*).
   (*c*) Graph the points representing these ordered pairs on a set of axes and join them with a straight line.
   (*d*) How far was Explorer II from Earth after 5 h?
   (*e*) How long did it take for Explorer II to reach a point 76 000 km from Earth?

1 Common factor

2 Difference of squares

3 Trivial

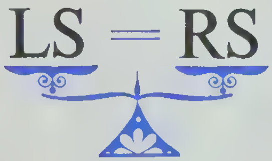
Factoring

2. Factor completely.
   (*a*) $x^2 - 3x$
   (*b*) $a^2 - 25$
   (*c*) $3m^2 - 27$
   (*d*) $k^2 + 2k - 15$
   (*e*) $x^2 - 9x + 18$
   (*f*) $b^2 - 13b - 48$

Equation Solving

Remember ?

(1) Multiply to clear fractions.
(2) Remove any brackets and collect like terms.
(3) Add terms to each side to isolate the variable.
(4) If necessary, multiply each side by the same number to solve for the variable.



Remember to keep your balance!

3. Check to see if the number given belongs to the solution set of the given equation.
   (*a*) $3y = 2y + 1$; 1
   (*b*) $-5z = -2 - 6z$; -2
   (*c*) $4 - 5y = -2 - 6y$; 2
   (*d*) $3 - 2x = 4 - x$; -1

4. Solve.
   (*a*) $-2y = -3y - 5$
   (*b*) $-9x = 63$
   (*c*) $0.4a = -14$
   (*d*) $\dfrac{2 - 2t}{5} = \dfrac{-4 - 3t}{4}$
   (*e*) $x + 5 = 2 - 4(x + 1)$
   (*f*) $2(a - 5) - 3 = 4(a + 1)$
   (*g*) $2(x + 3) = -7$
   (*h*) $\frac{2}{3}(y - 1) = \frac{5}{6}(y + 2)$

5. Express x in terms of y.
   (*a*) $x + 2y = 7$
   (*b*) $3x = 9y - 12$
   (*c*) $6 - x = 3y$
   (*d*) $3y = 2x - 7$

6. Express y in terms of x.
   (*a*) $4 + y = 3x$
   (*b*) $2y = x - 5$
   (*c*) $5 - y = 3x$
   (*d*) $2x = 7 + 2y$

2(a) $x(x - 3)$
(d) $(k + 5)(k - 3)$

3(a) Yes (d) Yes

4(a) $y = -5$ (e) $x = -\frac{7}{5}$
(c) $a = -35$ (h) $y = -14$

5(a) $x = 7 - 2y$
(c) $x = 6 - 3y$

6(a) $y = 3x - 4$
(c) $y = 5 - 3x$

12.1 Solving Quadratic Equations

A Recall that linear equations are of the first degree (no squared terms) while quadratic equations are of the second degree (one or more squared terms). A quadratic equation will have only one squared term. For example,

$$x^2 - 4 = 0, \qquad x^2 - 7x + 12 = 0, \qquad x^2 - x - 5 = 0$$

are quadratic equations in the variable x.

LINEAR
→ FIRST DEGREE
→ GRAPH IS A STRAIGHT LINE

QUADRATIC
→ SECOND DEGREE
→ GRAPH IS A CURVE

All quadratic equations are of the form

$$ax^2 + bx + c = 0, \qquad a \neq 0^{*}$$

where a is the coefficient of x^2
b is the coefficient of x
c is the constant term.

* If $a = 0$, there is no squared term, and the equation is not quadratic.

For the equation $2x^2 - 3x + 5 = 0$,
$a = 2$
$b = -3$
$c = 5$

1. State the values of a, b, and c for each quadratic equation.
   - (*a*) $3x^2 + 4x + 2 = 0$
   - (*b*) $x^2 - 5x + 3 = 0$
   - (*c*) $2x^2 + x - 7 = 0$
   - (*d*) $-17x^2 - 8x + 1 = 0$
   - (*e*) $5x^2 - 3x = 0$
   - (*f*) $4x^2 + 9 = 0$

$$\begin{array}{ccccccc} ax^2 & + & bx & + & c & = & 0 \\ \updownarrow & & \updownarrow & & \updownarrow & & \\ -17x^2 & - & 8x & + & 1 & = & 0 \end{array}$$

B *Solving Quadratic Equations by Factoring*

Recall that

$$\text{if } ab = 0, \quad \text{then} \begin{cases} a = 0, \\ \text{or } b = 0, \\ \text{or } a = b = 0. \end{cases}$$

If the product of 2 numbers is 0, then at least 1 of the numbers is 0.

If $(x-2)(x+3) = 0$
then $(x-2) = 0$
or $(x+3) = 0$
$\therefore x = 2$ or $x = -3$

1. State the value(s) of the variable for which each of the following represents a true statement.
   - (*a*) $3x = 0$
   - (*b*) $4(y - 1) = 0$
   - (*c*) $m(m + 1) = 0$
   - (*d*) $(k - 3)(k - 4) = 0$
   - (*e*) $(a + 5)(a + 4) = 0$
   - (*f*) $(b - 7)(b + 1) = 0$

Example 1

Solve by factoring.

(*a*) $y^2 + 2y = 0$ (*b*) $m^2 - 7m + 12 = 0$

Solution

(*a*)
$$\begin{aligned} &\because\ y^2 + 2y = 0 \\ &\therefore\ y(y+2) = 0 \\ &\therefore\quad y = 0 \text{ or } (y+2) = 0 \\ &\therefore\quad y = 0 \qquad\quad y = -2 \end{aligned}$$

(*b*)
$$\begin{aligned} &\because\ m^2 - 7m + 12 = 0 \\ &\therefore\ (m-4)(m-3) = 0 \\ &\therefore\quad (m-4) = 0 \text{ or } (m-3) = 0 \\ &\therefore\qquad m = 4 \qquad\quad m = 3 \end{aligned}$$

2. Solve by factoring.
   - (*a*) $k^2 - 25 = 0$
   - (*b*) $3t^2 - 6 = 0$
   - (*c*) $h^2 + 5h + 4 = 0$
   - (*d*) $x^2 - x - 12 = 0$
   - (*e*) $m^2 + 7m + 10 = 0$
   - (*f*) $z^2 + z - 20 = 0$

C *Solving Quadratic Equations by Formula*

Solutions to quadratic equations may be found using the quadratic formula.

If $ax^2 + bx + c = 0$, then
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

There are 2 answers here!

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$
or
$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Example 2

Solve using the quadratic formula.

(*a*) $x^2 - 7x + 12 = 0$ (*b*) $k^2 - 4k - 3 = 0$

Solution

(*a*) $x^2 - 7x + 12 = 0,$ where $a = 1$
$b = -7$
$c = 12$

$$\begin{array}{cccccc} ax^2 & + & bx & + & c & = 0 \\ \updownarrow & & \updownarrow & & \updownarrow & \\ 1x^2 & - & 7x & + & 12 & = 0 \end{array}$$

$$
\begin{aligned}
\therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(12)}}{2(1)} \\
&= \frac{7 \pm \sqrt{1}}{2} \\
&= \frac{7+1}{2} \text{ or } \frac{7-1}{2}
\end{aligned}
$$

$\therefore x = 4$ or $x = 3$

I can also solve this by factoring!
$\because x^2 - 7x + 12 = 0$
$\therefore (x-4)(x-3) = 0$
$\therefore x = 4$ or $x = 3$

(*b*) $k^2 - 4k - 3 = 0,$* where $a = 1$
$b = -4$
$c = -3$

*
Whoops! – This one won't factor!
$(k - ?)(k + ?) = 0$

$$
\begin{aligned}
\therefore k &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)} \\
&= \frac{4 \pm \sqrt{28}}{2} \\
&= \frac{4 \pm 2\sqrt{7}}{2} \\
&= 2 \pm \sqrt{7}
\end{aligned}
$$

Here you cannot simplify further, so leave your answer in this simplified radical form.

$k = 2 \pm \sqrt{7}$

If I use $\sqrt{7} \doteq 2.6$, then $k \doteq 2 \pm 2.6$
$\therefore k \doteq 4.6$ or $k \doteq -0.6$

D *Strategy for Solving Quadratic Equations*

1. Factorable? → factor and solve
(Remember: if $ab = 0$, then . . .)

This is quicker but not always possible!

2. Not factorable? → use quadratic formula
If $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This always works!

Exercise 12.1

Discussion

1. State the values of a, b, and c.
   - (a) $3x^2 - 4x + 1 = 0$
   - (b) $-x^2 + 2x - 3 = 0$
   - (c) $m^2 + 7m - 2 = 0$
   - (d) $2k - 3k^2 + 5 = 0$
   - (e) $4y^2 - 8 = 0$
   - (f) $2a + 5a^2 = 0$

Careful in (d)!

2. Form quadratic equations using each set of values for a, b, and c.
   - (a) $a = 2, b = 1, c = 5$
   - (b) $a = -5, b = 7, c = -1$
   - (c) $a = 6, b = 5$
   - (d) $a = 3, c = -8$

$ax^2 + bx + c = 0$

3. Which method of solving quadratic equations (factoring or formula) do *you* prefer? Why?

4. Given any quadratic equation, outline your approach to finding its roots.

"Roots" are members of the solution set.

Written Solutions

5. Solve by (*i*) factoring, (*ii*) using the quadratic formula. Compare your answers.
   - (a) $y^2 - 6y + 5 = 0$
   - (b) $k^2 + 4k + 3 = 0$
   - (c) $m^2 + 11m + 18 = 0$
   - (d) $x^2 - 2x - 8 = 0$

6. Solve.
   - (a) $x^2 + 3x - 10 = 0$
   - (b) $2y^2 + 5y - 3 = 0$
   - (c) $2m^2 - 50 = 0$
   - (d) $x^2 + x - 56 = 0$
   - (e) $r^2 - 3r = 0$
   - (f) $w^2 + 3w + 1 = 0$
   - (g) $2t^2 + 10t + 8 = 0$
   - (h) $k^2 - 6k + 9 = 0$
   - (i) $p^2 + 4p - 7 = 0$
   - (j) $3x^2 - 2x - 6 = 0$

7. Using the quadratic formula and the table of square roots on pages 444 and 445, solve for x, correct to 2 decimal places.
   - (a) $2x^2 + 4x - 1 = 0$
   - (b) $3m^2 - 6m + 2 = 0$

8. Use the quadratic formula to solve.
   - (a) $3x^2 - 2x + 1 = 0$*
   - (b) $2y^2 + y + 1 = 0$

* Remember that $\sqrt{-4}$ is not a real number!

9. Determine a computer program that will provide solutions for equations of the form $ax^2 + bx + c = 0$, using the quadratic formula.

5(a) $y = 5$ or $y = 1$
(c) $m = -2$ or $m = -9$
6(a) $x = 2$ or $x = -5$
(c) $m = 5$ or $m = -5$
(e) $r = 0$ or $r = 3$
(g) $t = -4$ or $t = -1$
(i) $p = -2 \pm \sqrt{11}$
7(a) $x = 0.23$ or $x = -1.23$
8(a) No real solution.

12.2 Solving Equations Graphically

You have learned how to solve one equation in 1 variable. If there are 2 variables in *one* equation, there will be more than one possible solution.* If we have 2 linear equations in 2 variables, however, only *one pair* of values will satisfy *both* equations at the same time. Equations of this type are called *simultaneous equations.*

* If $x + y = 7$
then $x = 1$ and $y = 6$
or $x = 2$ and $y = 5$
or $x = 3$ and $y = 4$
or $x = 4$ and $y = 3$
or ...

Example 1

Two cars travel along the same road. The equations relating distance travelled (d kilometres) after a certain time (t hours) are as follows.

For car A, $d = 40t$ ①
For car B, $d = 60t - 180$ ②

Determine graphically when the 2 cars are together.

The ① and the ② are used to name the equations.

Solution

We first form a table of values for each equation.

$d = 40t$ ①

t	0	1	2	3	4
d	0	40	80	120	160

$\therefore$ (0, 0), (1, 40), (2, 80), (3, 120), (4, 160) are points on the graph of ①.

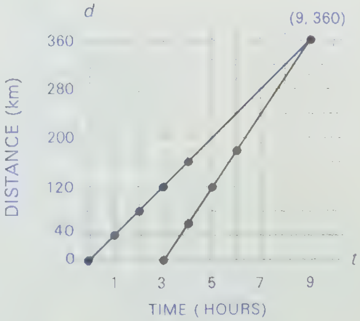
$d = 60t - 180$ ②

t	3	4	5	6
d	0	60	120	180

$\therefore$ (3, 0), (4, 60), (5, 120), (6, 180) are points on the graph of ②.

Why don't we use $t = 0, 1$ or 2 for equation ②?

Then we graph these points on the same set of axes. The final graph is shown in the margin.



From the graph, we see that the point common to both lines is (9, 360). Finding t and d from this point is called *solving* equations ① and ② graphically. Hence, the cars are together 9 h after car A started and after both have travelled 360 km.

We verify* our answer by checking to see that $t = 9$ and $d = 360$ satisfy both equations ① and ②.

* "Verify" means to check!

①

LS	RS
d	$40t$
$= 360$	$= 40(9)$
	$= 360$

②

LS	RS
d	$60t - 180$
$= 360$	$= 60(9) - 180$
	$= 540 - 180$
	$= 360$

$\because$ Both equations are satisfied
$\therefore$ the 9 h time is correct.

Example 2

Solve graphically these simultaneous equations.

$$2x + y = -2 \quad ①$$
$$\frac{x}{3} - \frac{y}{2} = -3 \quad ②$$

Solution

Before making a table of values, we can make the equations easier to work with as follows.

$2x + y = -2$ ①

$y = -2 - 2x$ ③

x	0	−1	−2
y	−2	0	2

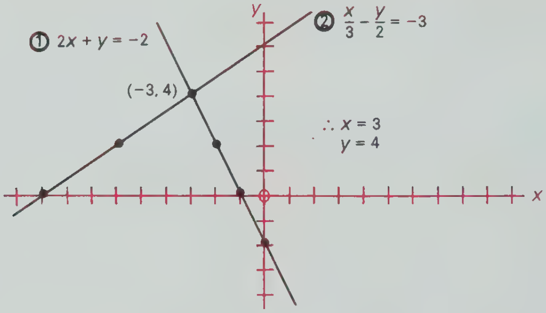
$\frac{x}{3} - \frac{y}{2} = -3$ ②

$x = -9 + \frac{3}{2}y$ ④

x	−6	−9	0
y	2	0	6

Multiply ② by 6 to clear fractions.

$6\left(\frac{x}{3}-\frac{y}{2}\right)=6\times(-3)$
$2x-3y=-18$
$2x=-18+3y$
$x=-9+\frac{3}{2}y$

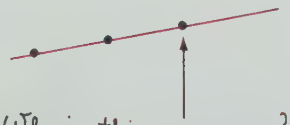


∴ The 2 lines intersect at the point $(-3, 4)$, and $x = -3$, $y = 4$ is the solution.

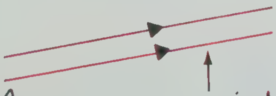
Exercise 12.2

Discussion

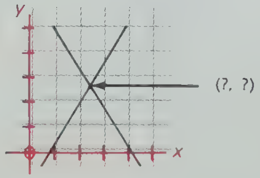
1. If only 2 points are required to fix the position of a line, why do we find 3 points and plot them?
2. Could this graphical method be used to solve *any* pair of simultaneous equations?
3. What limitations can you see to solving pairs of equations graphically?
4. Why is it necessary to substitute in the *original* equations when checking an answer?



Why is this necessary?



Any common points here?



Written Solutions

5. In each equation, express one variable in terms of the other. (Try to pick the "easier" variable!)
   (*a*) $x + y = 5$ (*c*) $2x + y = 2$ (*e*) $x - y = 8$
   (*b*) $2x - y = 1$ (*d*) $3x - y = -7$ (*f*) $3x + 2y = -1$

6. Make a table of values for each equation of Question 5. (Find at least 3 ordered pairs for each equation.)

7. Solve the following pairs of simultaneous equations graphically, using the information from Questions 5 and 6. Check your solutions.
   (*a*) $x + y = 5$, $2x - y = 1$ (*b*) $2x + y = 2$, $3x - y = -7$ (*c*) $x - y = 8$, $3x + 2y = -1$

8. Solve the following pairs of equations. Check your solutions.
   (*a*) $y = -1 - x$
   $y = 2x - 7$
   (*b*) $x - 3y = -2$
   $2x + y = -4$
   (*c*) $y = x$
   $5x + 4 = 7y$
   (*d*) $x = -1 - 6y$
   $2x + 5y = 5$
   (*e*) $-4x + y = 2$
   $-9 = 3x - 2y$
   (*f*) $x + y = -7$
   $3x - 4y = 0$
   (*g*) $11x + y = 8$
   $2x + 3y = -7$
   (*h*) $x - 5 = y$
   $2x = y + 12$
   (*i*) $\dfrac{x}{3} - y = -6$
   $\dfrac{x + 2y}{7} = 1$
   (*j*) $2x - 3y = 1$
   $6y = 4x - 10$

9. Graph the following pairs of equations to find *approximate* solutions for x and y.
   (*a*) $x + 4y = 5$
   $3x - 2y = 8$
   (*b*) $x - y = -3$
   $3x + 5y = 11$

10. The height of water, h centimetres, in 2 containers after time, t seconds, is given by the equations

$$h = 4t \qquad \text{for container A}$$
$$h = -2t + 12 \qquad \text{for container } B$$

Graphically find the time when the 2 containers contain the same height of water.

Careful in 8(j)!

5(a) $y=5-x$ (e) $y=x-8$
(c) $y=2-2x$

6(a)

x	1	3	5
y	4	2	0

(c)

x	-1	0	1
y	4	2	0

7(a) (2, 3) (b) $(-1, 4)$

8(a) $(2, -3)$ (g) $(1, -3)$
(d) $(5, -1)$ (i) $(-3, 5)$

9(a) (3, 0.5) (b) $(-0.5, 2.5)$

10. After 2 s

11. 360 tickets

11. The cost (dollars) of renting a baseball park is given by the formula $R = 900$. The gate receipts (dollars) for a game are given by the formula $R = 2.5n$, where n is the number of tickets sold. Graph these two equations and determine the number of tickets that must be sold before a profit may be realized.

12. Write a computer program that will produce ordered pairs for graphing $17x - 37y + 291 = 0$. Choose intervals of 0.2 for x.

12.3 Solving Equations by Elimination

Graphical solution
↓
approximate answers

Algebraic solution
↓
exact answers

The main disadvantage to solving two simultaneous equations graphically is that the answers are obtained from a diagram and the diagram may not be accurate. If fractions are involved, for instance, the solution will only be approximate. For this reason, an algebraic method, as explained below, can be used to obtain exact answers.
Study the following examples carefully.

Example 1

Solve algebraically.

$x + y = 5$ ①
$x - y = 1$ ②

Solution

Since you know how to solve a single equation in only one variable, we must reduce equations ① and ② to a single equation in x or y. This is done by eliminating *one* of the variables.

$x + y = 5$ ①
$\underline{x - y = 1}$ ②
Add: $2x = 6$
∴ $x = 3$
Substitute $x = 3$ in ①
∴ $3 + y = 5$
∴ $y = 2$

Check: If $x = 3$, $y = 2$

①

LS	RS
$x + y$	5
$= 3 + 2$	
$= 5$	

②

LS	RS
$x - y$	1
$= 3 - 2$	
$= 1$	

$\boxed{\therefore x = 3, y = 2}$

By adding we eliminate y and get one equation in x. Of course, we might just as well have subtracted to eliminate x.

$x + y = 5$ ①
$\underline{x - y = 1}$ ②
Subtract. $2y = 4$
∴ $y = 2$

This is called the Method of Elimination.

Example 2

Solve by elimination.

$$5x - 2y = -3 \qquad \text{①}$$
$$3x + 4y = 6 \qquad \text{②}$$

Solution

Here, neither addition nor subtraction eliminates either variable. In order to eliminate the x terms, the terms in x must be opposites (adding will cancel them) or they must be identical (subtracting will eliminate them). The same reasoning applies to the y terms. Hence, we proceed as follows.

Better check this out!

$$\begin{array}{rl} & 5x-2y=-3 \\ & \underline{3x+4y=6} \\ \text{Add.} & 8x+2y=3 \end{array}$$

$$\begin{array}{rl} & 5x-2y=-3 \\ & \underline{3x+4y=6} \\ \text{Subtract.} & 2x-6y=-9 \end{array}$$

She's right!

$$\begin{array}{lcl} 5x - 2y = -3 \quad \text{①} & \overset{\times 2}{\Longleftrightarrow} & 10x - 4y = -6 \quad \text{③} \\ 3x + 4y = 6 \quad \text{②} & \Longleftrightarrow & \underline{3x + 4y = 6} \quad \text{②} \\ & \textit{Add}: & 13x \qquad = 0 \\ & & \therefore x = 0 \end{array}$$

Substitute $x = 0$ in ②*

$\therefore 3(0) + 4y = 6$

or $\therefore 0 + 4y = 6$

$\therefore y = \frac{6}{4} = \frac{3}{2}$

* We could just as well have substituted in ①

Check: If $x = 0$, $y = \frac{3}{2}$

①

LS	RS
$5x - 2y$	-3
$= 5(0) - 2(\frac{3}{2})$	
$= 0 - 3$	
$= -3$	

②

LS	RS
$3x + 4y$	6
$= 3(0) + 4(\frac{3}{2})$	
$= 0 + 6$	
$= 6$	

$$\boxed{\therefore (x, y) = (0, \tfrac{3}{2})}$$

Example 3

Solve algebraically.

$$\begin{array}{rl} \frac{3x}{2}+\frac{5y}{4}=1 & ① \\ \frac{1}{2}-\frac{3y}{4}=\frac{7x}{8} & ② \end{array}$$

Solution

Once again, addition or subtraction are of no immediate use. First, clear the fractions.

$$\begin{array}{rlcrl} \frac{3x}{2}+\frac{5y}{4}=1 & ① & \overset{\times 4}{\Longleftrightarrow} & 6x+5y=4 & ③ \\ \frac{1}{2}-\frac{3y}{4}=\frac{7x}{8} & ② & \overset{\times 8}{\Longleftrightarrow} & 4-6y=7x & ④ \end{array}$$

If 4−6y=7x
then 4 =7x+6y
or 7x+6y=4

Next, rearrange the terms in ④ to "line up" with those in ③.

$$\begin{array}{rl} 6x+5y=4 & ③ \\ \rightarrow 7x+6y=4 & ⑤ \end{array}$$

Suppose we choose to eliminate y.

$$\begin{array}{rlcrl} 6x+5y=4 & ③ & \overset{\times 6}{\Longleftrightarrow} & 36x+30y=24 & ⑥ \\ 7x+6y=4 & ⑤ & \overset{\times(-5)}{\Longleftrightarrow} & \underline{-35x-30y=-20} & ⑦ \\ & & & \textit{Add:}\quad x \qquad\quad =4 & \end{array}$$

Pretty good, eh!!
Why −5 instead of 5?
(I'd rather add than subtract!)

Substitute $x = 4$ in ③

$$\begin{array}{rl} & 6(4)+5y=4 \\ \text{or} & 24+5y=4 \\ \text{and} & 5y=-20 \\ \therefore & y=-4 \end{array}$$

Check: $x = 4$, $y = -4$

①

LS	RS
$\frac{3x}{2}+\frac{5y}{4}$ $=\frac{3(4)}{2}+\frac{5(-4)}{4}$ $=\frac{12}{2}+\frac{-20}{4}$ $=6-5$ $=1$	1

②

LS	RS
$\frac{1}{2}-\frac{3y}{4}$ $=\frac{1}{2}-\frac{3(-4)}{4}$ $=\frac{1}{2}+\frac{12}{4}$ $=\frac{1}{2}+3$ $=3\frac{1}{2}$	$\frac{7x}{8}$ $=\frac{7(4)}{8}$ $=\frac{28}{8}$ $=\frac{7}{2}$

$$\boxed{\therefore\ x=4,\ y=-4}$$

Exercise 12.3

Discussion

1. Does it matter which variable is eliminated from a pair of simultaneous equations? Explain.
2. In checking your answers, why is it always necessary to go back and check in the *original* pair of equations?
3. State the first step you would take in solving each pair of equations.

(*a*) $\begin{aligned} x + y &= -3 \\ -x + y &= 7 \end{aligned}$ (*b*) $\begin{aligned} 3x + y &= 1 \\ 7x - 2y &= -2 \end{aligned}$ (*c*) $\begin{aligned} x - y &= 4 \\ 3x + 2y &= 7 \end{aligned}$

4. Solve for x and y.

(*a*) $\begin{aligned} x + y &= 7 \\ x - y &= 1 \end{aligned}$

(*b*) $\begin{aligned} 2x + y &= 11 \\ 3x - y &= 14 \end{aligned}$

(*c*) $\begin{aligned} x + 2y &= 5 \\ 5x - 2y &= 1 \end{aligned}$

(*d*) $\begin{aligned} 2x + y &= 11 \\ x + y &= 7 \end{aligned}$

(*e*) $\begin{aligned} 2x + 3y &= 13 \\ 2x - y &= 9 \end{aligned}$

(*f*) $\begin{aligned} 3x - 4y &= 7 \\ 3x - 7y &= 1 \end{aligned}$

Written Solutions

5. Solve and check.

(*a*) $\begin{aligned} x + y &= 4 \\ 2x - y &= 8 \end{aligned}$

(*b*) $\begin{aligned} 3x + 2y &= 1 \\ x - 3y &= 4 \end{aligned}$

(*c*) $\begin{aligned} 2x - y &= -7 \\ x - 3y &= 4 \end{aligned}$

(*d*) $\begin{aligned} 6x &= 3y \\ 4x - y &= 1 \end{aligned}$

(*e*) $\begin{aligned} 0.5x + 1.5y &= 4.3 \\ 3.5x - 2.5y &= 6.7 \end{aligned}$

(*f*) $\begin{aligned} 5x - 3y &= 0 \\ 3x + 4y &= 0 \end{aligned}$

(*g*) $\begin{aligned} y - x &= 15 \\ 3x &= 3 - y \end{aligned}$

(*h*) $\begin{aligned} -2 &= 6x + 5y \\ \frac{2x + y}{2} &= \frac{1}{15} \end{aligned}$

(*i*) $\begin{aligned} 2x - 3y &= 1 \\ 6y &= 4x - 10 \end{aligned}$

(*j*) $\begin{aligned} 0.7x + 2y &= 6.65 \\ 3x - 10y &= -23.5 \end{aligned}$

Do these in your head!

5(a) (4, 0) (g) (−3, 12)
(c) (−5, −3) (j) (1.5, 2.8)
(e) (3.2, 1.8)

6(a) $x=-1-y$
(b) $3(-1-y)-2y=-8$
(c) $y=1$
(d) $x=-2$

7(a) $(x, y)=(1, 3)$
(c) $(x, y)=(1, -2)$

8(a) $y=5-2x$
$y=3x-15$
(b) $x=4$
(c) $y=-3$

9(a) $(x, y)=(-2, 2)$
(c) $(x, y)=(5, 3)$

6. Solve $x + y = -1$ ①
$3x - 2y = -8$ ②
according to the following instructions.
   (*a*) From ① express x in terms of y.
   (*b*) Substitute this value for x in equation ② to obtain a single equation in y.
   (*c*) Solve the resulting equation for y.
   (*d*) Substitute this value of y into the equation from part (*a*), and solve for x.
   (*e*) Check your answers.

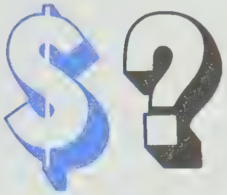
This method of solving 2 simultaneous equations is called the method of *substitution.*

7. Now use the method of substitution to solve each pair of equations.
   (*a*) $x - y = -2$
   $4x + 5y = 19$
   (*b*) $x + 3y = -5$
   $5x - 2y = -42$
   (*c*) $2x + y = 0$
   $4x - 3y = 10$
   (*d*) $3x - 5y - 21 = 0$
   $-2x + 6y + 22 = 0$

8. Solve $2x + y = 5$
$3x - y = 15$
according to the following instructions.
   (*a*) In each equation, express y in terms of x.
   (*b*) Equate the two values for y obtained in part (*a*) and solve for x.
   (*c*) Substitute this value of x back into one of the equations from part (*a*) and solve for y.
   (*d*) Check your answers.

This method of solving 2 simultaneous equations is called the method of *comparison.*

9. Use the method of comparison to solve each of the following.
   (*a*) $3x - 2y = -10$
   $3x + y = -4$
   (*b*) $x - 2y = 3$
   $3x + 4y = 14$
   (*c*) $2x - y = 7$
   $x + 4y = 17$
   (*d*) $x - 6y + 4 = 0$
   $2x - 3y + 5 = 0$



The Missing Dollar

Three representatives of the Hardsell Haberdashery Company checked into a hotel to spend the night; each paid $10 toward the cost of the room.

Not long after the 3 had been shown to their suite, the hotel manager, Nimrod Noble, discovered that the rate for the room the representatives had been given was in fact only $25 per night. Being honest and upright, the manager gave the bellhop five $1 bills and sent him up to make the refund.

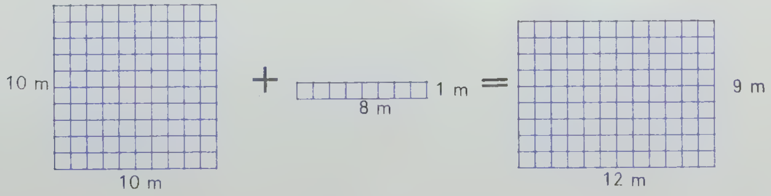
Now the bellhop, Quincy Quickthinker by name, felt that it would be rather awkward to refund $5 to 3 people. So to make his job easier, he pocketed $2 and refunded each rep $1.

This means that it cost each rep only $9 for the night's accommodation. Which brings us to the heart of the matter . . . . (Ready?) If each of the 3 representatives paid $9 for the room, a total of $27, and the bellhop kept $2 for himself . . . . Let's see now: $27 + 2 = $29! *Where did the other $1 get to*?

The Cut-Up Caper

Zelda Zigzag, owner of the Arabian Knights Rug Company, was in distress over a situation involving her best client, Meriwell Moneybags. Meriwell had ordered a very expensive ($69.96/m²!), 9 m by 12 m, hand-woven piece of broadloom rug to be installed in the family room of his house. By some quirk of fate, the carpeting arrived — but in 2 pieces: one 10 m by 10 m and the other 1 m by 8 m.

Imagine Zelda's utter amazement and profound joy when her teen-age daughter (living up to the *family name* — this is a clue!) managed to produce the required 9 m by 12 m rug from the 10 m by 10 m and the 1 m by 8 m pieces. This feat was all the more surprising since it was done by making a *single cut* in the 10 m by 10 m rug. Can you explain how it was done?



13

Unit 13
Word Problems

Recall that an algebraic solution for a word problem consists of five main steps.

(1) Draw a diagram (if possible).
(2) Write a "Let" statement.
(3) Write an equation.
(4) Solve the equation.
(5) Make a final statement.

Steps ② and ⑤ are in English; units are needed.
Steps ③ and ④ are in algebra; no units are needed.

Get-Set Exercise — Word Problems

1. Convert the following expressions into mathematical terms.
   (*a*) 7 less than the number
   (*b*) 40 divided by the number
   (*c*) 9 plus the square of the number
   (*d*) the product of 2, and the number increased by 3
   (*e*) the product of 2 and the number, increased by 3
   (*f*) the sum of 16 and the number, divided by 8
   (*g*) the difference between 7 and the number, plus 2

1(a) Like this!
$x - 7$

Commas are important!

How is this for starters? "3 more than the number"

2. State in words.
   - (*a*) $x + 3$
   - (*b*) $5 - x$
   - (*c*) $2x + 4$
   - (*d*) $3y - 7$
   - (*e*) $0.5m$
   - (*f*) $6t \div 3$
   - (*g*) $4(a - 16)$
   - (*h*) $3 + 2k$
   - (*i*) $(4 - 7) \div 3$
   - (*j*) $2 + \frac{3}{r}$

3. Solve.
   - (*a*) $x + 7 = 12$
   - (*b*) $2x - 3 = 19$
   - (*c*) $5x + 2 = -3$
   - (*d*) $x - 3 = 2x + 4$
   - (*e*) $3(x + 5) = -9$
   - (*f*) $2(x - 1) = 3(x + 5)$
   - (*g*) $4 - x = 7 - 4x$
   - (*h*) $\frac{x}{2} + \frac{x}{3} = 5$

4. Give a complete algebraic solution for each of the following.
   - (*a*) The product of 4 and a number, plus 7, is 15. Find the number.
   - (*b*) If 16 is decreased by the product of 3 and a number, the result is 22. Find the number.
   - (*c*) The product of 2, and a number increased by 3 is 16. Find the number.

5. State the value in cents.
   - (*a*) 2 nickels
   - (*b*) 19 nickels
   - (*c*) *x* nickels
   - (*d*) 5 quarters
   - (*e*) 9 dimes
   - (*f*) *m* dimes
   - (*g*) 7 dimes and 11 nickels
   - (*h*) *x* dimes and *y* nickels

3(a) $x=5$ **(e)** $x=-8$
(c) $x=-1$ **(g)** $x=1$
4(a) 2 **(c) 5**
5(a) 10¢ **(e) 90¢**
(c) 5*x*¢ **(g) 125¢**

13.1 Number Problems

Sherman Sweetooth stops at the candy store and purchases some Dentist's Delight candy chews and some Molar Masher jawbreakers, receiving 42 candies in all. If Sherman bought eight more jawbreakers than candy chews, how many of each did he buy?

Using simultaneous equations of the type found in Unit 13, you will find out in this unit how to solve word problems like the above. We call these problems *number problems* because they deal with numbers only (no units are involved).

Example 1

There are 33 students in form 10M. If there are twice as many boys as girls, determine how many of each are in the class.

Bet I can get the answer by guessing!

Girls	Boys	Total
9	18	27
10	20	30
11	22	33 Aha!

Solution

Let x represent the number of boys and y the number of girls.

$$\begin{array}{rrcl}
 & \therefore\ x + y = 33\ ① & \Longleftrightarrow & x + y = 33\ ① \\
\text{and} & x = 2y\ ② & \Longleftrightarrow & x - 2y = 0\ ③ \\
\hline
 & \textit{Subtract.} & & 3y = 33 \\
 & & & \therefore\ y = 11
\end{array}$$

Here we eliminate x

Substitute $y = 11$ in ② *

$\therefore x = 2(11) = 22$

* Could we have substituted in ① instead?

Check If $x = 22$, $y = 11$,

(i) total number of students is $22 + 11 = 33$

and (ii) $2(11) = 22$, so there are twice as many boys.

$\therefore$ There are 22 boys and 11 girls in 10M.

Notice that we check in the word problem itself, not in equations ① and ②. Why is this?

Example 2

Sam Slik slyly slips a paper with 2 numbers written on it to Terri Kloth. The numbers differ by 8, and 3 times the smaller is 10 more than the larger. What are the numbers?

Try a table again!

Numbers		3 Times the Smaller
3	11	9 too
4	12	12 small!
15	23	45 too big!
10	18	30 closer
9	17	27 Aha!

Solution

Let x represent the larger number and y the smaller number.

$$\begin{array}{rrcl}
 & \therefore\ x - y = 8\ ① & \Longleftrightarrow & x - y = 8\ ① \\
\text{and} & 3y = x + 10\ ② & \Longleftrightarrow & -x + 3y = 10\ ③ \\
\hline
 & & \textit{Add.} & 2y = 18 \\
 & & & \therefore\ y = 9
\end{array}$$

Substitute $y = 9$ in ①

$\therefore \quad x - 9 = 8$

$\therefore \quad x = 17$

Check If $x = 17$, $y = 9$

(i) the difference between the numbers is $17 - 9 = 8$.

(ii) $3(9) = 27$, and 27 is 10 more than 17.

$\therefore$ The numbers are 17 and 9.

4. **11, 16**
6. **10, 12**
8. **10 golfballs, 24 tees**
9. **Betty, 5 parts**
   Billy, 14 parts

Remember to distinguish between the smaller number and the larger number!

Exercise 13.1

Discussion

1. Why is it necessary to check a word problem by verification in the word problem itself, rather than in the equations formed from the problem?

2. Make up a word problem from which each pair of equations might have been formed.

   (*a*) $x + y = 5$
   $x - y = 1$

   (*b*) $x = 3y$
   $x + y = 8$

3. Solve each pair of equations in Question 2.

Written Solutions

(Give an algebraic solution for each problem.)

4. Two numbers have a sum of 27 and differ by 5. Find them.

5. One number is 3 times as large as another and exceeds the smaller by 14. Find the numbers.

6. The sum of two numbers is 22. If the smaller number is doubled and then added to the larger number, the result is 32. Find the numbers.

7. In a class of 30 students, four times as many passed a math exam as failed it. How many failed the exam?

8. Patti Parthemall purchased some golf balls and tees at the pro shop before she started her first game of the season. If she bought 34 items altogether and if there were 14 more tees than balls, how many of each did Patti buy?

9. In rebuilding their motorbikes after a recent mishap, Billy Byways and Betty Buzzalong together purchased 19 repair parts. If Billy bought 9 more parts than Betty, how many parts did each of them buy?

10. In her capacity as summer playground supervisor, Hilda Healthy placed an order for 143 items, consisting of tennis balls and badminton birds only. If there were 2 more badminton birds than twice the number of tennis balls, how many of each item did Hilda order?

11. Harry Hikemore, in planning a week's trek through the Rocky Mountains, packed 125 items, consisting of perishable or non-perishable goods, into his rucksack. If there were 5 more non-perishable items than 3 times the number of perishable items, how many items of each kind did Harry pack?

12. Comparing babysitting chores over the past year, Fiona Firmstand and Leslie Letemgo find that their combined sitting total is 97 sittings. If Fiona sat 17 more times than Leslie, find the number of sitting assignments for each student.

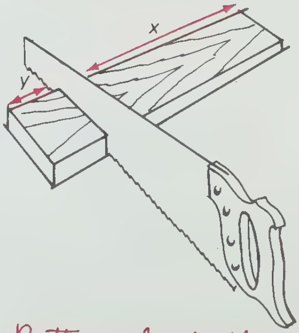
10. **96 badminton birds, 47 tennis balls**
11. **95 non-perishable, 30 perishable**
12. **Fiona 57, Leslie 40**

13.2 Measurement Problems

These problems deal with measured quantities for which *units* are required.

Example 1

In helping her father build a sauna, Sally Sawstraight cuts a board 5.5 m long so that the longer part is 0.1 m greater than twice the shorter part. Find the length of each part.



Solution

Let x metres represent the length of the longer part and y metres represent the length of the shorter part.

$$\begin{array}{llcrl}
\therefore & x+y=5.5 \ \textcircled{1} & \overset{\times(-2)}{\Longleftrightarrow} & -2x-2y= & -11.0 \ \textcircled{3} \\
\text{and} & 2y+0.1=x \ \textcircled{2} & \Longleftrightarrow & -x+2y= & -0.1 \ \textcircled{4} \\
\hline
 & & \textit{Add.} & -3x= & -11.1 \\
 & & \therefore & x= & 3.7
\end{array}$$

Substitute $x = 3.7$ in ①:

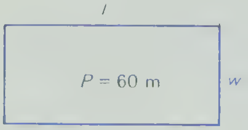
$$3.7 + y = 5.5$$
$$\therefore \quad y = 1.8$$

$\therefore$ The parts are 3.7 m and 1.8 m in length.

Better check this!
(i) Total length is 3.7 m + 1.8 m = 5.5 m
(ii) Twice the shorter part is 2 × 1.8 m or 3.6 m, and the longer part (3.7 m) is 0.1 m longer than this.

Example 2

The perimeter of a rectangular lawn is 60 m. If the length is 4.2 m shorter than twice the width, find the dimensions.



Solution

Let the length of the lawn be l metres and the width of the lawn be w metres.

$\therefore\ 2(l+w)=60$ ① $\Leftrightarrow$ $2l+2w=60.0$ ③

and $l+4.2=2w$ ② $\overset{\times 2}{\Leftrightarrow}$ $2l-4w=-8.4$ ④

Subtract. $6w = 68.4$

$\therefore\ w = 11.4$

Substitute $w = 11.4$ in ②:

$$l+4.2=2(11.4)$$
$$l=22.8-4.2$$
$$\therefore\ l=18.6$$

Can you check this?

∴ The dimensions of the lawn are 18.6 m by 11.4 m.

Exercise 13.2

Discussion

1. How do you check the answer to a word problem?

2. Complete the following steps in solving a measurement problem.

What steps may involve units?

(*a*) Draw a ....
(*b*) Write a .... statement.
(*c*) .... the equation.
(*d*) .... the equation.
(*e*) Make a ....

Written Solutions

3. A board 4.3 m long is cut into 2 parts so that 1 part is 0.9 m longer than the other. Find the length of each part.

4. A rectangular piece of sheet metal is 3 cm longer than twice its width. If its perimeter is 72 cm, find its dimensions.

5. A piece of cardboard is 0.2 m longer than it is wide. If its perimeter is 2.8 m, find its dimensions.

3. 1.7 m, 2.6 m
4. 25 cm by 11 cm
5. 0.6 m by 0.8 m

6. To raise money for the local Crippled Children's Fund, Nora and Nellie Neverquit walked a total of 67.2 km. If Nora walked 6.4 km farther than Nellie, how far did each girl walk?

7. The perimeter of a basketball court is 88 m, and the length is 10 m greater than the width. Determine the dimensions of the court.

8. In a cosy tête-à-tête in the ski lodge after skiing one brisk winter's day, Sam Slopeslider and Helen Hotdogger discover that the difference between their ski lengths is only 15 cm and that the sum of the lengths of the 2 skis is 365 cm. Calculate the length of each skier's skis.

9. After their takeoff from the dock, waterskiers Philip Flipp and Flora Flopp have a combined "stay-up" time of 8.0 min. If Flora stayed up 0.6 min longer than Phil, find their individual "stay-up" times.

10. Ruth and Rita Rackemup, stars of the Rallytown ringette team, skate a total of 4.1 km in one game. However, since Ruth plays forward, she skates 0.3 km farther than Rita, who plays defence. How far did each girl skate in the game?

11. Billy and Benny walk to and from school each day; the sum of the distances of their homes from the school is 2.0 km. If the difference between these two distances is 1.6 km and Billy walks the greater distance, how far does each boy live from the school?

12. A certain number of square wall tiles are placed side by side to form a rectangle that is 1 tile wide. The perimeter of this rectangle is 100 cm. When the rectangle is made 1 tile longer, the perimeter increases to 120 cm. How many tiles were used to form the original rectangle, and what are the dimensions of each tile?

6. **Nora, 36.8 km**
Nellie, 30.4 km

7. **27 m by 17 m**

8. **Sam's skis, 190 cm**
Helen's skis, 175 cm

9. **Flora, 4.3 min**
Phil, 3.7 min

10. **Ruth, 2.2 km**
Rita, 1.9 km

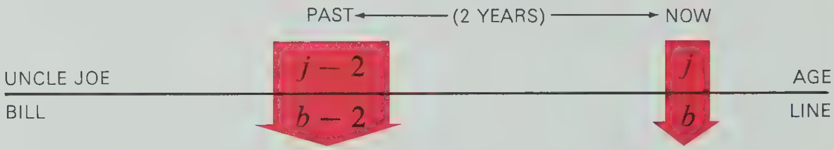
11. **Billy, 1.8 km**
Benny, 0.2 km

12. **4 tiles; each tile**
10 cm by 10 cm

13.3 Age and Mass Problems

Sam: "Bill, how old is your Uncle Joe?"
Bill: "Well, he's 25 years older than I am now, and 2 years ago the sum of our ages was 55."
Sam: "You sure like to complicate things, don't you! Give me a piece of paper and a pencil, and I'll work it out."

Here is Sam's solution to the problem.
Let Bill's present age be b years, and his uncle's present age be j years.



$\therefore (j-2)+(b-2)=55$ ① and $j-b=25$ ②

From ① $j+b=59$ ③
and $j-b=25$ ②
Add. $2j=84$
$\therefore j=42$

Substitute $j=42$ in ②:
$42-25=b$
$\therefore b=17$

I'll unscramble this for you!

If $(j-2)+(b-2)=55$ ①
then $j-2+b-2=55$
and $j+b-4=55$
$\therefore j+b=59$ ③

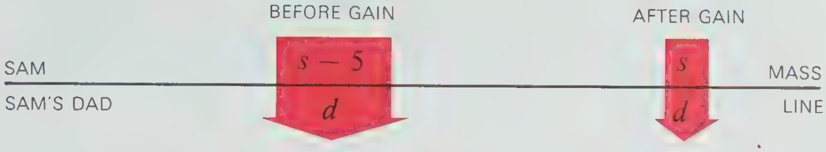
$\therefore$ Bill is 17 years old and his Uncle Joe is 42 years old.

Bill: "Not bad, not bad! But say, you seem to lean heavily on that pencil! Gained a bit lately?"
Sam: "Oh, a bit, I guess."
Bill: "Bet your mass is greater than mine! What *is* your mass?"
Sam: "My dad and I together have a mass of 165 kg and, although his mass hasn't changed, before I gained 5 kg his mass was 50 kg more than mine."
Bill: "Now it's my turn — let's see the back of that piece of paper!"

And Bill wrote the solution that appears on the next page.

Revenge is sweet...

Let Sam's present mass be s kilograms and his dad's be d kilograms.



$$\therefore d - (s - 5) = 50 \quad ① \quad \text{and} \quad s + d = 165 \quad ②$$

From ① $\quad -s + d = 45 \quad ③$

and $\quad s + d = 165 \quad ②$

Add. $\quad 2d = 210$

$$\therefore d = 105$$

Substitute $d = 105$ in ②:

$$s + 105 = 165$$

$$\therefore s = 60$$

$\therefore$ Sam's present mass is 60 kg.

Exercise 13.3

See if you can do as well as Sam and Bill in the following questions.

Written Solutions

1. Alphonse is 5 years younger than Brunhilda, and in 7 years the sum of their ages will be 47 years. How old is each one now?
2. Before he went on a diet, Peter's mass was 16 kg less than Joe's. After each had lost 5 kg, their combined mass was 102 kg. Determine their original masses.
3. Heidi, the performing killer whale at Sealand in Victoria, B.C., has a mass 22 times greater than her trainer's. If the combined mass of Heidi and her trainer is 1955 kg, find their individual masses.
4. Mr. Rightis is twice as old as his daughter, Artha. If the sum of their ages in 9 years will be 81 years, what are their present ages?
5. Darlene Dragster's beefed-up Cougar Special has mass 225 kg greater than Wally Wheelspin's stripped-down Mustang. If the 2 cars have a combined mass of 2775 kg, find the mass of each.

1. Alphonse, 14 years
Brunhilda, 19 years

2. Peter, 48 kg
Joe, 64 kg

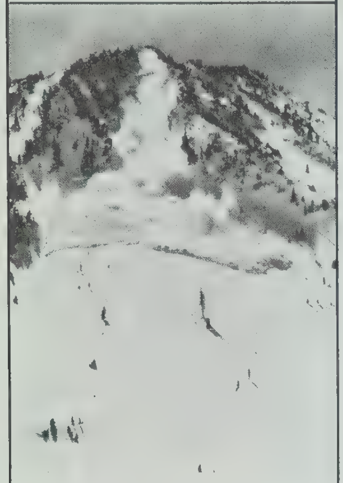
3. Heidi, 1870 kg
trainer, 85 kg

4. Mr. Ritis, 42 years
Artha Ritis, 21 years

5. Cougar, 1500 kg
Mustang, 1275 kg

6. Sally, 32 years
Bertie, 24 years

7. Helicopter, 1980 kg
TNT, 180 kg



6. Sally Single and Bertie Bachelor, stars of the recent motion picture, "Moonlight and Roses," are asked for their ages for the local newspaper. Knowing Bertie's age, Sally replies, "The sum of our present ages is 56 years, and 10 years ago I was 2 years younger than Bertie is now." How old are Sally and Bertie?

7. In order to provide safe skiing areas atop Whistler's Mountain north of Vancouver, an Avalanche Control Team "bombs" potential danger areas with TNT dropped by helicopter. If the 'copter's mass is 180 kg more than 10 times the mass of its TNT payload and the total mass of 'copter and TNT is 2160 kg, calculate the masses of the 'copter and the TNT.

13.4 Number-Value Problems

Quite often we are concerned not only with the *number* of items, but also with their *worth* or *value*. For instance, the statements

6 articles costing \$12
5 books worth \$30
9 coins worth 45¢
4 bricks with mass 6 kg

all deal with value as well as number. The solution of problems of this type is illustrated by the following examples.

Example 1

As Stan entered the machine shop, his instructor handed him a box of ball bearings and said, "Yesterday Betty counted these bearings, but did not have time to determine how many were 50 g size and how many were 40 g size. There are 210 altogether, and the total mass is 9.33 kg. Can you find out how many there are of each size?"

Looks like a tough problem!

Stan's work follows.

Solution

9.33 kg = 9330 g

	Number	*Mass* (g)
Let the number of 50 g ball bearings be	x	$50x$
Let the number of 40 g ball bearings be	y	$40y$
Totals	210	9330

Notice that all masses are changed to a common unit!

$$\begin{array}{rlcrl}
\therefore & x+y=210 \ \text{①} & \overset{\times 4}{\Longleftrightarrow} & 4x+4y= & 840 \ \text{③}\\
\text{and} & 50x+40y=9330 \ \text{②} & \overset{\div 10}{\Longleftrightarrow} & 5x+4y= & 933 \ \text{④}\\
 & & \textit{Subtract.} & -x = & -93\\
 & & & \therefore x = & 93
\end{array}$$

Substitute $x = 93$ in ①:

$$933 + y = 210$$
$$\therefore \quad y = 117$$

Check

(i) $93 \times 50\text{ g} = 4650\text{ g}$
$117 \times 40\text{ g} = \underline{4680\text{ g}}$
9330 g
$= 9.33\text{ kg}$

(ii) $93 + 117 = 210$

∴ There are 93 50-g ball bearings and 117 40-g ball bearings.

Example 2

On her paper route, Candie Carrimore collects $8 in quarters and half-dollars only. If there are 11 more quarters than half-dollars, how many of each type of coin did she collect?

Solution

	Number	*Value* (¢)
Let the number of quarters be	q	$25q$
Let the number of half-dollars be	h	$50h$
		800

Notice the unit!

$$\begin{array}{rlcrl}
\therefore & q-h=\overset{*}{11} \ \text{①} & \overset{\times 50}{\Longleftrightarrow} & 50q-50h= & 550 \ \text{③}\\
 & 25q+50h=800 \ \text{②} & \Longleftrightarrow & 25q+50h= & 800 \ \text{②}\\
 & & \textit{Add.} & 75q = & 1350\\
 & & & \therefore q = & 18
\end{array}$$

* q is larger than h.

Substitute $q = 18$ in ①:

$$\therefore \quad 18 - h = 11$$
$$\therefore \quad h = 7$$

Check

(i) $18 \times 25¢ = \$4.50$
$7 \times 50¢ = \underline{\ \ 3.50}$
$\$8.00$

(ii) $18 - 7 = 11$

∴ Candie collected 18 quarters and 7 half-dollars.

Exercise 13.4

Discussion

1. Make up "let" statements for each of the following problems.
   (*a*) Charlie Cashem has $115 in $2 and $1 bills only. If there are 14 more $2 bills than $1 bills, how many of each does Charlie have?
   (*b*) Phyllis Philatelist goes to the store to buy some 10¢ and 20¢ postage stamps. If there are twice as many 10¢ stamps as 20¢ stamps, how many of each kind did Phyllis buy with $3.60?
   (*c*) Tickets to the Sports Night Sock Hop sell for $1.50 a couple or $1.00 single. If the number of "couple" tickets is 30 more than the number of "singles" and total ticket sale receipts were $95, how many "singles" were sold?

Written Solutions

2. Give complete solutions for each part of Question 1.

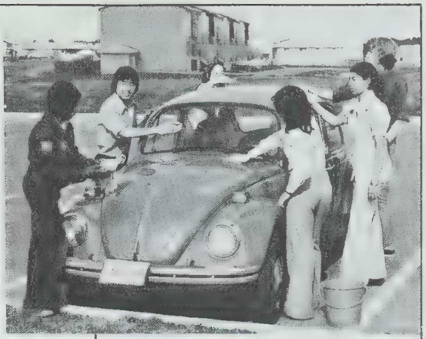
3. A bill for $4.15 is paid in dimes and quarters only. If there are 4 fewer dimes than quarters, find the number of each kind of coin used to pay the bill.

4. Donna Dunkem sets a conference record of 63 points in a high school basketball game, scoring on a total of 40 shots that were either floor shots (2 points) or free throws (1 point). If Donna sank 6 more floor shots than free throws, determine how many of each she sank.

5. Orville Overdraft cashes a cheque for $216.00 and receives $10 and $2 bills only. If the number of $2 bills is 6 less than the number of $10 bills, how many of each kind did Orville receive?

2(a) 43 $2 bills
29 $1 bills
(b) 18 10¢ stamps
9 20¢ stamps
(c) 20 singles
3. 13 quarters, 9 dimes
4. 23 floor shots,
17 free throws
5. 19 $10 bills
13 $2 bills

6. The Klunker Kleaners, a group of concerned teen-agers, held a Car-Wash Carousel at a local service station to raise money for the treatment of muscular distrophy. They charged \$2 to wash a full-size car and \$1.50 to wash a compact car. If they washed 60 cars and took in \$106.50, determine how many of each kind of car they washed.



7. In a metal shop, Garfield Gearbox pours a box of nuts, each with mass 15 g, into a box of bolts and finds the total mass to be 5.3 kg. If there are 20 fewer bolts than nuts and each bolt has mass 35 g, how many nuts and how many bolts are there in the box?

5.3 kg = 5300 g

8. In a recent year-long "Build Our Bleachers" fund drive, Anytown High School sold boxes of candies and received a profit of 50¢ on each box of Peppermint Patties and 70¢ on each box of Almonds in Chocolate. If the fund drive resulted in a profit of \$3525.00 and 750 more boxes of almonds than patties were sold, how many boxes of each type of candy were sold?

9. A Honeybowl football game is attended by 50 000 paying customers, and the tickets are either \$5 each or \$3.50 each. If there are 1000 fewer \$5 tickets than twice the number of \$3.50 tickets, how many of each were sold?

10. Henry Hearmore purchases 3 records and 2 tapes for \$19.25, and Allysa Listener buys 2 records and 4 tapes for \$25.50. Find the cost per record and per tape.

11. Rockus Rythum's Record Shop orders 60-min cassette tapes at \$1.49 and 8-track stereo tapes at \$4.98 from a wholesaler. If a total of 451 tapes were ordered and the invoice for the order is \$1240.86, determine the number of each kind of tape ordered.

◆

6. 33 full-size cars, 27 compacts
7. 120 nuts, 100 bolts
8. 2500 boxes mints, 3250 boxes almonds
9. 33 000 \$5 tickets 17 000 \$3.50 tickets
10. \$3.25/record \$4.75/tape
11. 288 cassette tapes, 163 stereo tapes

Maintaining Basic Skills

Exercise 3

All variables are real numbers.

1. Find 3 ordered pairs for each equation, and graph.
   (*a*) $x + y = 7$ (*b*) $y = 5x$ (*c*) $x - y = 1$

2. Find the x- and y-intercepts in each case, and graph.
   (*a*) $3x + 2y = 6$ (*c*) $2x - 5y = 8$
   (*b*) $y = -4x + 2$ (*d*) $y = 3x$

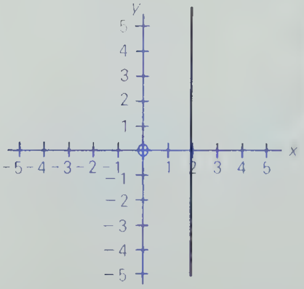
3. Express each equation in the form $y = mx + b$, state the slope and the y-intercept, and graph.
   (*a*) $y - 2x = 0$ (*d*) $3x - y = 6$
   (*b*) $y + 4 = 3x$ (*e*) $4 + 2y = 3x$
   (*c*) $x + 2y = 6$ (*f*) $5x - 2y - 4 = 0$

4. By graphing, find the intersection point of the lines $x - y + 5 = 0$ and $x + 2y = 1$.

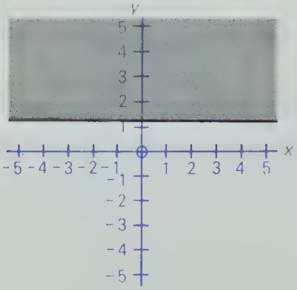
5. Name and graph each curve.
   (*a*) $x^2 + y^2 = 81$ (*c*) $4x^2 + y^2 = 16$
   (*b*) $y = \frac{1}{4}x^2$ (*d*) $3x^2 + 3y^2 = 27$

6. State the equation or inequation of each line or region.

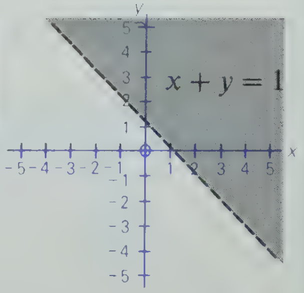
(*a*)



(*b*)



(*c*)



7. Draw the graph of each inequation.
   (*a*) $x \geq 3$ (*d*) $2x + y < 4$
   (*b*) $y < -1$ (*e*) $x^2 + y^2 \leq 100$
   (*c*) $y > 3x + 1$ (*f*) $4x^2 + 9y^2 > 36$

2(a) $2, 3$ **(c)** $4, -\frac{8}{5}$
3(a) $y = 2x;\ 2, 0$
(c) $y = -\frac{1}{2}x + 3;\ -\frac{1}{2}, 3$
(e) $y = \frac{3}{2}x - 2;\ \frac{3}{2}, -2$
5(a) Circle **(c)** Ellipse
(b) Parabola **(d)** Circle
6(c) $x + y > 1$

8. Solve by factoring, or by using the quadratic formula.
   (*a*) $x^2 - 16 = 0$
   (*b*) $x^2 + 5x = 0$
   (*c*) $m^2 - 3m - 28 = 0$
   (*d*) $y^2 - 3y - 1 = 0$
   (*e*) $3k^2 + 4k - 1 = 0$
   (*f*) $6x^2 + x - 2 = 0$

9. Solve.
   (*a*) $4a - 5 = 3a + 2$
   (*b*) $0.75y = -5$
   (*c*) $(t - 2)(t + 3) = t(t - 1) + 4$
   (*d*) $\frac{3}{4}(w + 1) - 5 = \frac{2}{3}(5 + w)$

10. Check by substitution to see if the number given belongs to the solution set of the given equation.
    (*a*) $2m - 3(m + 1) = 7; \quad 2$
    (*b*) $5(m + 1) = 4(m - 3) + 2; \quad -15$

11. Solve.
    (*a*) $-x + y = 2$
    $x + y = 4$
    (*b*) $x + y = 4$
    $2x - y = 5$
    (*c*) $m - 2n = 4$
    $2m + n = 5$
    (*d*) $2y = 5x - 4$
    $10x + 3y = 1$

12. In buying tinned goods for a group of teens going on a weekend hiking trip, Bartholemew Buywell buys only baked beans and fruit cocktail. If Bart purchases 82 tins in all and there are 14 more tins of fruit than of beans, determine how many tins of each item he bought.

13. At Grand River Collegiate, students organized a basketball marathon to raise funds for the Canadian Cancer Society. Ralph Runalong and Hattie Hoopster played for a total of 36 h and raised a total of \$46.40. If Ralph was sponsored for \$1.40/h and Hattie was sponsored for \$1.20/h, find how many hours each played.

14. The famous mathematician, Diophantus of Alexandria (300 AD), spent $\frac{1}{6}$ of his life in childhood, $\frac{1}{12}$ in youth, and $\frac{1}{7}$ more as a bachelor. Five years after he married, his son was born, who died 4 years before his father at half the age his father finally attained. How old was Diophantus when he died?

FACTORING

1 Common factor

2 Difference of squares

3 Trinomial

FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

8(a) $x = 4$ or $x = -4$
(c) $m = 7$ or $m = -4$
(e) $k = \frac{-2 \pm \sqrt{7}}{3}$

9(a) $a = 7$ (d) $w = 91$

11(a) $(x, y) = (1, 3)$
(c) $(m, n) = (\frac{14}{5}, -\frac{3}{5})$

12. 34 tins of beans,
48 tins of fruit

13. Ralph, 16 h
Hattie, 20 h

14

Unit 14
Ratio, Proportion, and Variation

Get-Set Exercise — Ratio

A ratio is a means of comparing 2 or more quantities.

1. Express each ratio in lowest terms.
   - (*a*) 8:12
   - (*b*) $\frac{27}{18}$
   - (*c*) 45:10
   - (*d*) 19:57
   - (*e*) 42:36
   - (*f*) $\frac{72}{18}$
2. In each case, state whether the ratios are *equivalent ratios*.
   - (*a*) 4:2, 12:6
   - (*b*) $\frac{27}{18}$, $\frac{9}{12}$
   - (*c*) 7:21, 0.15:0.45
   - (*d*) 7:8, 16:14
   - (*e*) $\frac{3.5}{7}$, 8:16
   - (*f*) 15:20, 39:52

$a:b \Leftrightarrow \frac{a}{b}\ (b \neq 0)$

3. Rewrite in a different form, if possible.
(*a*) 2:3 (*c*) 42:11 (*e*) $\frac{17}{8}$
(*b*) $\frac{5}{9}$ (*d*) 9:5:2 (*f*) 2:5 = 4:10

4. Express each of the following as a fraction in lowest terms, if possible.
(*a*) 2:3 (*b*) 5.5:0.75 (*c*) 5:7:9 (*d*) 28:49

5. State the missing term (or terms).
(*a*) $\frac{1}{6} = \frac{3}{\square}$ (*d*) $\frac{5}{\square} = \frac{15}{24} = \frac{\square}{16}$
(*b*) $\frac{7}{3} = \frac{\square}{12} = \frac{35}{\square}$ (*e*) 7:▨ = 42:54
(*c*) 9:36 = ▨:24 (*f*) $\frac{3}{0.5} = \frac{18}{\square} = \frac{\square}{2}$

Can you do this in more than one way? →

6. Tell how to test if 3:5 is equivalent to 9:15.

7. State the ratio (in lowest terms) for each of the following.
(*a*) 40 cm, 1 m
(*b*) 100 m, 1 km
(*c*) 5000 g, 1 kg
(*d*) 2500 m, 2 km
(*e*) 60 cm, 0.6 m, 0.006 km

8. To set up a business, Mr. Greene, Ms. Jones, and Mr. Smith invest $1000, $1500, and $1200, respectively.

Why can't this be done using fraction notation? →

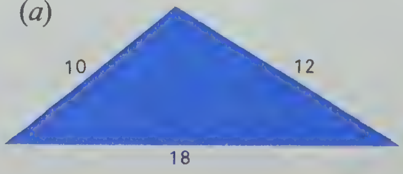
(*a*) What is the ratio of their investments, in lowest terms?
(*b*) Who should receive the largest share of any profit?

9. If Penny Paylater borrows $600 and pays $30 interest, what is the ratio of interest to amount borrowed?

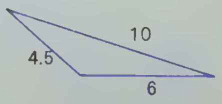
10. An alloy contains 28 kg of tin and 16 kg of lead. What is the ratio of tin to lead? Of lead to tin?

11. Determine the ratio of the sides for each triangle.

(*a*)



(*b*)



7(a) 2:5 (e) 1:1:10
(c) 5:1
8(a) 10:15:12
(b) Ms. Jones
9. 1:20
10. 7:4, 4:7
11(a) 5:9:6 (b) 9:12:20

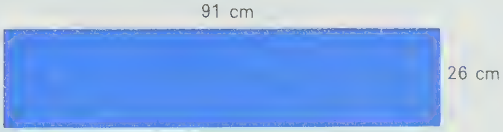
12. Use the test for equivalent ratios to find the value of each variable.
(*a*) $6:5 = r:25$
(*b*) $\frac{3}{10} = \frac{24}{p} = \frac{q}{200}$
(*c*) $r:7 = 81:63 = 45:s$
(*d*) $\frac{5}{12} = \frac{35}{x} = \frac{y}{144} = \frac{150}{z}$

$$6:5 = r:25$$

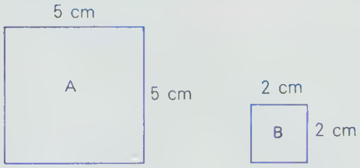
$$\therefore 5r = 6 \times 25$$

13. A recipe calls for 3 parts flour, 0.25 part salt, and 1.5 parts water. What is the ratio of salt to water to flour? (Answer in lowest terms.)

14. State the ratio of length to width.



15. Consider 2 squares with measurements as shown. Calculate the ratio of the
(*a*) length of side
(*b*) perimeter
(*c*) area
of square A to that of square B.



16. Two circles have diameters of 16 cm and 12 cm. Find the ratio of their diameters, radii, perimeters, and areas.

17. Calculate the ratio of 100 km/h to 50 m/s.

12(a) $r = 30$
(b) $p = 80$, $q = 60$
(c) $r = 9$, $s = 35$
(d) $x = 84$, $y = 60$, $z = 360$
13. 1:6:12
14 7:2
15(a) 5:2 **(c)** 25:4
(b) 5:2
16. 4:3, 4:3, 4:3, 16:9
17. 5:9

14.1 Ratio and Proportion

Recall that $\frac{2}{3} = \frac{4}{6}$ is a statement involving equivalent ratios. An equation like this, stating that one ratio is equal to another, is called a *proportion*. The numbers 2, 3, 4, and 6, taken in order, are said to be in *proportion*.

2 is the *first proportional*
3 is the *second proportional*
4 is the *third proportional*
6 is the *fourth proportional*

Also, if $\frac{8}{4} = \frac{4}{2}$, then 4 is said to be the *middle* proportional, or the *mean* proportional of 8 and 2.

Example 1

Find the fourth proportional to 7, 15, and 12.

Solution

Let the fourth proportional be x.

$$\therefore \frac{7}{15} = \frac{12}{x}$$

and $7x = 15 \times 12$

$$\therefore \quad x = \frac{15 \times 12}{7}$$

$$= \frac{180}{7}$$

$\therefore$ The fourth proportional to 7, 15, and 12 is $\frac{180}{7}$.

If $\frac{2}{3} = \frac{4}{6}$ or $2:3 = 4:6$

then $2 \times 6 = 3 \times 4$

Example 2

Find the mean proportional of 9 and 16.

Solution

Let the mean proportional be m.

$$\therefore \frac{9}{m} = \frac{m}{16}$$

and $m^2 = 9 \times 16$

$$= 144$$

$$\therefore \quad m = \sqrt{144} \text{ or } -\sqrt{144}$$

$$= 12 \quad \text{or } -12$$

$\therefore$ The mean proportional of 9 and 16 is 12 or -12.

Let's check. If $m = 12$, $\frac{9}{12} = \frac{12}{16}$ is true.

If $m = -12$ $\frac{9}{-12} = \frac{-12}{16}$ is true,

Example 3

Use 2-term ratios to express the proportion $2{:}a{:}3 = 6{:}15{:}b$. Find a and b.

Solution

$2{:}a{:}3 = 6{:}15{:}b$ is equivalent to $\frac{2}{6} = \frac{a}{15} = \frac{3}{b}$.

If $\frac{2}{6} = \frac{a}{15}$ and if $\frac{2}{6} = \frac{3}{b}$

then $6a = 30$ then $2b = 18$

$\therefore \quad a = 5$ $\therefore \quad b = 9$

$\therefore a = 5$ and $b = 9$

$\because 1:2:3 = 2:4:6$

$\therefore \frac{1}{2} = \frac{2}{4} = \frac{3}{6}$

Exercise 14.1

Discussion

1. Find the missing proportional.

   (*a*) $\frac{p}{9} = \frac{2}{3}$ (*b*) $\frac{15}{18} = \frac{5}{m}$ (*c*) $\frac{7}{k} = \frac{2}{10}$ (*d*) $\frac{r}{4} = \frac{25}{r}$

2. State the missing proportionals.

   (*a*) $\frac{2}{7} = \frac{m}{14} = \frac{k}{21}$
   (*b*) $1:4:5 = 2:x:y$
   (*c*) $3:7:r = 12:p:20$
   (*d*) $a:d:6 = 27:42:18$

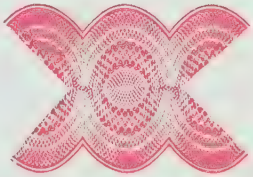
If $3:7:r = 12:p:20$,
$\therefore \frac{3}{12} = \frac{7}{p} = \frac{r}{20}$

Now that's much easier!

Written Solutions

3. Determine the missing proportionals.

   (*a*) $\frac{5}{3} = \frac{15}{r}$
   (*b*) $\frac{2}{6} = \frac{m}{42}$
   (*c*) $\frac{x}{3} = \frac{39}{9}$
   (*d*) $5:y = 25:35$
   (*e*) $\frac{8}{q} = \frac{q}{18}$
   (*f*) $\frac{7}{2} = \frac{t}{5}$
   (*g*) $78:n = 3:4$
   (*h*) $\frac{5}{2} = \frac{35}{a} = \frac{b}{22}$
   (*i*) $\frac{4}{w} = \frac{6}{5} = \frac{9}{z}$
   (*j*) $3:9:k = 8:y:112$
   (*k*) $11:5:100 = 9:x:y$
   (*l*) $18:p = p:4$



4. Two lengths are in the ratio $1\frac{1}{3}:5$. If the first length is 16 cm, find the second length.

5. The ratio of 2 areas is 5:6. If the second area is 42 m², determine the first area.

6. Two metal castings have the ratio of their masses equal to the ratio of their volumes. The first casting has mass 170 g and volume 30 cm³. Find the volume of the second casting if its mass of 510 g.

7. The ratio of the diameters of 2 tires is 9:11. If the larger diameter is 38 cm, find the smaller diameter, correct to 1 decimal place.

3(a) $r = 9$
(c) $x = 13$
(e) $q = 12$ or $q = -12$
(g) $n = 104$
(i) $w = \frac{10}{3}, z = 7.5$
(k) $x = \frac{45}{11}, y = \frac{900}{11}$
4. 60 cm
5. 35 m²
6. 90 cm³
7. 31.1 cm

14.2 Direct Variation

Consider the following situations.

$400:600=30:45$

or $\frac{400}{600} = \frac{30}{45}$

1. The quantity of cookie batter is increased from 400 ml to 600 ml to increase the number of cookies from 30 to 45.

Here, $\frac{16}{12} = \frac{800}{600}$

2. The electric current to a motor drops from 16 A to 12 A as speed decreases from 800 r/min to 600 r/min.

In each of these situations, a change in one quantity brings about a corresponding change in the other quantity. We say there is a *direct variation* of the second quantity caused by the change in the first.

You should note two important aspects of direct variation. Both quantities change

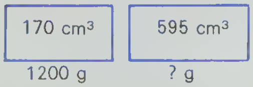
Both increase or both decrease.

The rates of change are proportional.

(i) in the *same direction*
(ii) at the *same rate*

Study the following examples carefully.

Example 1



The mass of a substance varies directly as its volume. If 170 cm³ of the substance has mass 1200 g, find the mass of 595 cm³.

Solution

$$\because \frac{M_1}{M_2} = \frac{V_1}{V_2} \qquad \left(\text{that is, } \frac{\text{Mass 1}}{\text{Mass 2}} = \frac{\text{Volume 1}}{\text{Volume 2}}\right)$$

$$\therefore \frac{1200}{M_2} = \frac{170}{595}$$

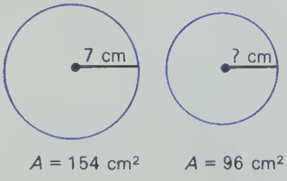
$$\therefore M_2 = \frac{1200 \times 595}{170}$$

$$= 4200$$

$\therefore$ 595 cm³ of the substance has mass 4200 g or 4.2 kg.

What is the difference between M^2 and M_2?

Example 2



The area of a circle varies directly as the square of its radius. If a circle of radius 7 cm has an area of 154 cm², what is the radius of a circle whose area is 96 cm²? (Answer to 2 decimal places.)

Solution

$$\frac{A_1}{A_2} = \frac{{r_1}^2}{{r_2}^2}$$

Read this as "r-one-squared".

$$\therefore \frac{154}{96} = \frac{7^2}{{r_2}^2}$$

$$\therefore {r_2}^2 = \frac{96 \times 7^2}{154}$$

$$= \frac{\overset{48}{\cancel{96}} \times \overset{7}{\cancel{49}}}{\cancel{154}_{\cancel{77}_{11}}}$$

$$= 30.6$$

$$\therefore r_2 = \sqrt{30.6}$$

$$\doteq 5.53$$

Why not −5.53 also?

∴ The circle with area 96 cm² has radius 5.53 cm.

Exercise 14.2

Discussion

1. Which of the following are instances of direct variation? Why?
   - (*a*) The taller an object, the longer its shadow.
   - (*b*) The deeper under water, the greater the pressure.
   - (*c*) The higher above sea level, the less the pressure.
   - (*d*) The greater the assessment on a property, the higher the taxes.
   - (*e*) The more you turn on the tap, the less time it takes to fill the pail.
   - (*f*) The faster you travel, the less time it takes to complete your journey.

Remember:
same direction!
same rate!

For direct variation of 2 quantities, A and B,
$\frac{A_1}{B_1} = \frac{A_2}{B_2} = \frac{A_3}{B_3} = \frac{A_4}{B_4} = \dots$

(*g*)

Interest Rate	5%	6%	7%	8%	9%	10%
Interest	\$25	\$30	\$35	\$40	\$45	\$50

(*h*)

x	1	2	3	4	5	6	7
y	4	8	12	16	20	24	28

(*i*)

Speed (km/h)	50	60	70	80	90
Gas Consumption (ℓ/h)	5	10	15	20	25

(*j*)

Size of roast (kg)	1	2	3	4	5
Cooking time (h)	2	4	6	8	10

Are the ratios constant?

2. 59.7 cm
3. $481
4. 124 V
5. 96 N
6. 140 kPa
7. 58.1 m
8. 82 500 W

Written Solutions

2. The circumference of a circle varies directly as its diameter. If the circumference is 44 cm when the diameter is 14 cm, find the circumference when the diameter is 19 cm.

3. Taxes on a property are directly proportional to the assessment. If the taxes are $550 on a property assessed at $7200, what will be the taxes on a property assessed at $6300?

4. The voltage in an electric circuit varies directly as the current. If a current of 10.4 A flows when the voltage is 90 V, find the voltage when the current is 14.3 A.

5. The increase in length of a spring varies directly as the applied force. What force will increase the length by 7.2 cm, if a force of 38 N will increase the length by 2.85 cm?

6. Pressure under water is directly proportional to depth. 300 cm below the surface, the pressure is 30 kPa. What will be the pressure 1400 cm below the surface?

7. If the distance required to stop a car varies directly as the square of its speed, and a car travelling at 50 km/h requires 12 m to stop, how much distance is required to stop a car travelling at 110 km/h?

8. The power of the engines needed to drive an airplane is directly proportional to the cube of the speed. If 660 W is required to obtain a speed of 420 km/h, find the power of the engines needed to give a speed of 2100 km/h.

A means "amperes"
V means "volts"
W means "watts"
N means "newtons"
kPa means "kilopascals"

14.3 Inverse Variation

Consider the following situations.

1. A train travelling 150 km/h takes 4 h to complete a journey, while another train travelling 100 km/h requires 6 h to complete the same trip.

2. Vegetables in a pressure cooker cook in 15 min at 140 kPa. At 210 kPa, cooking time is only 10 min.

$$\frac{150}{100} = \frac{6}{4}$$
$$\therefore 150 \times 4 = 6 \times 100$$
$$\boxed{\therefore s_1 \times t_1 = s_2 \times t_2}$$

$$\frac{140}{210} = \frac{10}{15}$$
$$\therefore 140 \times 15 = 210 \times 10$$
$$\boxed{\therefore p_1 \times t_1 = p_2 \times t_2}$$

In each case, a change in one quantity brings about an *opposite* change in the other. We say there is an *inverse variation* of the second quantity resulting from a change in the first. It is important to note that the quantities change

(*i*) in *different directions*, and
(*ii*) so that the *product* of the 2 quantities is *constant*.

If one increases, the other decreases.

If $p_1 \times t_1 = p_2 \times t_2$ then $p \times t$ is constant.

Example 1

The time to complete a journey varies inversely as the speed. If it takes 5 h to complete the journey at a speed of 65 km/h, how long would it take at 97.5 km/h?

5 h at 65 km/h
? h at 97.5 km/h

Solution

Since this is inverse variation, speed × time is constant.

Makes sense, since distance = speed × time.

That is,
$$s_1 \times t_1 = s_2 \times t_2$$
$$\therefore\ 65 \times 5 = 97.5 \times t_2$$
$$\therefore\ t_2 = \frac{65 \times 5}{97.5}$$
$$= \frac{10}{3}$$

∴ It would take 3 h 20 min at 97.5 km/h.

Check using $d = s \times t$.

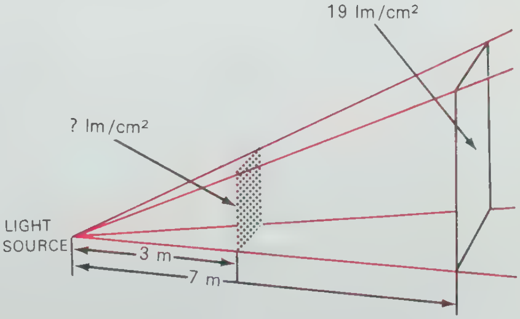
Example 2

The brightness per square centimetre of an illuminated object is inversely proportional to the square of the distance from the source of light. If the brightness is 19 lm/cm² when the distance is 7 m, find the brightness at a distance of 3 m.

Solution

By inverse variation,
$$b_1 \times d_1^2 = b_2 \times d_2^2$$
$$\therefore\ 19 \times 7^2 = b_2 \times 3^2$$
$$\therefore\ b_2 = \frac{19 \times 7^2}{3^2}$$
$$= \frac{19 \times 49}{9}$$
$$= 103.4$$

∴ The brightness is 103.4 lm/cm² at a distance of 3 m.



lm
means "lumens"

Exercise 14.3

Discussion

1. Distinguish between T_2 and T^2.

Somebody drop a "2"?

2. Classify each of the following as instances of direct or inverse variation. Explain your answer.
   - (*a*) The longer you let a kettle boil, the less water there is left in it.
   - (*b*) The more you eat, the less hungry you become.
   - (*c*) The bigger they are, the harder they fall.
   - (*d*) The greater the wing area of an aircraft, the slower its landing speed.
   - (*e*) The faster you try to write, the harder it is to read the writing.
   - (*f*)

x	2	3	4	6	10	20	40
y	24	16	12	8	4.8	2.4	1.2

   - (*g*)

Distance	80	120	180	240	400
Time	2	3	4.5	6	10

   - (*h*)

Pressure	100	200	300	400	500
Volume	12	6	4	3	2.4

If $\frac{x}{y}$ is constant, variation is *direct*.

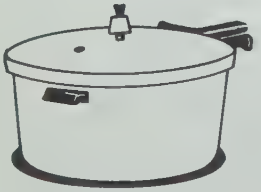
● ● ●

If xy is constant, variation is *inverse*.

3. In an electric circuit, the product of current and resistance is constant. Make a statement concerning the variation of current with resistance. What algebraic equation is equivalent to this statement?

Written Solutions

4. Food in a pressure cooker takes 25 min to cook at a pressure of 128 kPa. If the cooking time varies inversely as the pressure, how long would it take to cook the food at 200 kPa?

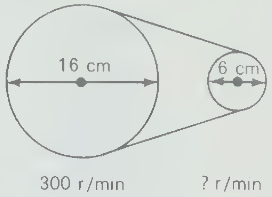


5. The time required to vacuum a particular rug varies inversely as the width of the pick-up nozzle of the vacuum cleaner. If it takes Sylvester Stayatome 18 min to vacuum the rug with a nozzle 20 cm wide, how wide a nozzle must he use to clean the rug in 12 min?

4. 16 min **5.** 30 cm

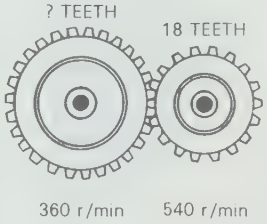
6. In her basement workshop, Ima Handygal has a grinding machine with 2 pulleys connected by a belt, and pulley speed (r/min) is inversely proportional to the diameter.
   (*a*) If the 2 pulleys are 6 cm and 16 cm in diameter and the larger pulley turns at 300 r/min, find the speed of the smaller pulley.
   (*b*) If one pulley turns at 540 r/min and has a diameter of 8 cm, determine the diameter of the driven pulley turning at 960 r/min.

6(a)



7. When 2 gears mesh, the speed varies inversely as the number of teeth.
   (*a*) Find the number of teeth on a gear turning at 360 r/min, if it is meshing with an 18-tooth gear whose speed is 540 r/min.
   (*b*) The number of teeth on 2 gears is 55 and 22, respectively. If the first gear's speed is 80 r/min, find the speed of the second gear.

7.



8. The value of Flying Fingers Flannigan's pipe organ varies inversely as its age. If it was worth \$4260 when it was 2 years old, find its worth after 8 years.

9. Current and electrical resistance in an electric circuit are inversely proportional.
   (*a*) If a current of 24 A flows when the resistance is 840 Ω, find the resistance when the current is 18 A.
   (*b*) If the current is 4.5 A when the resistance is 40 Ω, find the current when the resistance is 100 Ω.

r/min means "revolutions per minute"

Ω means "ohms"

10. In an automobile engine, the volume of air in the cylinder varies inversely as the pressure. In testing out her Dragster Special, Claudia Cloudadust finds that the pressure is 80 kPa when the volume is 70 cm^3. Find the pressure when the volume is 100 cm^3.

11. An aircraft's landing speed (km/h) is inversely proportional to the wing area (m^2). Find the landing speed of an aircraft with wing area 40 m^2, if an aircraft with wing area 105 m^2 has a landing speed of 150 km/h.

6(a) 800 r/min
  (b) 4.5 cm

7(a) 27 teeth
  (b) 200 r/min

8. \$1065

9(a) 1120 Ω (b) 1.8 A

10. 56 kPa

11. 393.75 km/h

14.4 Partial Variation

Sales representatives for the Charisma Cosmetics Company are paid a salary of \$100/week plus a commission of 25% of weekly sales. The weekly pay of a sales rep, therefore, consists of 2 parts.

(*i*) a *constant* sum (the basic salary)
(*ii*) the commission, which *varies directly* as the sales

Such a variation, involving both a *constant term* and *direct* or *inverse variation* is said to be *partial variation.*

Example 1

Calculate Sally Sellem's first weekly pay cheque if she sells \$110 worth of cosmetics.

Solution

$$\begin{aligned}\text{Amount of pay cheque} &= \text{Basic salary} + \text{Commission}\\ &= \$100 + (25\% \text{ of } \$170)\\ &= \$100 + \$42.50\\ &= \$142.50\end{aligned}$$

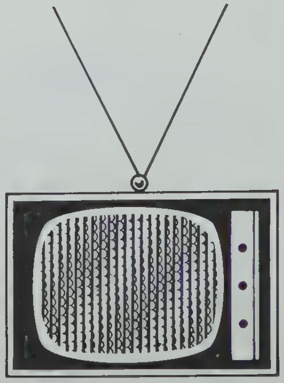
Before deductions of course!

Example 2

To fix Cheryl Channelchanger's broken TV set, the technician charged her \$19.50 for the service call, plus $1\frac{3}{4}$ h labor at \$10/h. What was the cost of repairing the TV set?

Solution

$$\begin{aligned}\text{Total cost} &= \text{Service Charge} + \text{Labor}\\ &= \$19.50 + (1\tfrac{3}{4}\text{ h @ }\$10/\text{h})\\ &= \$19.50 + \$17.50\\ &= \$37.00\end{aligned}$$



Exercise 14.4

Discussion

1. The cost (\$) of manufacturing a particular article depends upon a fixed amount and a variable amount, according to the formula

$$\text{Cost} = 90 + 10n$$

where n is the number of articles made.

   (*a*) Find the cost of 2 articles; 4 articles; 18 articles.
   (*b*) How many articles can be made for \$160? For \$300? For \$50?
   (*c*) What is the cost before any articles are produced?

2. Make up at least 5 examples of partial variation different from those mentioned so far.

Written Solutions

3. A car rental agency charges \$12.50/day plus 7¢/km. Find the cost of renting a car for one day and driving 245 km.

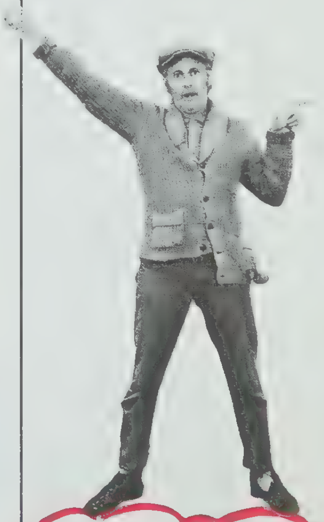
4. Sales rep Sam Sellalot earns a salary of \$145/week, plus a commission of 30% on sales. In 3 successive weeks his sales were \$320, \$295, and \$380. Determine his pay cheque each week.

5. Mr. and Mrs. Seymour went out to a show for the evening. If the tickets cost \$5.50 per couple and they had to pay a babysitter 75¢/h, what did the evening cost if they left home at 19 30 and returned home at 01 30?

6. Farmer Cy Low charges a fixed rate of \$50 to thresh grain with a combine-harvester. In addition, he charges \$6.25/ha and 15¢/hl of grain harvested. Find the total cost of harvesting 130 ha of grain if the yield is 3580 hl.

7. In the equation $y = 3x + 5$, y varies partially as x. If x increases from -2 to 3, find the corresponding change in y.

3. \$29.65
4. \$241, \$233.50, \$259
5. \$10
6. \$1399.50
7. 15



ha means "hectares"
hl means "hectolitres"

His printing set cost $10, so 5¢ a card isn't bad!

8. Wee Willie prints Christmas cards and sells them to his neighbors. Willie determines the cost of his cards by adding his fixed cost of \$2.50 to the cost of \$0.05 for each card printed.
   (*a*) Write an equation to determine the total cost of printing n cards.
   (*b*) Find the cost of printing 40 cards; 75 cards; 114 cards.
   (*c*) By calculating the cost per card in each part of (*b*) show that this is not an instance of direct variation.

9. The equation $C = nk + h$ gives the cost of producing n articles when the labor cost is k dollars per article, and there is a fixed amount (h dollars) called the overhead.
   (*a*) Find the cost of producing 219 articles, if labor costs \$2.75/article and the overhead is \$84.50.
   (*b*) Find the cost per article, if 1100 articles are produced and the labor rate is \$2.45/article with overhead of \$79.75.

10. Barbara Bridgem is a consultant for the Maykem Longur Engineering firm, specializing in bridge construction. In her calculations, Barbara uses the formula

$$l = b + 0.000\,494(C - 4)$$

where l is the length of the bridge deck (metres)
b is the basic deck length (metres)
C is the temperature (degrees Celsius)

If the basic deck length is 58 m, determine
   (*a*) the temperature at which the deck is 58 m long,
   (*b*) the deck length (in metres) at a temperature of 21°C,
   (*c*) the change in deck length (in centimetres) from a hot summer's day of 32°C to a cold winter's day of −26°C.

8(a) $\$C = \$2.50 + \$0.05\,n$
(b) \$4.50, \$6.25, \$8.20
(c) 11.25¢/card, 8.3¢/card, 7.2¢/card

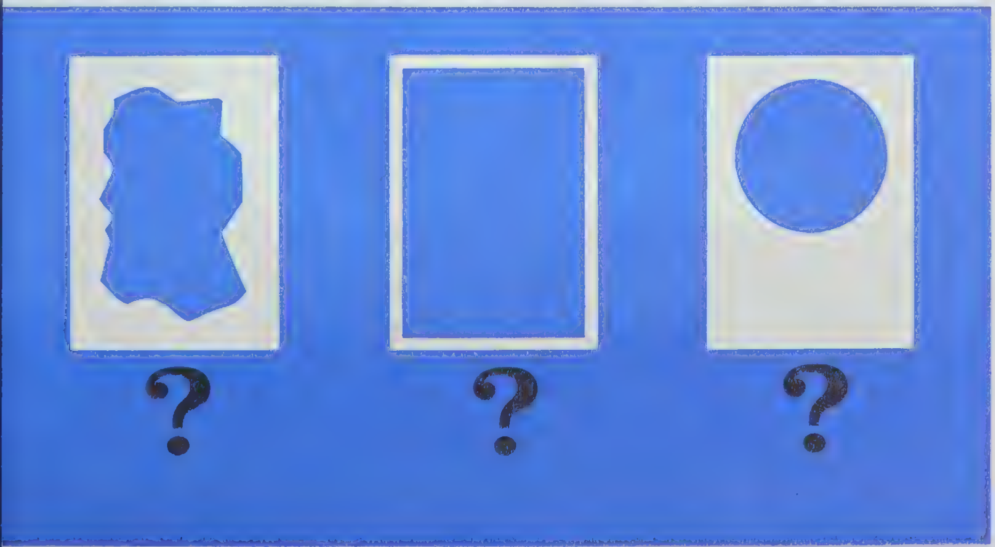
9(a) \$686.75
(b) \$2.52/article

10(a) 4°C
(b) 58. 008 398
(c) 2.8652 cm

* * *

Puzzlers

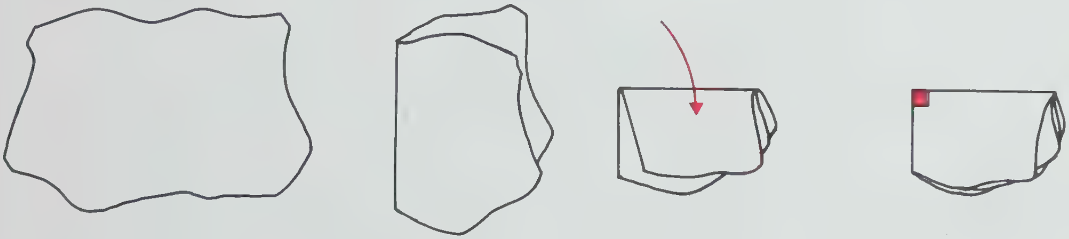
1. What are the next terms in the following sequence?
   1, 4, 9, 61, 52, 63, 94, ___, ___
2. A farmer brought 5 pieces of chain of 3 links each to a blacksmith and asked him to form a single piece of chain. If the blacksmith charges 20¢ to cut a link and 20¢ to weld a link, how much should he charge for the job?
3. Three clowns standing in a row are wearing the numbers 1, 6, and 8. Arrange the 3 clowns so that they form a number divisible by 9.
4. I have many coins in my pocket, but I can't make change for a nickel, a dime, a quarter, a 50¢ piece, or a dollar. What is the largest sum of money I can have in my pocket?
5. We can simplify $\frac{26}{65}$ as follows: $\frac{2\cancel{6}}{\cancel{6}5} = \frac{2}{5}$, which is true! Can you find any other similar cases?
6. Evaluate: $1 - 2 + 3 - 4 + 5 - 6 + \ldots + 99 - 100 = ?$
7. Using all of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 only once, write a numeral for 100.
8. If a concrete block balances exactly with 0.75 kg and $\frac{3}{4}$ of a block, what is the mass of a whole block?
9. Can you cut a hole in a sheet of notepaper so that you can pass through the hole? At first glance, this would seem an impossible task, but it *can* be done! Can you do it?



15

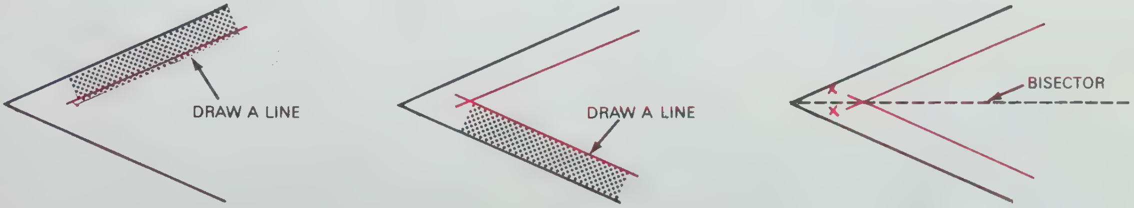
Unit 15
Basic Ideas of Geometry

You can use your knowledge of geometry to solve many practical problems. Suppose you have to draw a straight line on your page and you have no ruler. You could use the edge of your book, or you could fold a piece of paper. The crease is a straight line.



Folding the paper a second time will produce a right angle or square corner.

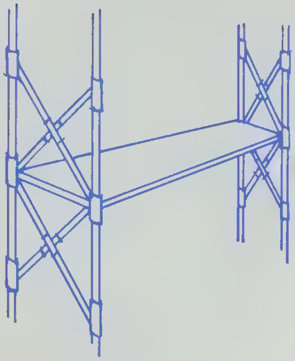
Did you know that you can bisect an angle using a piece of wood that has parallel sides? (A ruler would do!)

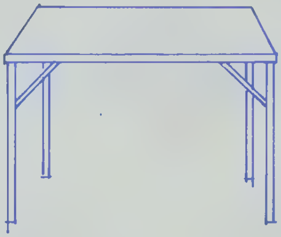


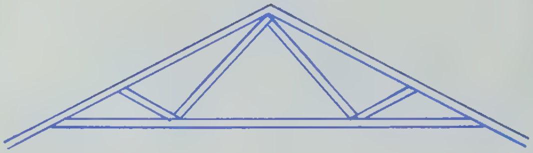
A basic knowledge of geometry enables us to improvise such working techniques and to understand why they work.

15.1 Triangles

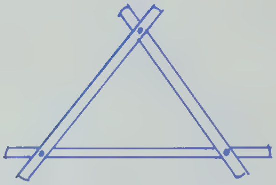
The triangle is a very common shape in structures that we see around us.



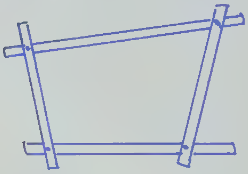




Why is the triangle shape used? Nail three sticks together as pictured. Can you easily change the shape? We say that the triangle is a *rigid* figure.



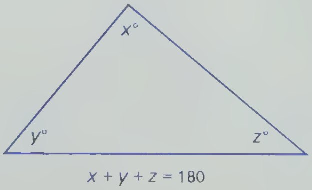
Now nail four sticks together as pictured. Is this shape rigid?



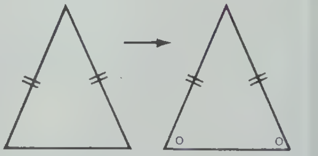
Look for the rigid triangles in the structures pictured above.

Facts About Triangles That You Should Know

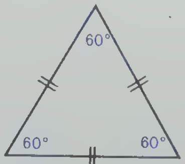
The sum of the angles of any triangle is 180°.



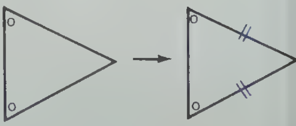
Two angles of an isosceles triangle are equal.



Each angle of an equilateral triangle is 60°.

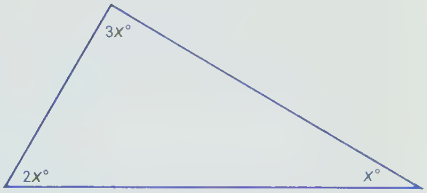


A triangle with two angles equal is isosceles.



Example 1

Find the measure of each angle in the given triangle.



Solution

Since the sum of the angles of a triangle is 180°,

$\therefore\ x + 2x + 3x = 180$
or $6x = 180$
$\therefore\ x = 30$
$\therefore\ 2x = 60$ and $3x = 90$

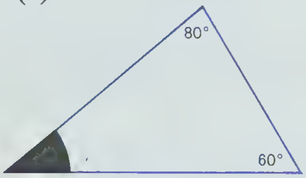
∴ The measures of the angles of the triangle are 30°, 60°, 90°.

Exercise 15.1

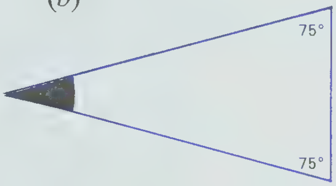
Discussion

1. State the measure of the third angle in each of the following.

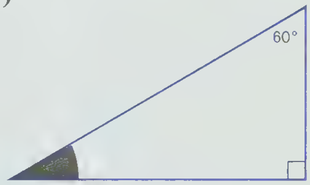
(*a*)



(*b*)

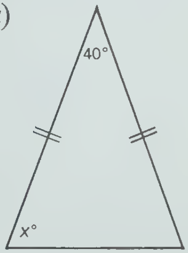


(*c*)

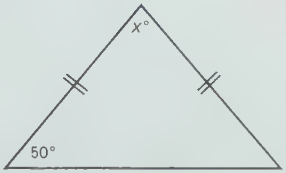


2. Find x in each of the following.

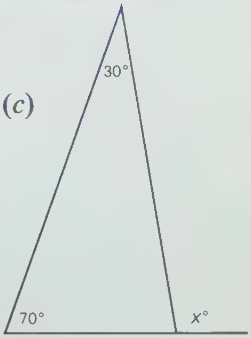
(*a*)



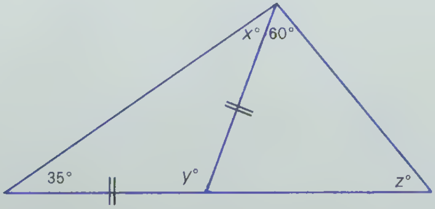
(*b*)



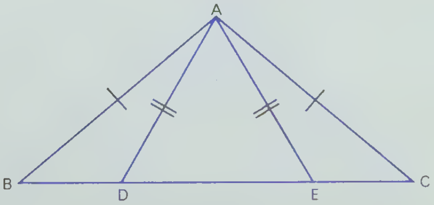
(*c*)



3. Find x, y and z.

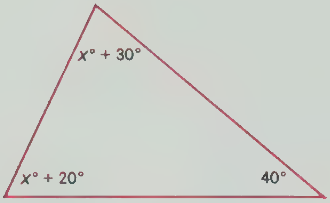


4. Name as many pairs of equal angles as you can.



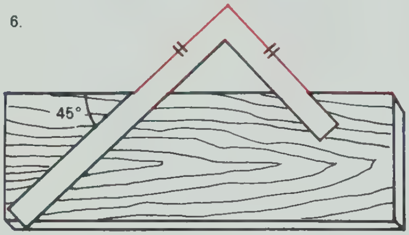
Written Solutions

5.



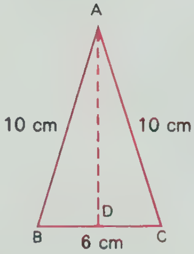
5. Find x and the measure of each angle of the triangle.

6.



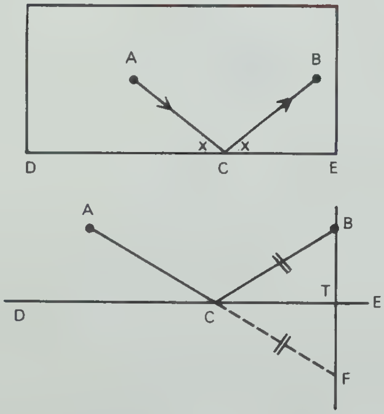
6. A carpenter's square is used to mark a 45° angle on a board. Explain why the method illustrated here will work.

7.



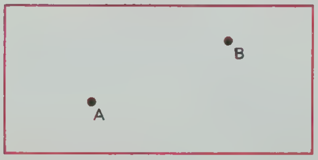
7. (*a*) Draw an isosceles triangle with sides 10 cm, 10 cm, and 6 cm.
   (*b*) Cut out the isosceles triangle shape.
   (*c*) Fold along the line indicated. Is the crease the bisector of $\angle BAC$?
   (*d*) Compare $\angle ABD$ and $\angle ACD$.
   (*e*) Compare the lengths of BD and CD.
   (*f*) What is the measure of $\angle ADC$?

8.



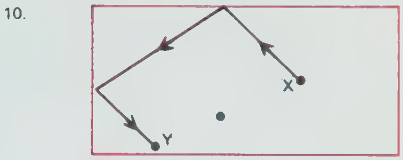
8. A billiard ball striking a cushion rebounds such that $\angle ACD = \angle BCE$.
   (*a*) Extend AC to F so that CF = CB.
   (*b*) Join B to F.
   (*c*) What kind of triangle is $\triangle BCF$?
   (*d*) How is $\angle FCT$ related to $\angle ACD$? to $\angle BCT$?
   (*e*) How is BT related to TF?
   (*f*) What is the measure of $\angle BTC$?

9.

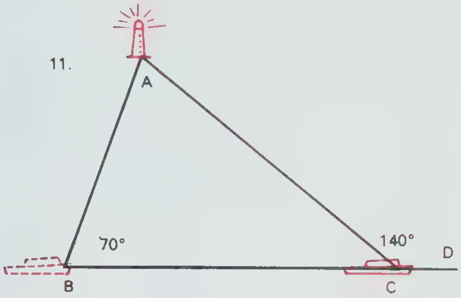


9. (*a*) How many different shots using a one-cushion rebound could you use to direct ball A to ball B?
   (*b*) Describe a method for finding one of these paths.

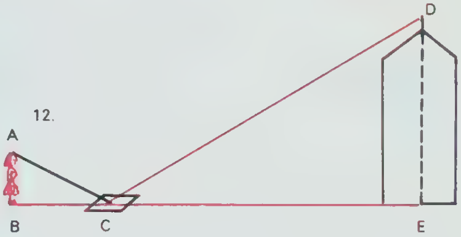
10. Describe a method for executing a 2-cushion shot from X to Y.

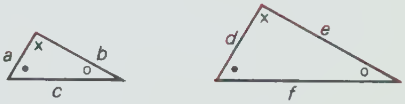


11. "Doubling the angle at the bow" is a method of estimating distance at sea. Finmore Flounder, captain of the good ship *Pickled Herring*, is at B moving in a direction BD when he sights a lighthouse and measures ∠ABC to be 70°. Later at C he measures ∠DCA to be 140°.
   (*a*) Calculate the measure of ∠ACB and ∠BAC.
   (*b*) What kind of triangle is △ABC?
   (*c*) If Finmore calculates the distance BC from the ship's log, how can he then determine the distance CA?



12. To find the height of a tower, Paula placed a mirror on level ground. She walked away until she could see the top of the tower in the mirror. Paula's eyes are 170 cm above the ground and she stands 272 cm from the mirror. She measures the distance CE to be 28 m. How high is the tower?

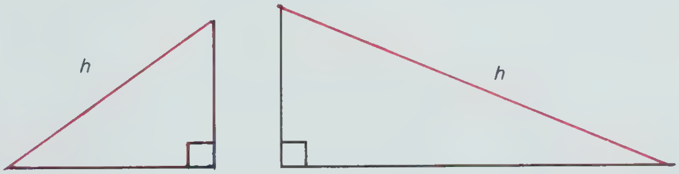


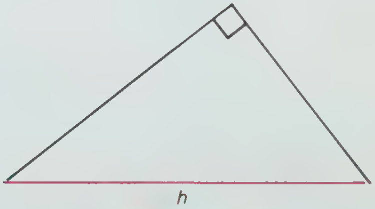


$$\frac{a}{d}=\frac{b}{e}=\frac{c}{f}$$

15.2 The Right-Angled Triangle

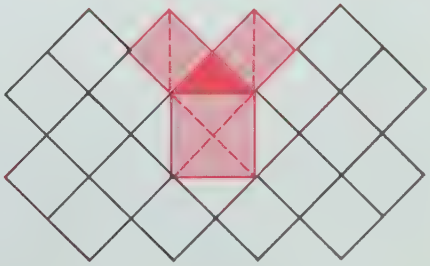
A The side opposite the right angle in a right-angled triangle is called the *hypotenuse*.



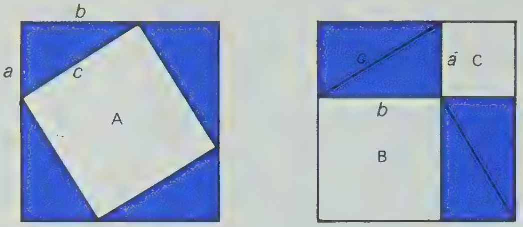


Right-angled triangles have an important property that is illustrated by this tile pattern. Notice that

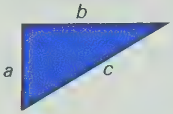
> The area of the square on the hypotenuse equals the sum of the areas of the squares on the other 2 sides.
>
> —*Law of Pythagoras*



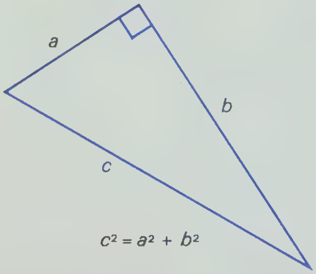
Here's another way to show the same idea.



1. Begin with a square containing 4 right-angled triangles as shown.
2. Imagine that the colored triangles are moved to a second square the same size and area as the first and placed as pictured.
3. If one of the triangles has sides *a*, *b*, and *c*, what is the area of white square A? Of square B? Of square C?
4. Compare the area of square A with the areas of squares B and C.

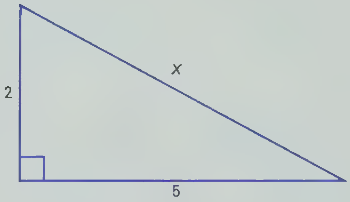


Remember Pythagoras!



B If 2 sides of a right-angled triangle are given, we can find the length of the third side.

Example
Find *x*.

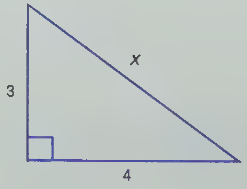


Solution

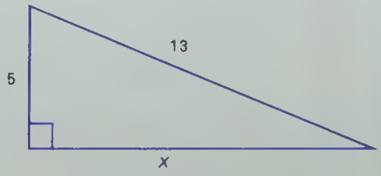
$$\begin{aligned} x^2 &= 5^2 + 2^2 \\ &= 25 + 4 \\ &= 29 \\ \therefore\ x &= \sqrt{29} \\ &\doteq 5.39 \qquad \text{(From tables)} \end{aligned}$$

1. Find the length of the third side.

(*a*)



(*b*)



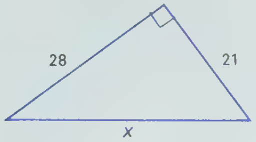
Careful in (b)!

Exercise 15.2

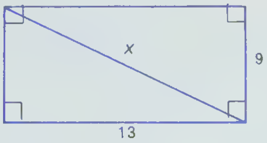
Written Solutions

1. Find the length x in each of the following.

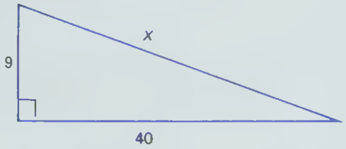
(*a*)



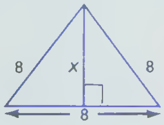
(*c*)



(*b*)



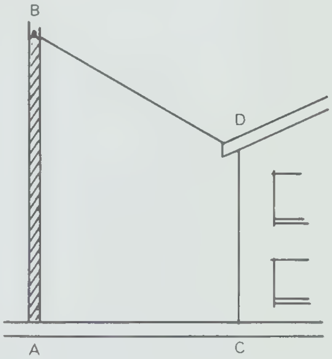
(*d*)



2. What distance is saved by walking along the diagonal of a rectangular lot 20 m by 15 m, instead of walking along 2 adjacent sides?

3. How much wire is needed to connect the top of a pole AB with a point D on the side of a house if the pole is 12 m high, DC is 7 m, and AC is 9 m?

3.



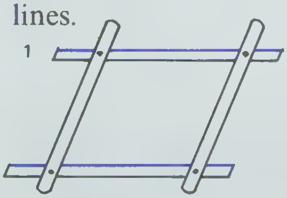
4. If you travel 4 km east, then 5 km north, and then 7 km east, you will travel 16 km. But how far will you be from your starting point?

1(a) 35 (c) 15.8
2. 10 m
3. 10.3 m
4. 12.1 km

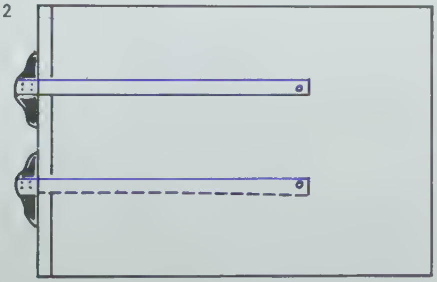
15.3 Parallel Lines and Parallelograms

Here are some instruments you may have used to draw parallel lines.

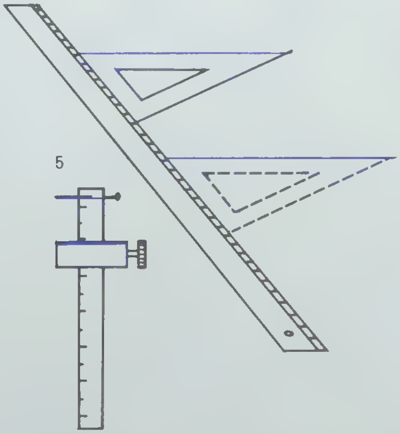
1



2

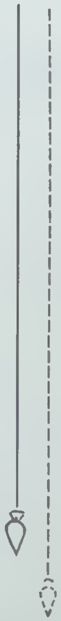


3



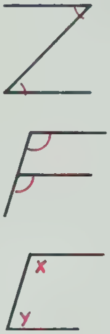
5

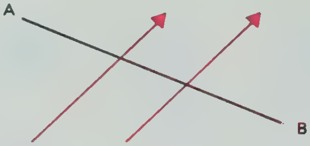
4.



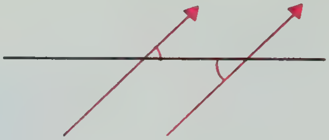
1 PARALLEL RULERS
2 T-SQUARE ON A DRAWING BOARD
3 SLIDE A TRIANGLE ON A STRAIGHTEDGE
4 PLUMB LINES
5 CARPENTER'S GAUGE

Facts About Parallel Lines That You Should Know

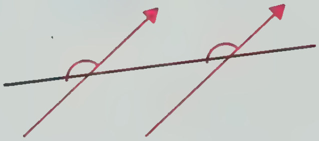




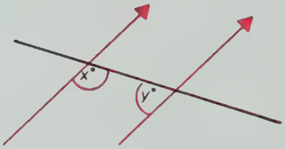
AB is a transversal



alternate angles are equal



corresponding angles are equal



$x + y = 180$

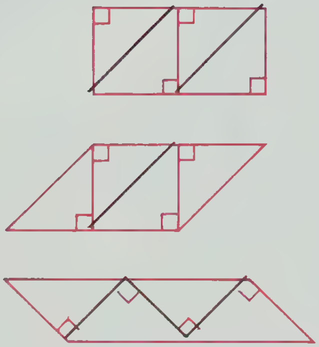
A parallelogram is a 4-sided figure with opposite sides parallel.

1. (*a*) From paper cut out a square with sides 15 cm long.
   (*b*) Draw the diagonals of the square.
   (*c*) What kind of angles are formed by the diagonals?

2. Cut out 4 triangles and rearrange them to form the figures illustrated.

3. (*a*) Is each figure from Question 2 a parallelogram? Why?
   (*b*) Compare the lengths of the opposite sides of a parallelogram.
   (*c*) How are the opposite angles of a parallelogram related?

2.

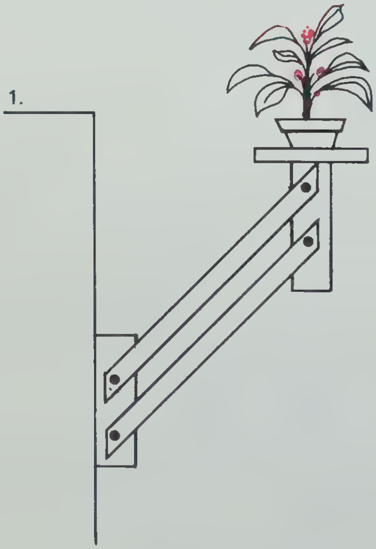


Exercise 15.3

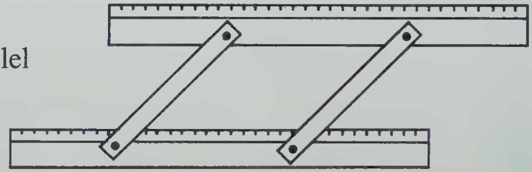
Discussion

1. A wall bracket supports a shelf. Why does the shelf remain level?

1.



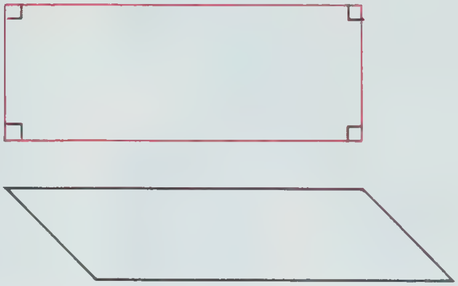
2. Explain how a parallel ruler works.



Written Solutions

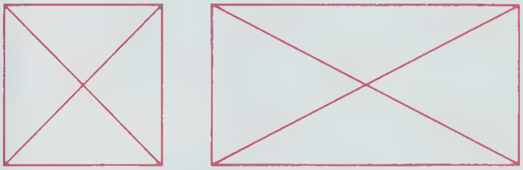
3. (*a*) Measure the lengths of the diagonals of a parallelogram that is also a rectangle.
   (*b*) Measure the lengths of the diagonals of a parallelogram.
   (*c*) State how the diagonal property is used to determine if a parallelogram is also a rectangle.

3.



4. (*a*) Measure the angles formed by the diagonals of a square.
   (*b*) Measure the angles formed by the diagonals of a rectangle.
   (*c*) State how the diagonal property is used to determine if a rectangle is a square.

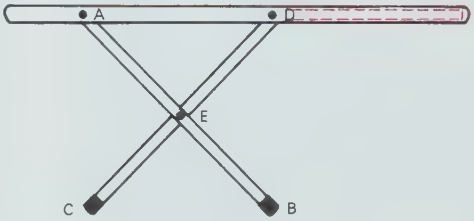
4.



5. Naomi Nailer is making a rectangular box 180 cm by 60 cm. She nails the boards together and wishes to test whether the box is rectangular. Suggest a method of using a string to "square" the box.

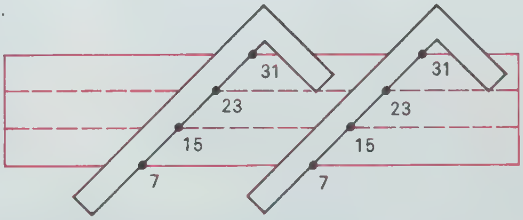
6. A collapsible ironing board has its legs hinged at E, and D can slide in a slot and moves along the board. If AE = EB and CE = ED, why is the ironing board parallel to the floor for all positions of D?

6.



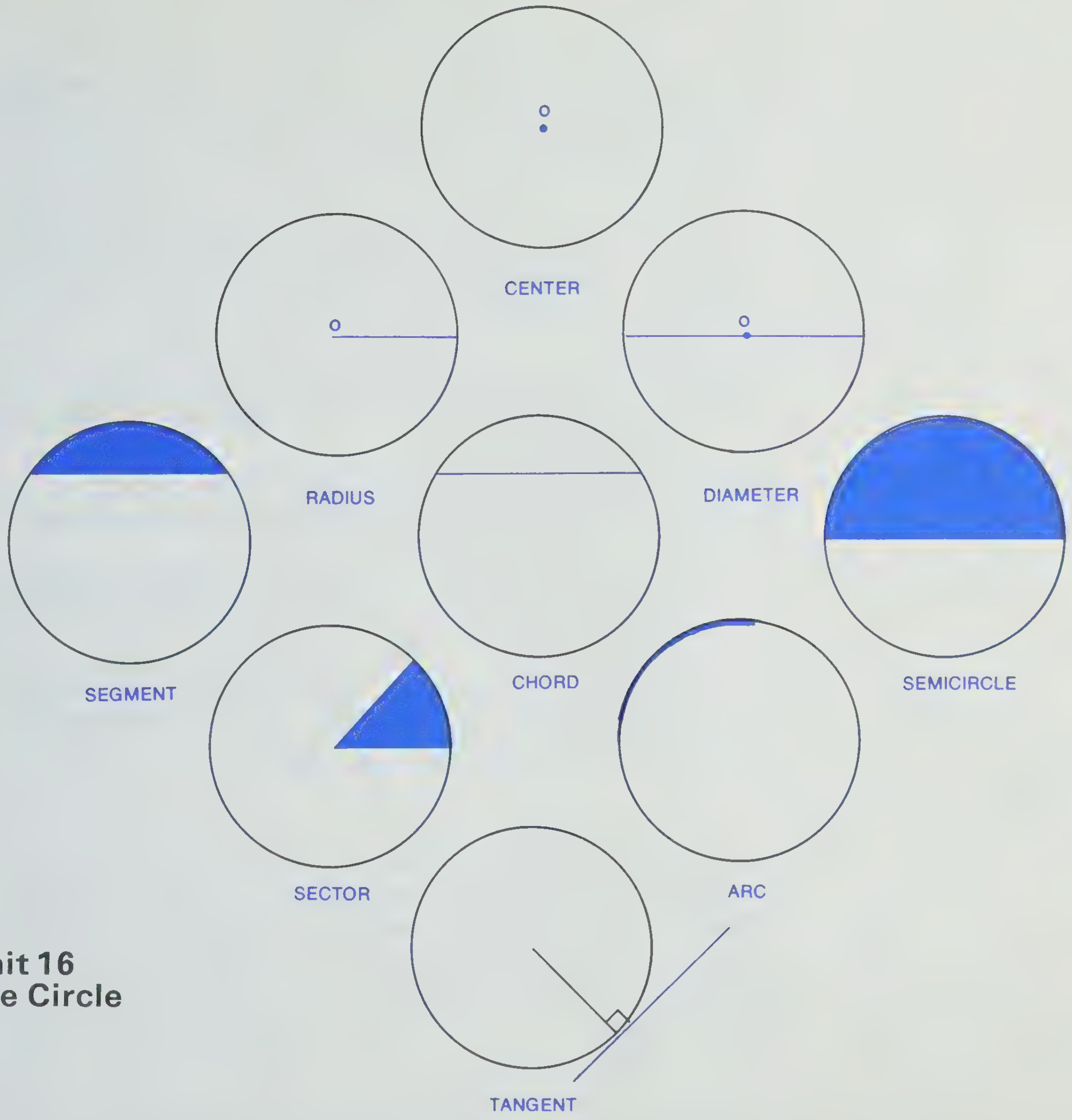
7. Rodney Rafter wishes to mark a board in 3 strips of equal width. This is how he does it. He places his square so that the 7 cm and 31 cm marks come at the outer edge of the board. He marks the points at the 15 cm and 23 cm marks and draws the lines as indicated. Can you explain why this method works? Could you use other numbers on the square?

7.



16

Unit 16
The Circle

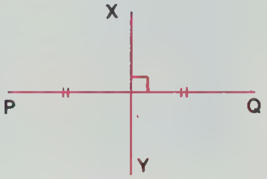

O
CENTER
O
RADIUS
O
DIAMETER
SEGMENT
CHORD
SEMICIRCLE
SECTOR
ARC
TANGENT

One of the most important shapes in geometry is that of the circle. You have only to look around you to observe its usefulness and its application in everyday living.

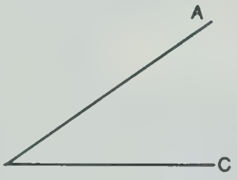
In this unit, you will examine some of the more important properties associated with the circle. It is essential for you to have a good understanding of the terms used in talking about the circle and its properties.

16.1 Chord Property of a Circle

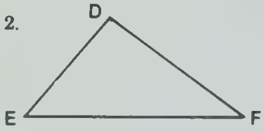
A The perpendicular bisector of a line segment is the line that bisects a line segment at right angles (that is, is perpendicular to it).



XY is the perpendicular bisector of PQ.

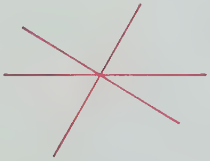
1.


1. Draw an angle like ABC. Draw the perpendicular bisectors of AB and BC.

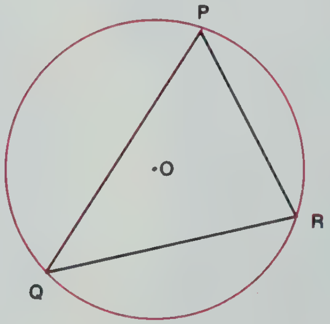
2.


2. Draw a triangle like △DEF. Draw the perpendicular bisectors of the 3 sides of △DEF.
   (*a*) What do you notice about the intersection of the bisectors? What name is given to lines that intersect in this way? *
   (*b*) With O, the intersection of the perpendicular bisectors, as center, draw a circle with radius OD. What do you notice?

* "Concurrent" means "run together".



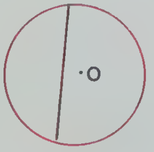
B In the diagram, we say that the circle with center O is the *circumscribed circle* for △PQR, or that △PQR is *inscribed* in the circle with center O. O is the *circumcenter* of △PQR.



1. What name is given to line segments PQ, PR, QR?

2. Draw 3 circles, all of 3 cm radius, and draw 3 chords (one in each circle) of different lengths.

3. Construct the perpendicular bisector of each chord of Question 2. Be accurate. Where does the center of the circle lie with respect to each bisector?



Exercise 16.1

Discussion

1. In your own words, state the relationship between the center of a circle and the perpendicular bisector of any chord in the circle.

2. In order to locate the circumcenter of a triangle, how many perpendicular bisectors must be drawn?

3. What is the length of the longest chord that can be drawn in a circle of radius 5 cm?

4. What special name is given to a chord that passes through the center of the circle?

Written Solutions

5. In a circle of radius 5 cm, construct a chord 7 cm long.

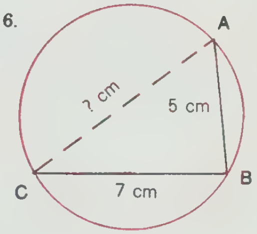
6. Chords AB and BC are drawn in a circle of radius 4 cm, as shown. Make a similar diagram and measure the length of AC.

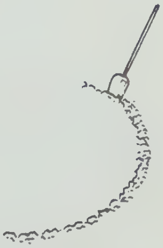
7. Draw a circle with a 4 cm radius. Inside the circle, draw an inscribed isosceles triangle with equal sides 7 cm long. Determine the length of the third side by measurement.

8. Draw the circumscribed circle of a triangle ABC with AB = 5 cm, BC = 6 cm, and AC = 7 cm.

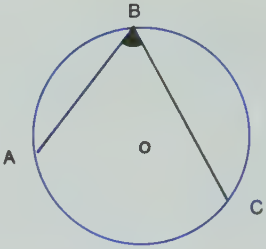
9. Greta Greenthumb was digging a circular flower bed 6 m in diameter by using a string tied to a fixed stake to locate the edge of the bed. While she was inside eating lunch, however, some small boys playing nearby took her stake and string to help fly their kite. This didn't bother Greta, for she soon had relocated the center of the flower bed by using the chord property of a circle. Using a scale of 1 cm = 1 m, make a drawing to show how Greta relocated the center of the flower bed.

6.





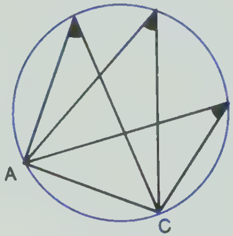
16.2 Angle Properties of a Circle



A Angle ABC is said to be an *inscribed angle* of the circle with center O. The vertex of an inscribed angle must lie on the circle.

∠ABC can be thought of as "standing on" arc or chord AC. We also say that angle ABC is *subtended by* arc AC (or chord AC).

1.

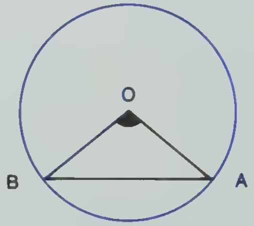


"Subtended by" means "standing on".

1. Draw a circle of radius 4 cm, and construct a chord 3 cm in length within the circle. Draw 3 different angles subtended by the chord, and measure each using a protractor. What do you notice?

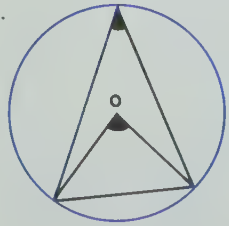
2. Repeat Question 1 using a radius of 5 cm and a chord 6 cm in length. What do you notice about the size of the inscribed angles?

3. Repeat Question 1 using a radius and chord length of your own choice. What do you notice about the inscribed angles?



B An angle whose vertex is at the center of the circle is called a *sector angle*. As with the inscribed angle, a sector angle may be considered to be subtended by the arc BA or the chord BA.

1.



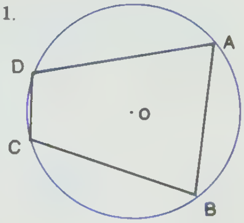
1. Construct a circle of radius 5 cm, with center O, and draw a chord in the circle 6 cm long. Draw an inscribed angle and a sector angle subtended by the chord, and find the degree measure of each. Draw 2 other inscribed angles subtended by the chord and compare their measure with that of the sector angle. What do you notice?

2. Repeat Question 1 using a different radius and different chord length. Are the results the same as for Question 1?

C 1. Draw 3 inscribed angles in a semicircle. Without measuring, tell how the degree measure of these 3 angles will be related. Use a protractor to find the measure of each angle. What do you notice?

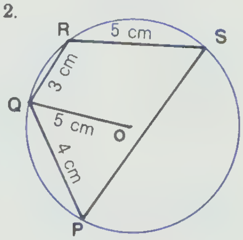
D 1. Construct a circle with center O having radius 4 cm. Draw chords AB, BC, and CD with lengths 5 cm, 6 cm, and 3 cm, respectively. Join A to D. Find the measure of ∠A, ∠B, ∠C, ∠D. Calculate ∠A + ∠C and ∠B + ∠D. What do you notice?

1.



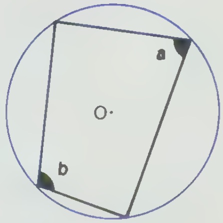
2. Inscribe quadrilateral PQRS in a circle with center O using the measurements shown. Find the measure of ∠P, ∠R, ∠Q, ∠S. Calculate ∠P + ∠R and ∠Q + ∠S. What do you notice?

2.



3. Check your results by drawing any circle of suitable radius and inscribing a quadrilateral in it. What is the sum of the opposite angles of the quadrilateral inscribed in a circle?

3.



$a + b = ?$

E Study the following examples carefully.

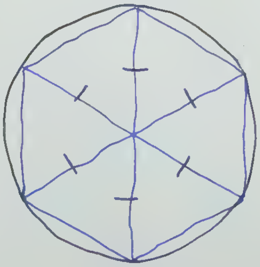
Example 1

Construct a regular hexagon with sides 2 cm long.

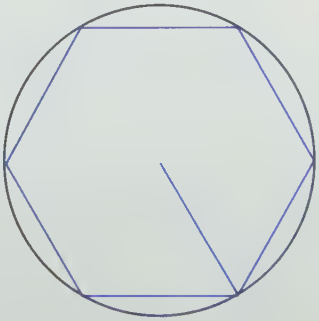
Solution

From the sketch, we see that a regular hexagon may be thought of in terms of 6 equilateral triangles arranged as shown. Notice that the outside edge of each triangle is a chord of the circumscribed circle. Also, since the triangles are equilateral, the chord length is equal to the radius.

Sketch

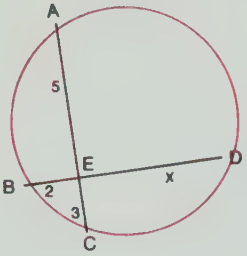


Scale Diagram



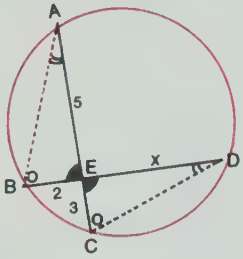
Example 2

Two chords of a circle intersect, as shown. Determine the length of ED.



Solution

If we join BA and CD, we see that $\angle A$ and $\angle D$ are subtended by arc BC, and hence $\angle A = \angle D$. Also, $\angle B$ and $\angle C$ are subtended by arc AD and thus $\angle B = \angle C$. Since $\angle AEB$ and $\angle DEC$ are opposite angles, $\angle AEB = \angle DEC$.



$$\therefore \quad \triangle ABE \mathbin{|||} \triangle DCE$$

$$\therefore \quad \frac{DE}{AE} = \frac{CE}{BE}$$

$$\text{or} \quad \frac{x}{5} = \frac{3}{2}$$

$$\text{and} \quad 2x = 15$$

$$\therefore \quad x = 7.5$$

Thus, the length of ED is 7.5 units.

Exercise 16.2

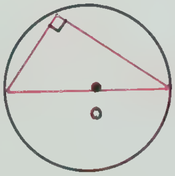
Discussion

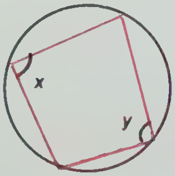
1. In your own words, state the relationship between
   (*a*) inscribed angles in a circle subtended by the same arc or chord,
   (*b*) a sector angle and an inscribed angle, subtended by the same chord or arc, and
   (*c*) opposite angles of a quadrilateral inscribed in a circle.

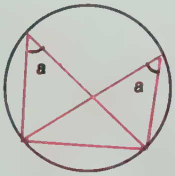
2. State the measure of any angle inscribed in a semicircle.

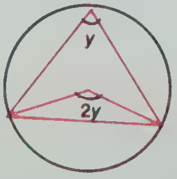
3. Match each of the diagrams with a part of Question 1 or 2.

3.



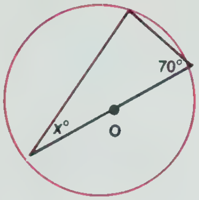




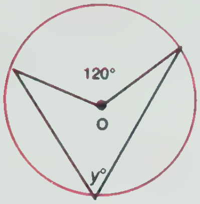


4. Determine the value of the variable in each of the following.

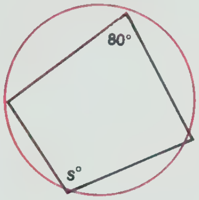
(*a*)



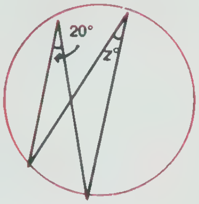
(*b*)



(*c*)



(*d*)

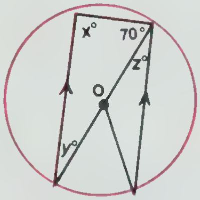


Written Solutions

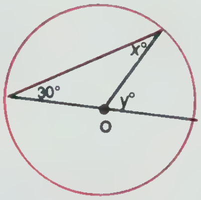
Use ruler and compasses only for construction problems.

5. Determine the value of the variable in each of the following.

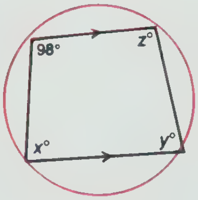
(*a*)



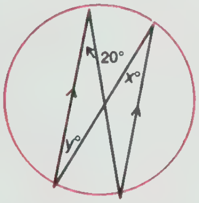
(*b*)



(*c*)



(*d*)



6. Construct a right-angled triangle with hypotenuse 10 cm and one side 4 cm long.

7. Draw a circle of radius 3 cm. Inside the circle, inscribe a rectangle having one side 4 cm long.

8. Inscribe a square in a circle of radius 5 cm.

9. Construct a square ABCD with diagonals 7 cm long.

10. Construct a regular hexagon with sides 4 cm long.

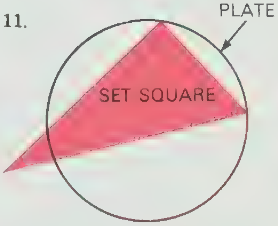
11. Trace the outline of a dinner plate on a piece of paper. Using a set square and your knowledge of the angle properties of a circle, locate the center of the plate.

12. Determine the value of the variables in the diagrams.

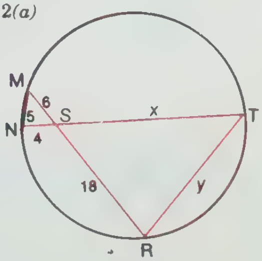
5(a) $x = 90$ $y = 20$ $z = 20$
(b) $x = 30$ $y = 60$
(c) $x = 82$ $y = 82$ $z = 98$
(d) $x = 20$ $y = 20$

12(a) $x = 27$ $y = 22.5$
(b) $a = 90$ $n = 60$
$f = 30$ $r = 30$
$k = 60$ $t = 60$

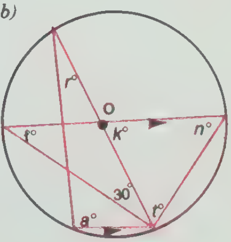
11.



12(*a*)



12(*b*)



16.3 Tangents

A Below are 3 circles, each with 2 tangents drawn to the circle from a point outside the circle.

1. Measure the lengths of PT and PK in Fig. 1. What do you notice?

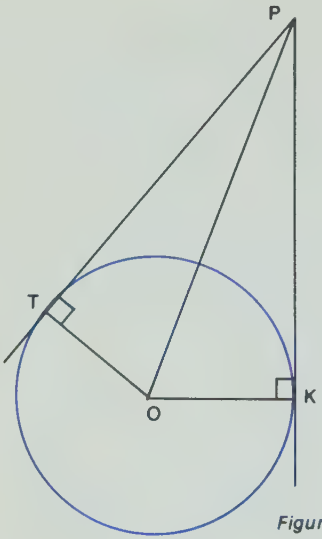


Figure 1

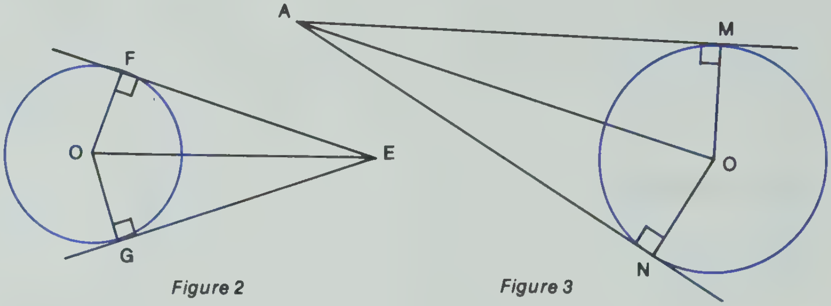
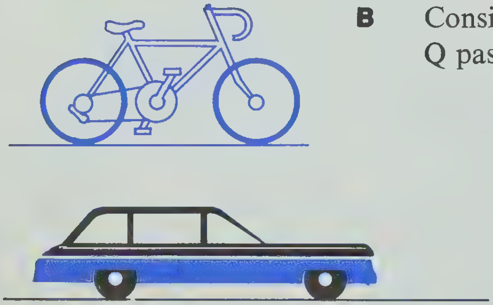


Figure 2

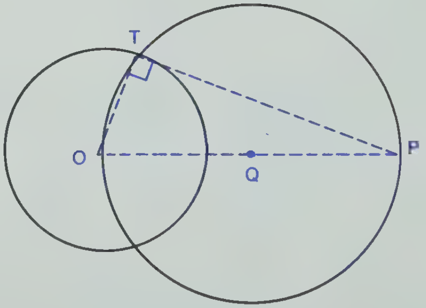
Figure 3

2. Find the measure of ∠TOP and ∠KOP. What do you notice?
3. Find the measure of ∠TPO and ∠KPO. What do you notice?
4. Repeat Questions 1, 2, 3 for Fig. 2 and Fig. 3, using corresponding tangents and angles.

Examples of Tangents



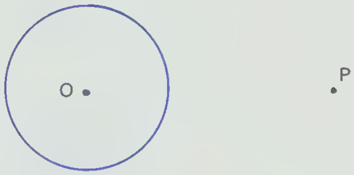
B Consider the diagram below, in which the larger circle with center Q passes through the center O of the smaller circle.



Since OP is a diameter of the larger circle, ∠OTP = 90°. But because OT is the radius of the smaller circle, PT must be a tangent to the smaller circle at T. This gives us a method for constructing a tangent to a circle from a point outside the circle.

Example

Construct a tangent from P to the circle with center O.



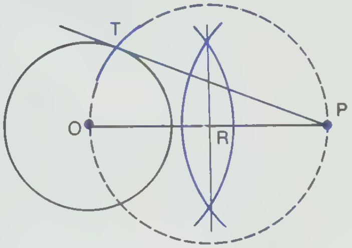
Solution

Step 1 Join O to P.

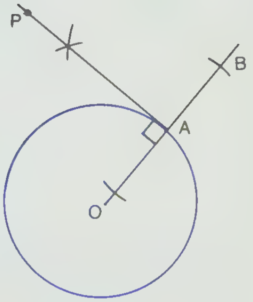
Step 2 Bisect OP at R.

Step 3 With center R and radius OR, draw an arc to cut the circle at T.

Step 4 Join PT to get the required tangent.

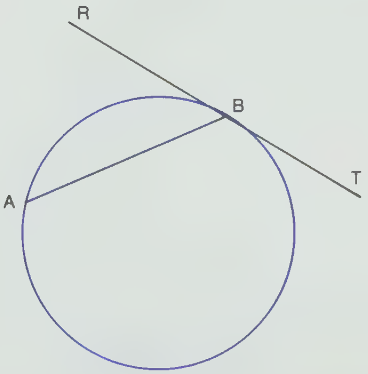


C 1. Draw a circle of radius 4 cm with center O. Join O to A, where A is any point on the circle, and extend OA to B, as shown. Draw PA ⊥ OB. What is the relationship of PA to the circle with center O? If PO = 6 cm, what is the length of the tangent from P to the circle?



2. Construct a tangent 6 cm long to the circle with center O and radius 5 cm.

D 1. Copy the diagram and measure the angle ABR formed by the tangent RT and chord AB.



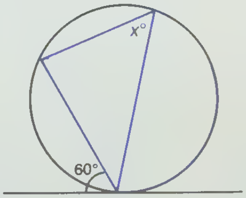
2. Draw 3 inscribed angles subtended by chord AB. Without measuring, tell how the angle measures of these 3 angles are related. Check by measurement.

3. What is the relationship between the angle formed by the tangent and the chord, and the inscribed angles?

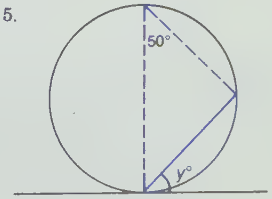
4. What value has x in the diagram?

5. What is the value of y in the diagram?

4.



5.

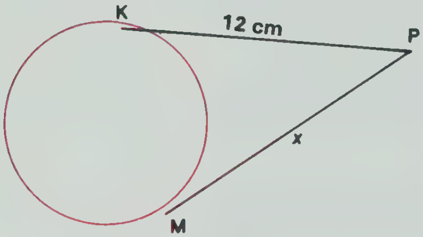


Exercise 16.3

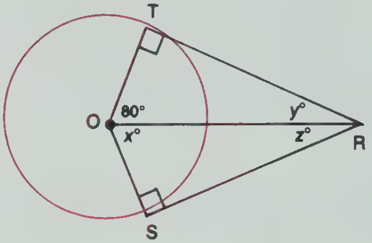
Discussion

1. What is the greatest number of tangents that can be drawn to a circle from a given point outside the circle?
2. How many tangents can be drawn to a circle from a given point inside the circle?
3. Find the value of each variable.

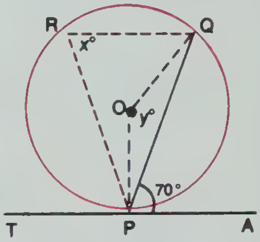
(*a*)



(*b*)



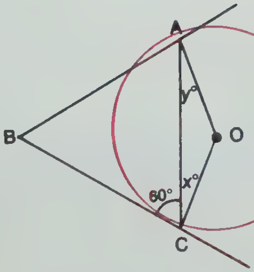
(*c*)


R
Q
$x°$
O
$y°$
70°
T
P
A

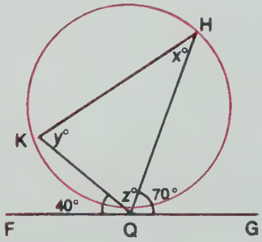
Written Solutions

4. Find the value of the variables.

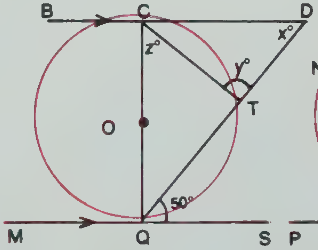
(*a*)



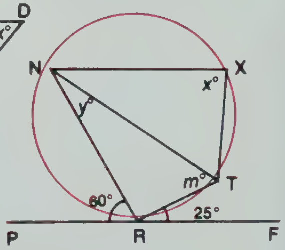
(*b*)


H
$x°$
K
$y°$
40°
$z°$
70°
F
Q
G

(*c*)


B
C
D
$z°$
$x°$
$y°$
T
O
50°
M
Q
S

(*d*)


N
X
$y°$
$x°$
$m°$
T
60°
25°
P
R
F

5. A circle has radius 4 cm. Using ruler and compasses only, find a point outside the circle from which tangents 5 cm long may be drawn to the circle.

6. Draw a circle with 3 cm radius. Locate a point P, 7 cm from the center of the circle, and draw 2 tangents to the circle from P.

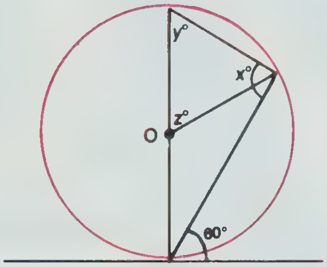
4(a) $x = 30$ $y = 30$
(b) $x = 40$ $y = 70$ $z = 70$
(c) $x = 50$ $y = 90$ $z = 50$
(d) $m = 60$ $x = 85$ $y = 25$

7. A worker lays a plank against an oil drum with a diameter of 1.2 m, as shown. If the distance from the center of the drum to the point at which the plank touches the ground is 1 m, determine the length of the plank from the ground to the point where it touches the oil drum.

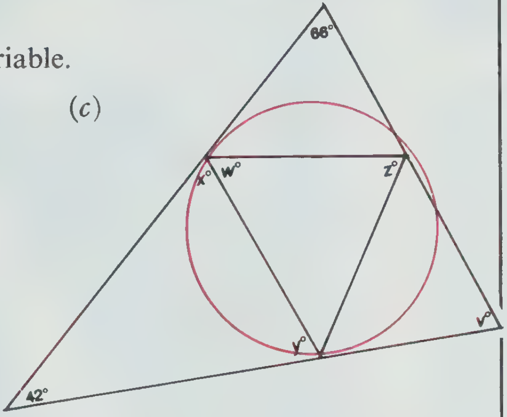
8. A pie plate 24 cm in diameter is set in a corner of a cupboard so that it touches each wall. How far is the edge of the pie plate from the corner?

9. Find the value of each variable.

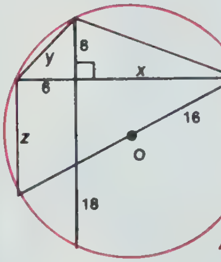
(*a*)



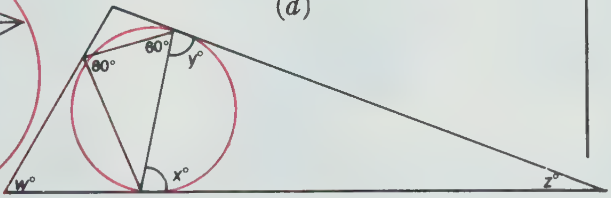
(*c*)



(*b*)

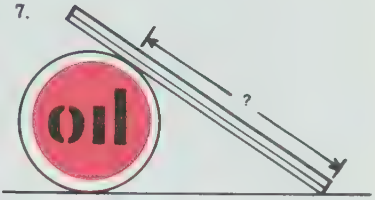


(*d*)

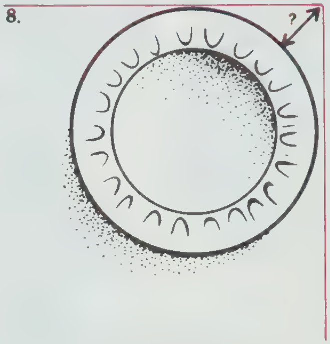


10. (*a*) Construct △ABC with AB = 8 cm, BC = 7 cm, and AC = 5 cm.
(*b*) Draw the angle bisectors of angles A, B, and C. What do you notice?
(*c*) From O, the point of intersection of the angle bisectors, draw a perpendicular to meet AB at P.
(*d*) With center O and radius OP, draw a circle. What do you notice? This circle is called the *inscribed* circle of △ABC. Your diagram should look like the one in the margin.

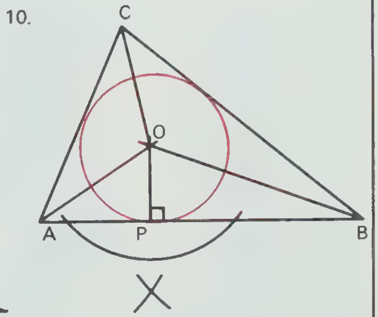
7.



8.



10.

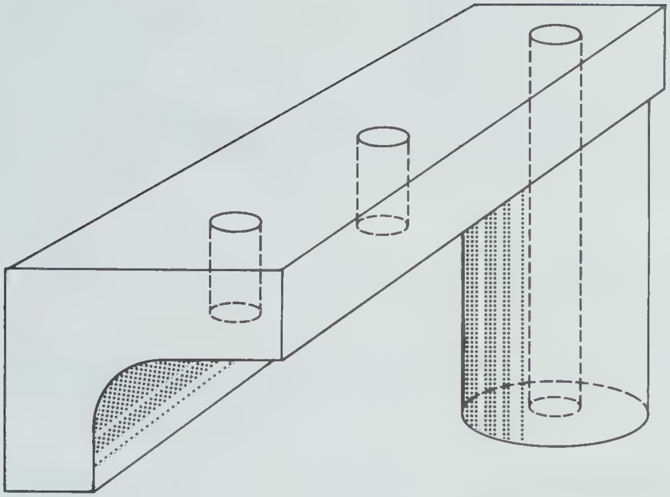


7. 0.8 m

8. 4.97 cm

9(a) $x = 90$ $y = 60$ $z = 60$
(b) $x = 24$ $y = 10$ $z = 11.1$
(c) $x = 69$ $y = 69$
$z = 69$ $w = 54$
$v = 72$
(d) $x = 80$ $y = 80$
$z = 20$ $w = 60$

17

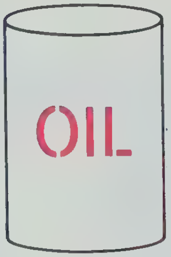


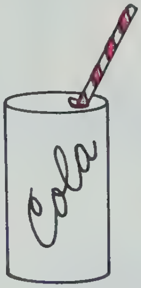
Unit 17
Constructing Solids

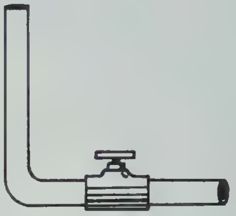
Objects in our real world have length, width and thickness. Such objects are called solids, even though some of them are hollow! In this unit you will learn how to construct some interesting solids and illustrate some of their properties.

17.1 Cylinders and Prisms

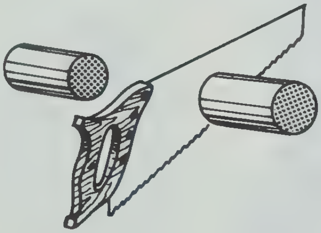
The cylinder has a shape found in many common objects.





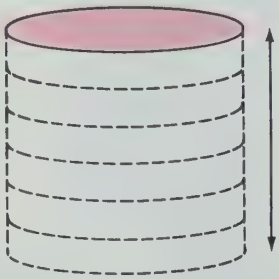


The cylinders we study have a circular cross section.

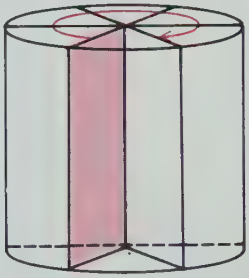


What is the shape of the top and bottom of each of these cylinders?

How can a cylinder be formed by the motion of a circular disc in space?



How can a cylinder be formed by the motion of a rectangle in space?



A cylindrical shape is used for containers and for tubes through which liquids flow smoothly. It is interesting to note that if a given quantity of sheet metal is used to make a pipe, the *cylindrical* pipe carries the greatest volume of liquid.

Another important property of a cylindrical shape is that it is perhaps the solid shape most easily constructed using flat materials. Try the following experiment.

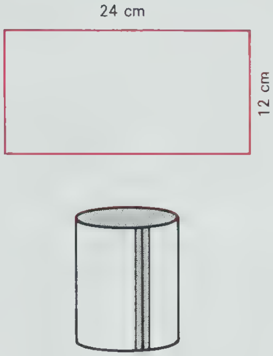
Project 1

Object: To construct a cylinder, a square prism, a rectangular prism, and a triangular prism, and to compare the volumes of each.

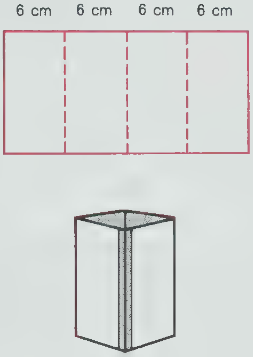
Equipment: Four pieces of light cardboard, each measuring 24 cm by 12 cm; cellulose tape; a light granular material like salt, sugar, or dry sand; container (cake or pie plate).

Method: Construct the shapes as indicated in the diagram. Score along the dotted lines. Fold the patterns and use tape to hold the shapes together.

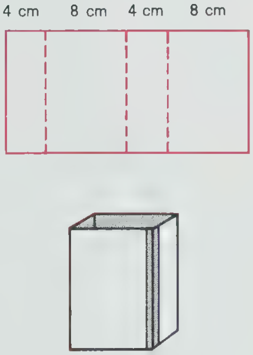
What measurements of each solid are the same?



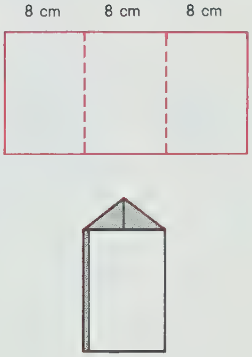
CYLINDER



SQUARE PRISM

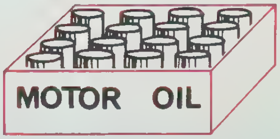


RECTANGULAR PRISM



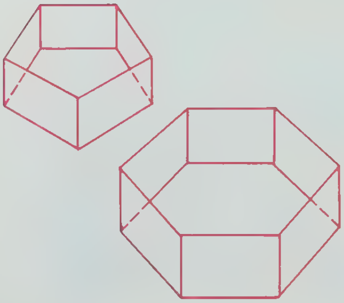
TRIANGULAR PRISM

Use the granular material to determine which shape holds the greatest amount of material.



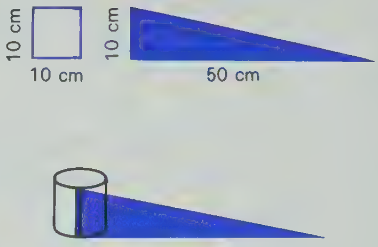
Discussion

1. Product containers such as coffee jars, oil cans, soft drink cans are shipped in cartons. Which of the following shapes fill a rectangular carton best?
   (*a*) cylindrical cans or square-prism shaped cans
   (*b*) square-prism shaped cans or triangular-prism shaped cans
2. Why would a manufacturer be wise to use a hexagonal (6-sided) prism rather than a pentagonal (5-sided) prism as a product container to be shipped in cartons?



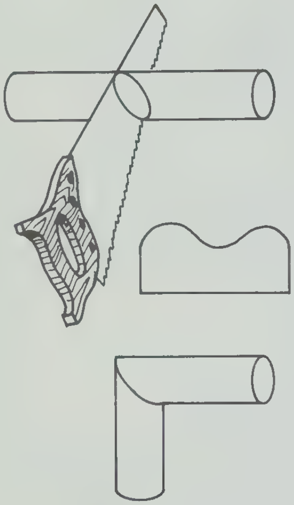
Project 2

Object: To study the nature of the thread of a screw.



Method: Cut out the paper shapes as indicated. Make a cylinder from the square shape.

Tape the triangular shape to the cylinder and wrap the shape about the cylinder.



Project 3

Object: To show the pattern for making pipes and elbows.

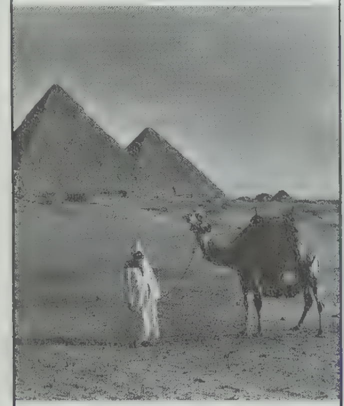
Equipment: Wooden cylinder (dowel), paper roll, hand saw and mitre box.

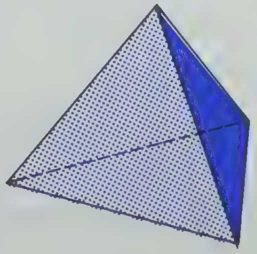
Method: Wrap the paper around the cylinder several times. Cut the cylinder through the paper at an angle of 45°. Unravel the paper and describe the shape. Put the two pieces of wood together to form a 90° elbow.

The unravelled paper has a wave shape. This is the shape one must cut a flat sheet of metal in order to form a section of a 90° elbow.

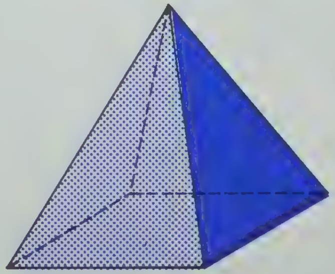
17.2 Pyramids

Pyramids were made famous by the ancient Egyptians. It is interesting to learn about some of the bigger pyramids of Egypt and how they were built. Use your library to determine how the ancients built these architectural wonders.





TRIANGULAR PYRAMID



SQUARE PYRAMID

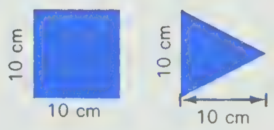
Discussion

1. What are the differences between a triangular and a square pyramid?
2. If the base of a pyramid is a five sided polygon (pentagon), give a possible name for the pyramid.
3. How many triangular faces has each of the following pyramids?
   (*a*) triangular pyramid
   (*b*) square pyramid
   (*c*) pentagonal pyramid
   (*d*) hexagonal pyramid

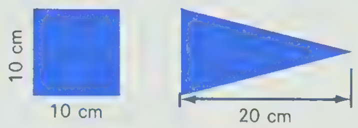
Project 4

1. Make a pyramid from cardboard using a square base and triangular faces with measurements as shown.
2. Repeat step 1 using the measurements indicated.
3. Make a pyramid using only equilateral triangles.
4. Make a square pyramid such that 6 of them form a cube.

1.

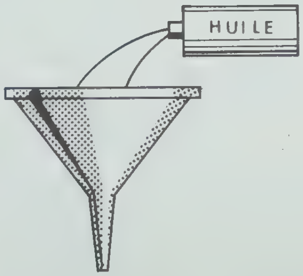


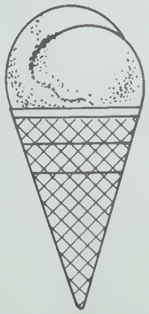
2.

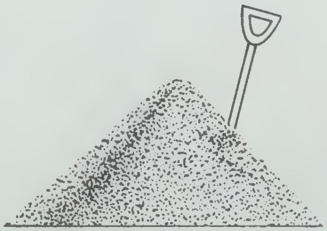


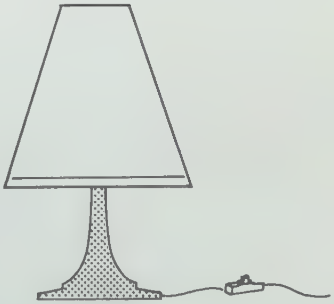
17.3 Cones

Cones are very common shapes that we encounter every day. Perhaps the most important property of a cone is the fact that a conical container can be made from *one piece* of flat material. Although easy to make, the cone has certain disadvantages for packaging of materials.





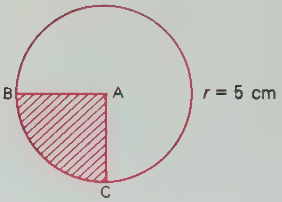


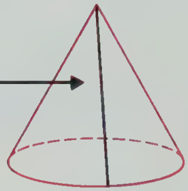


Project 5

Object: To construct a model of a cone.

Method: Cut a sector ABC (pie shape) from a circular disc. Tape edge AB to edge AC. On your cone mark a line segment that represents the radius of the original disc.

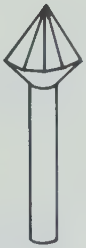




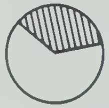
Discussion

1. Suppose you repeat the preceding project but remove a larger sector. Compare the properties of the resulting cone with the one you have already made.
2. Look at a *countersink* as used in a drill. What shape is it and why is it used? Name other tools and objects in a workshop that have a conical shape.
3. A lamp shade can be made from part of a cone. How might you construct a lamp shade?
4. What is a wind tunnel? How is a cone used to increase the speed of the incoming air?
5. Investigate the meaning of a *Venturi effect*. Is this effect related to a cone?
6. Discuss the reasons for using a cone for the serving of ice cream.
7. What are some advantages of making paper cups in a conical shape.
8. Why is a wind sock at an airfield conical in shape?

2.



1.



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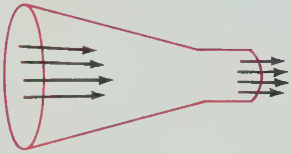
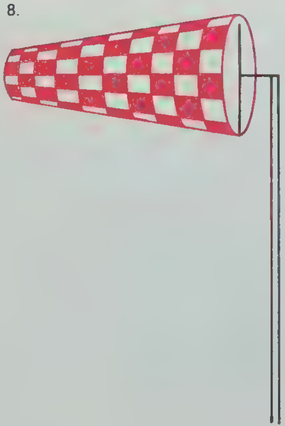


7.

Have you considered packaging?

8.



17.4 Polyhedra

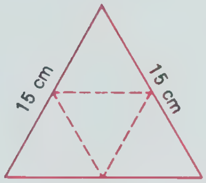
The study of polyhedra has occupied great minds since the beginning of history. The polyhedra studied in this section were well known in the time of Plato and Archimedes in 400 B.C.

poly = many
hedron = face

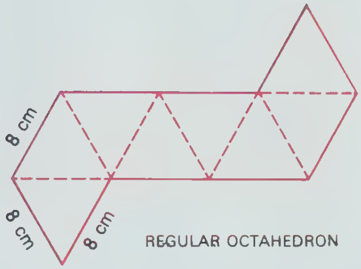
Project 6

Object: To construct the 5 Platonic solids (attributed to Plato).

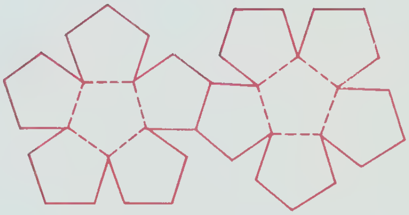
Method: Cut pieces of light cardboard as indicated in the diagrams. Fold about the dotted lines and tape the edges.



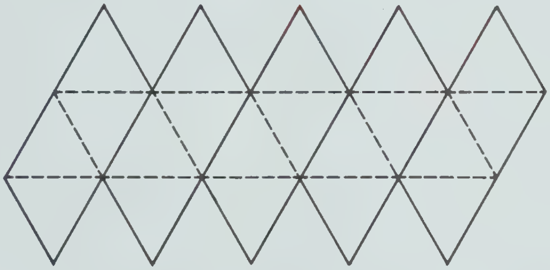
REGULAR TETRAHEDRON



REGULAR OCTAHEDRON



REGULAR DODECAHEDRON



REGULAR ICOSAHEDRON

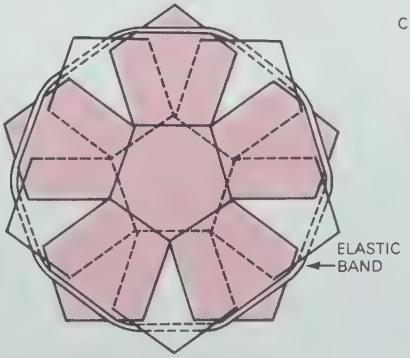
MAKE
YOUR OWN
PATTERN

CUBE

Project 7

Object: To construct a regular dodecahedron.

Method: Cut out 2 shapes, each consisting of 6 pentagons as shown. Place the parts one over the other as illustrated. Stretch an elastic band around the two parts as indicated. Stand back and let it go!



Project 8

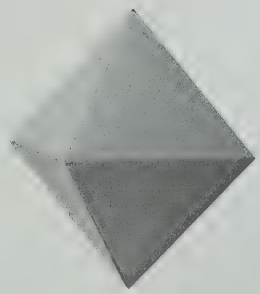
Make models of each of the Archimedean solids, or semi-regular polyhedra.

	Face angles at a vertex	*Number of faces*	*Name of polyhedron*
(*a*)	two 120° angles one 60° angle	4 regular hexagons 4 equilateral triangles	truncated tetrahedron
(*b*)	two 120° angles one 90° angle	8 regular hexagons 6 squares	truncated octahedron
(*c*)	two 90° angles two 60° angles	6 squares 8 equilateral triangles	cuboctahedron
(*d*)	one 90° angle four 60° angles	6 squares 32 equilateral triangles	snub cube

Discussion

"Space filling" means "may be packed so that there is no space between."

1. Which of the solids you have made are *space filling*?
2. Obtain several models of an octahedron and a tetrahedron. Show how they may be put together to be space filling.
3. Read the booklet *Looking at Solids* to learn how to construct an octahedron by *plaiting*.



References

Bell, Stuart E. *Looking at Solids*. London: Longmans, 1967.

Cundy, H. M. and Rollett, A. P. *Mathematical Models*. 2d. ed. (Chapter III, "Polyhedra") Oxford: Clarendon Press, 1961.

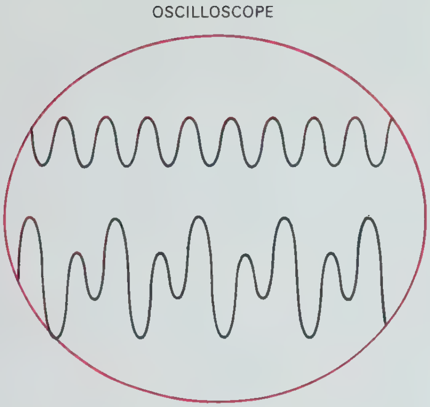
Ehrenfeucht, A. *The Cube Made Interesting*. New York: Macmillan, 1964.

Lewis, K. *Polyhedra*. London: Longmans, 1969.

Mold, Josephine. *Solid Models*. Cambridge: Cambridge University Press, 1967.

National Council of Teachers of Mathematics. *18th Yearbook*. New York: NCTM, 1945.

Wenniger, Magnus J. *Polyhedron Models*. Cambridge: Cambridge University Press, 1970.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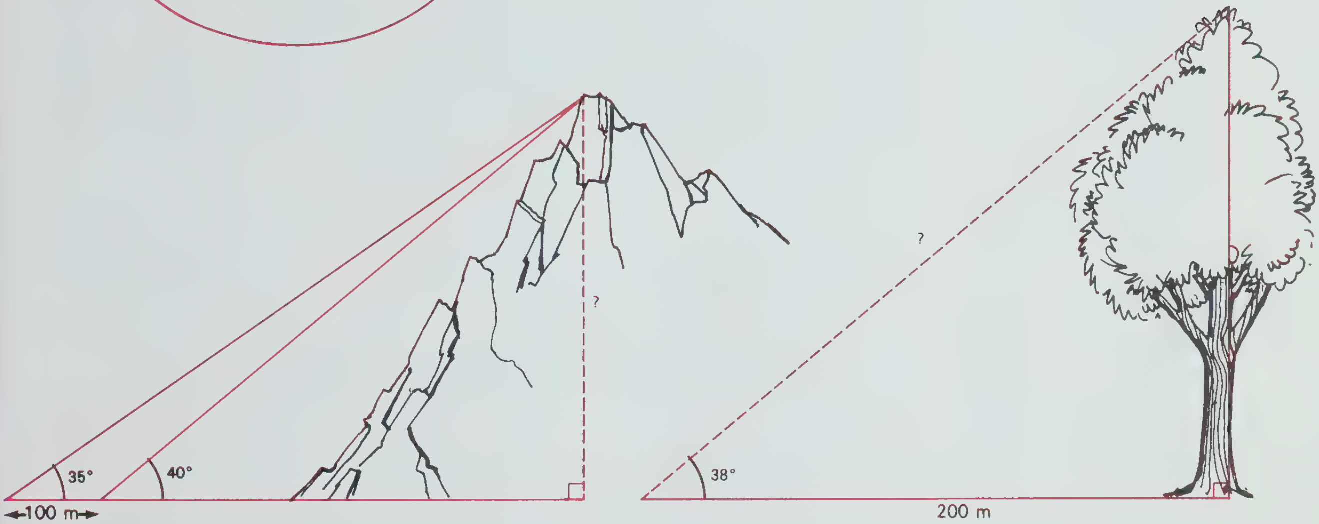

35°
40°
←100 m→
?
38°
?
200 m

Unit 18
Introduction to Trigonometry

Patterns on a TV screen . . . distances on a surveyor's map . . . how can these be related? And yet they are related, by the branch of mathematics called Trigonometry.

18.1 The Basic Trigonometric Values

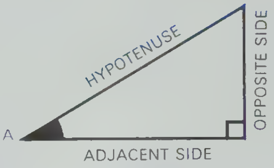
A Trig values may be defined in terms of the sides in a *right-angled triangle*. Consider an acute angle A and the sides of the triangle in relation to angle A.

We define

$$\text{sine of A} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\text{cosine of A} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\text{and tangent of A} = \frac{\text{opposite side}}{\text{adjacent side}}$$


HYPOTENUSE
OPPOSITE SIDE
A
ADJACENT SIDE

In short:

$$\sin A = \frac{opp}{hyp}$$

$$\cos A = \frac{adj}{hyp}$$

$$\tan A = \frac{opp}{adj}$$

Example 1

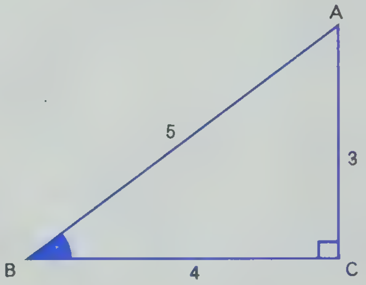
Find the basic trig values for angle B in the given triangle.

Solution

$$\sin B = \frac{AC}{AB} = \frac{3}{5}$$

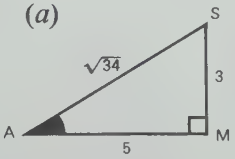
$$\cos B = \frac{BC}{AB} = \frac{4}{5}$$

$$\tan B = \frac{AC}{BC} = \frac{3}{4}$$

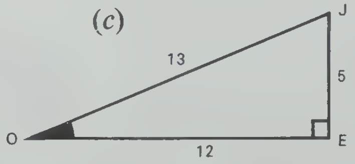


1. State
   (*i*) the opposite and adjacent sides and hypotenuse, and
   (*ii*) the sine, cosine, and tangent of each of the angles indicated.

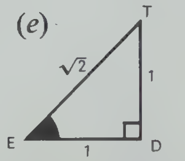
(*a*)


S
$\sqrt{34}$
3
A
M
5

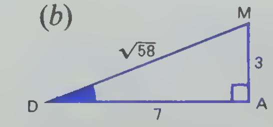
(*c*)



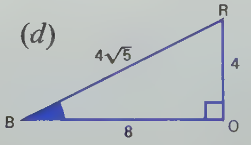
(*e*)


T
$\sqrt{2}$
1
E
1
D

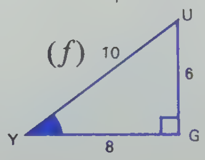
(*b*)


M
$\sqrt{58}$
3
D
7
A

(*d*)


R
$4\sqrt{5}$
4
B
8
O

(*f*)

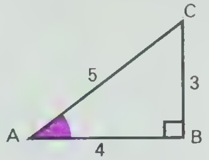


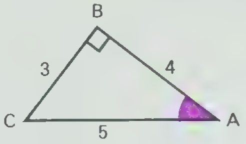
Sine
Opp
Hyp
Cosine
Adj
Hyp
Tangent
Opp
Adj
SOHCAHTOA ?

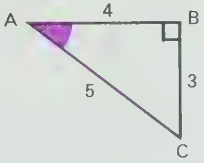
Hmm ?

2. Look back at the diagrams for Question 1 and state the sines and cosines of the remaining *acute* angles.

B Consider △ABC in 3 different positions.





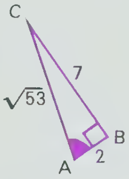


State the value of sin A in each case. Does the position of the triangle have any effect on the value of sin A?

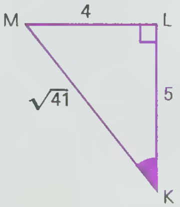
Example 2

State the basic trig values for the angle marked in each triangle.

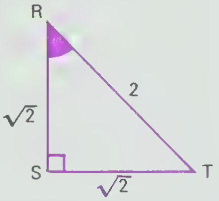
(*a*)


C
7
$\sqrt{53}$
B
A
2

(*b*)


M
4
L
$\sqrt{41}$
5
K

(*c*)


R
2
$\sqrt{2}$
S
T
$\sqrt{2}$

Solution

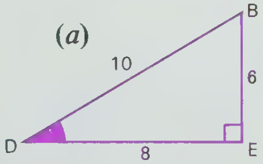
(*a*) $\sin A = \dfrac{7}{\sqrt{53}}$ (*b*) $\sin K = \dfrac{4}{\sqrt{41}}$ (*c*) $\sin R = \dfrac{\sqrt{2}}{2}$

$\cos A = \dfrac{2}{\sqrt{53}}$ $\cos K = \dfrac{5}{\sqrt{41}}$ $\cos R = \dfrac{\sqrt{2}}{2}$

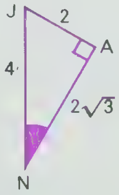
$\tan A = \dfrac{7}{2}$ $\tan K = \dfrac{4}{5}$ $\tan R = \dfrac{\sqrt{2}}{\sqrt{2}} = 1$

1. State the basic trig values for the angle marked in each triangle.

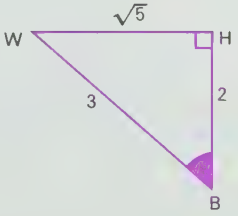
(*a*)



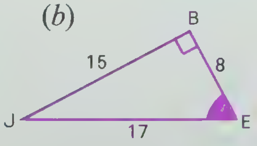
(*c*)


J
2
A
4
$2\sqrt{3}$
N

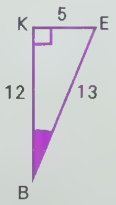
(*e*)


W
$\sqrt{5}$
H
3
2
B

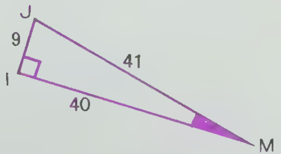
(*b*)



(*d*)



(*f*)



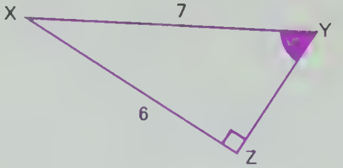
2. Repeat Question 1 for the remaining *acute* angles.

sin
cos
tan

SOH
CAH
TOA

Example 3

Find the basic trig values for ∠Y.



Solution

Before we can find all 3 values, we must know the measure of all sides of the triangle.

$$\begin{aligned} \because\ 6^2 + ZY^2 &= 7^2 \\ \therefore\ 36 + ZY^2 &= 49 \\ \text{and}\quad ZY^2 &= 13 \\ \therefore\quad ZY &= \sqrt{13} \end{aligned}$$

$$\therefore\ \sin Y = \frac{6}{7}$$

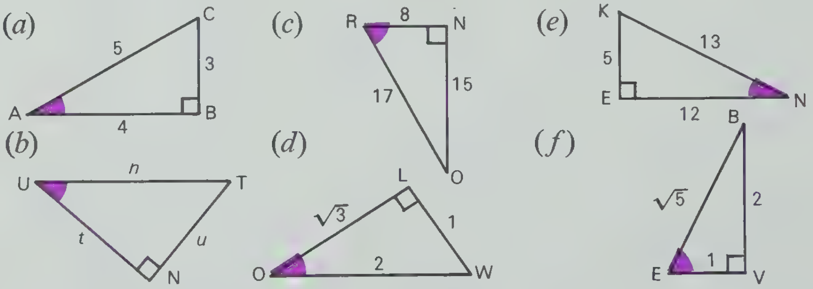
$$\cos Y = \frac{\sqrt{13}}{7}$$

$$\tan Y = \frac{6}{\sqrt{13}}$$

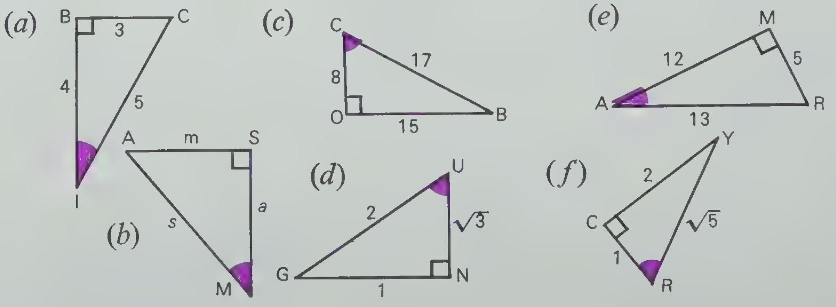
Exercise 18.1

Discussion

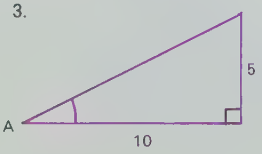
1. For each angle marked, state the opposite side, adjacent side, hypotenuse, and the sine, cosine, and tangent.


(a)
C
5
3
A
B
4
(c)
R
8
N
17
15
O
(e)
K
13
5
E
N
12
(b)
U
n
T
t
u
N
(d)
L
√3
1
O
2
W
(f)
B
√5
2
E
1
V

2. Repeat Question 1 for these triangles.


(a)
B
3
C
4
5
I
(c)
C
17
8
O
15
B
(e)
M
12
5
A
R
13
(b)
A
m
S
s
a
M
(d)
U
2
√3
G
1
N
(f)
Y
2
C
√5
1
R

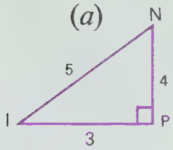
3.



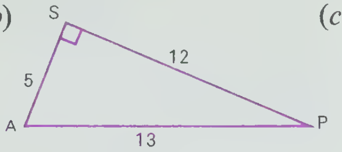
3. To find the sine and cosine of A, what step must you take first?

4. State an equation relating the 3 sides of each triangle.

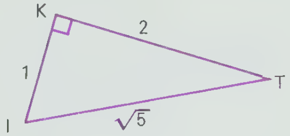
(a)



(b)



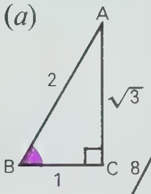
(c)


K
2
1
T
I
$\sqrt{5}$

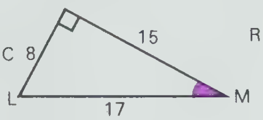
Written Solutions

5. Find the sine, cosine, and tangent of each marked angle.

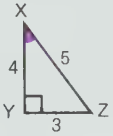
(a)


A
2
$\sqrt{3}$
B
C
1

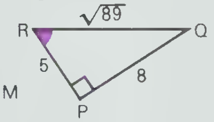
(b)



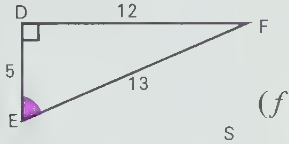
(c)



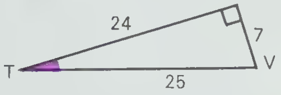
(d)


$\sqrt{89}$
R
Q
5
8
P

(e)

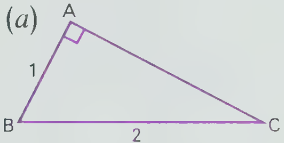


(f)

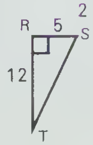


6. Use Pythagoras to help you determine the basic trig values for each acute angle.

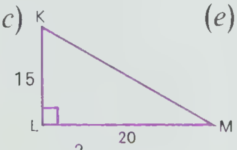
(a)



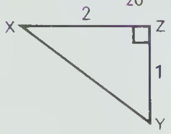
(b)



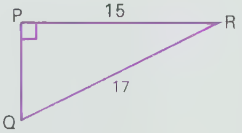
(c)



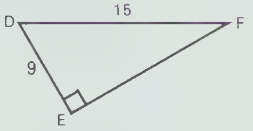
(d)



(e)



(f)

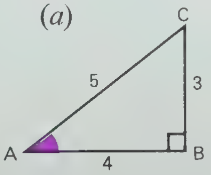


7. In $\triangle PQR$, $\angle Q = 90°$, $PQ = 4$ cm, and $RQ = 6$ cm. Find the sine and cosine of $\angle P$.

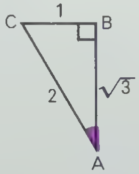
8. Diagonal WY is drawn in rectangle WXYZ. If the diagonal is 13 cm long and the short sides of the rectangle are 5 cm, find the sine of the angles at which the diagonal meets XY and YZ.

9. For the marked angle A in each triangle, evaluate sin A, cos A, and $\sin^2 A + \cos^2 A$. What do you notice?

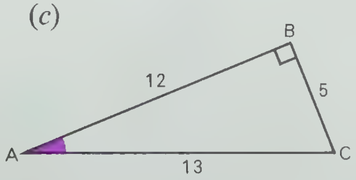
(a)



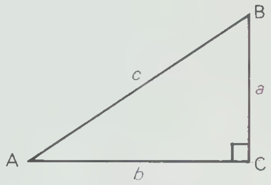
(b)


C
1
B
2
$\sqrt{3}$
A

(c)



Remember Pythagoras!



$c^2 = a^2 + b^2$

5(a) $\sin B = \frac{\sqrt{3}}{2}$

$\cos B = \frac{1}{2}$

$\tan B = \sqrt{3}$

6(b) $\sin T = \frac{5}{13}$

$\cos T = \frac{12}{13}$

$\tan T = \frac{5}{12}$

$\sin S = \frac{12}{13}$

$\cos S = \frac{5}{13}$

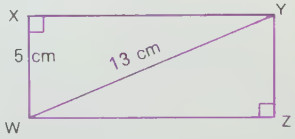
$\tan S = \frac{12}{5}$

7. $\sin P = \frac{3}{\sqrt{13}}$, $\cos P = \frac{2}{\sqrt{13}}$

8. $\sin \angle XYW = \frac{5}{13}$,
$\sin \angle WYZ = \frac{12}{13}$

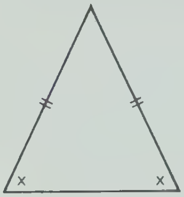
9(a) $\frac{3}{5}$, $\frac{4}{5}$, 1

8

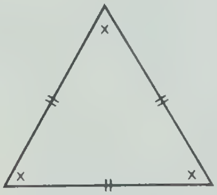

X
Y
5 cm
13 cm
W
Z

$\sin^2 A$ means $(\sin A)^2$

18.2 Trig Values of Any Acute Angle



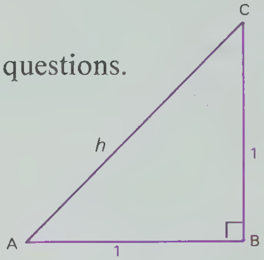
ISOSCELES



EQUILATERAL

A There are 3 acute angles that occur so frequently in trigonometry that they deserve our special consideration.

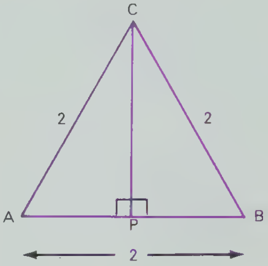
Trigonometric Values of 45°



Study the diagram and answer the following questions.

1. Use Pythagoras to find h in the isosceles triangle ABC.
2. Since sides AB and CB are equal what other equality must exist?
3. Using Question 2 what must be the measure of ∠A and ∠C?
4. State the value of
   (*a*) sin 45° (*b*) cos 45° (*c*) tan 45°

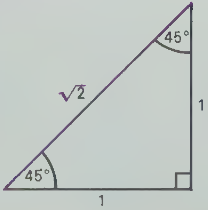
Trigonometric Values of 30° and 60°



An *equilateral* triangle ABC is drawn with sides each 2 units long. Then CP is drawn perpendicular to AB.

5. State the measure of ∠A, ∠B, and ∠ACP.
6. How long is AP?
7. Now use Pythagoras to calculate the length of CP.
8. Using △ACP, state the value of each of the following.
   (*a*) sin 60° (*b*) cos 60° (*c*) tan 60°
   sin 30° cos 30° tan 30°

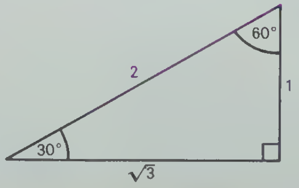
Summary



$\sin 45° = \dfrac{1}{\sqrt{2}}$

$\cos 45° = \dfrac{1}{\sqrt{2}}$

$\tan 45° = 1$



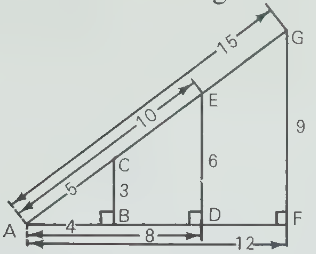
$\sin 30° = \dfrac{1}{2} = \cos 60°$

$\cos 30° = \dfrac{\sqrt{3}}{2} = \sin 60°$

$\tan 30° = \dfrac{1}{\sqrt{3}}$

$\tan 60° = \sqrt{3}$

B Consider the "nested" set of triangles with common vertex A.



1. Using $\triangle ABC$, evaluate sin A, cos A, and tan A.
2. Using $\triangle ADE$, evaluate sin A, cos A, and tan A.
3. Using $\triangle AFG$, evaluate sin A, cos A, and tan A.

> It would appear, then, that the trig values of an angle depend only upon the size of the angle and *not* upon the length of the arms of the angle.

C To find the trig values for *any* acute angle, say 23°, we proceed as in the following example.

Example 1

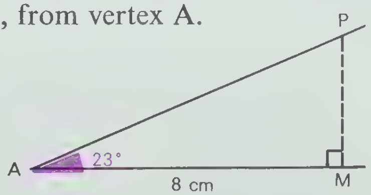
Find the basic trig values for 23°.

Solution

Step 1 Construct a 23° angle, as shown.

Step 2 Mark point M on one arm of the angle at a convenient distance, say 8 cm, from vertex A.

Step 3 Draw $PM \perp AM$.



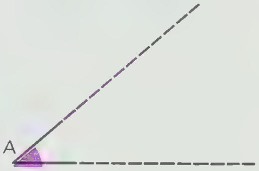
By measurement, we find $MP \doteq 3.4$ cm and $AP \doteq 8.7$ cm.

$$\therefore \sin A = \frac{PM}{AP} \doteq \frac{3.4}{8.7} \doteq 0.391 \qquad \cos A = \frac{AM}{AP} \doteq \frac{8}{8.7} \doteq 0.920 \qquad \tan A = \frac{PM}{AM} \doteq \frac{3.4}{8} = 0.425$$

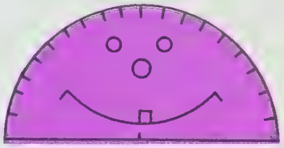
Do you see that triangles ABC, ADE, and AFG are similar?

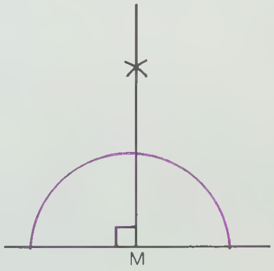
$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$$

What do you notice?



I can help out here!



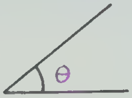


Remember?

What a relief!

Our values for 23° were pretty close to these, eh?

* θ is a letter from the greek alphabet, used to name angles.



D Now it would be very inconvenient if we had to draw a scale diagram each time a trig value was required. Fortunately, this will not be necessary. The table on page 443 gives trig values for angles from 0° to 90°, correct to 3 decimal places. A portion of the table is shown below.

θ	$\sin\ \theta$	$\cos\ \theta$	$\tan\ \theta$
20	0.342	0.940	0.364
21	0.358	0.934	0.384
22	0.375	0.927	0.404
23	0.391	0.921	0.424
24	0.407	0.914	0.445
25	0.423	0.906	0.466
26	0.438	0.899	0.488
27	0.454	0.891	0.510
28	0.469	0.883	0.532
29	0.485	0.875	0.554

Example 2

(*a*) Find cos 20°.

(*b*) If $\tan\ \theta = 0.510$, find θ *

Solution

(*a*) By looking under the cosine column ($\cos\ \theta$) and across the 20° row, we see that cos 20° = 0.940.

(*b*) From the table, we see than tan 27° = 0.510. Hence, $\theta = 27°$.

1. From the portion of the table above, find the required values.
   (*a*) sin 22° (*d*) cos 21° (*g*) tan 25°
   (*b*) cos 28° (*e*) tan 20° (*h*) cos 27°
   (*c*) tan 24° (*f*) sin 29° (*i*) sin 26°

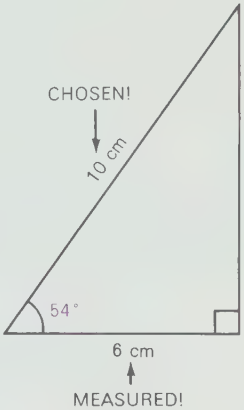
2. Use the table to find the angle θ in each case.
   (*a*) $\tan\ \theta = 0.488$ (*d*) $\tan\ \theta = 0.404$
   (*b*) $\cos\ \theta = 0.934$ (*e*) $\sin\ \theta = 0.485$
   (*c*) $\sin\ \theta = 0.470$ (*f*) $\cos\ \theta = 0.906$

3. Using the table on page 443, find each value.
   (*a*) cos 14° (*d*) sin 54° (*g*) tan 4°
   (*b*) tan 53° (*e*) cos 42° (*h*) sin 37°
   (*c*) sin 81° (*f*) tan 32° (*i*) cos 77°

Exercise 18.2

Discussion

1. The margin diagram shows how to construct a triangle to evaluate cos 54°. Explain how to evaluate cos 38° in a similar way.



$\therefore \cos 54° = \frac{6}{10}$
$= 0.6$

2. In the trig table on page 443, as θ gets larger and larger, what happens to $\sin \theta$? To $\cos \theta$? To $\tan \theta$?

3. State the value of each expression.

(*a*) $\frac{1}{2} + \frac{1}{4}$ (*c*) $\left(\frac{1}{\sqrt{2}}\right)^2$ (*e*) $\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

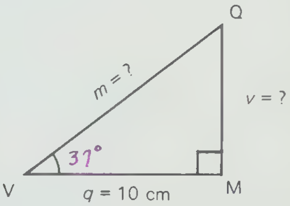
(*b*) $(\sqrt{2})(\sqrt{3})$ (*d*) $\sqrt{10} \times \sqrt{5}$ (*f*) $\frac{0.5}{5}$

Written Solutions

4. (*a*) Using a protractor, accurately construct an angle of 37°. Mark a point M on one arm of the angle at a distance 10 cm from the vertex V. At M, draw a perpendicular to cut the other arm of the angle at Q. Measure MQ and VQ as accurately as possible and calculate values for sin 37°, cos 37°, and tan 37°. Check your answers with values from the trig table.

(*b*) Using the same $\triangle$QVM, calculate values for sin 53°, cos 53°, and tan 53°. Again, check your answers in the trig table.

(*c*) Evaluate $\frac{\sin V}{v}$ and $\frac{\sin Q}{q}$. What do you notice?



5. Use the trig table to find each value.

(*a*) tan 80° (*d*) cos 41° (*g*) cos 11°
(*b*) sin 15° (*e*) tan 33° (*h*) sin 87°
(*c*) tan 12° (*f*) sin 39° (*i*) cos 74°

6. Use the trig table to find angle θ in each case.

(*a*) $\cos \theta = 0.454$ (*d*) $\tan \theta = 0.105$
(*b*) $\tan \theta = 4.332$ (*e*) $\sin \theta = 0.559$
(*c*) $\sin \theta = 0.875$ (*f*) $\cos \theta = 0.819$

7(a) 1 (c) $\frac{\sqrt{3}}{2}$ (e) 1
(b) 0 (d) 1 (f) 1

7. Evaluate each of the following expressions.
   (*a*) $\dfrac{\sin 45°}{\cos 45°}$ (*d*) $(\sin 60°)(\cos 30°) + (\cos 60°)(\sin 30°)$
   (*b*) $\sin 60° - \cos 30°$ (*e*) $(\sin 60°)^2 + (\cos 60°)^2$
   (*c*) $2(\sin 30°)(\cos 30°)$ (*f*) *$\sin^2 45° + \cos^2 45°$

* $\sin^2 45°$ means $(\sin 45°)^2$.

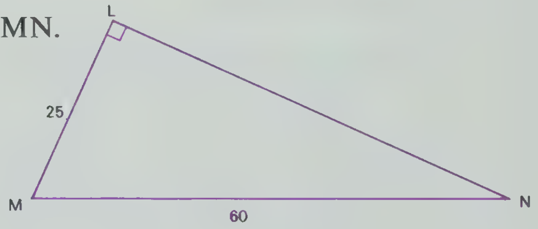
8. Evaluate $\sin^2 30° + \cos^2 30°$ and compare to 7 (*e*), (*f*). What appears to be the value of $\sin^2 A + \cos^2 A$ for any value of A? See if you can prove this using any angle A instead of a special angle like 30°, 45°, or 60°.

18.3 Applications of Trigonometry

One of the more important uses of the trigonometric ratios is to find unknown angles and sides of a triangle.

Example 1

Find the acute angles of △LMN.



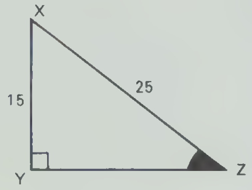
Solution

In △LMN, $\sin N = \dfrac{25}{60}$
$\doteq 0.417$

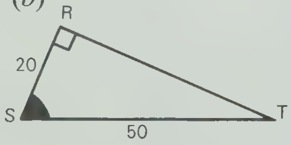
From the trig table $\angle N = 25°$
But $\angle M + \angle N = 90°$
$\therefore$ $\angle M = 65°$

1. Find the angle indicated in the diagrams on the left.

(*a*)



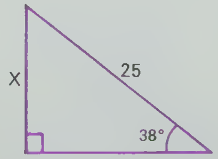
(*b*)



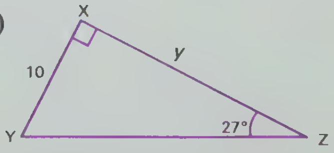
Example 2

Find the unknown side in each case.

(*a*)



(*b*)



Solution (*a*) $\sin 38° = \dfrac{x}{25}$

$\therefore x = 25 \times \sin 38°$
$\doteq 25 \times 0.616$
$= 15.4$

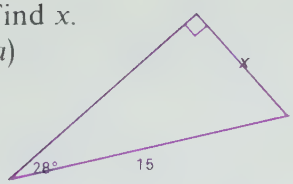
(*b*) $\tan 27° = \dfrac{10}{y}$

$\therefore y = \dfrac{10}{\tan 27°}$
$\doteq \dfrac{10}{0.510} \doteq 19.6$

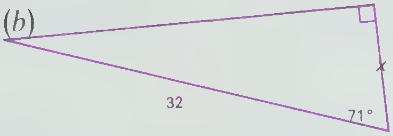
Here's a short cut for (*b*)!
$\because \angle Y = 90° - 27° = 63°$
$\therefore \tan 63° = \dfrac{y}{10}$
$\therefore y = 10 \times \tan 63°$
$= 10 \times 1.963$
$= 19.63$
(Multiplying by 10 is easier than dividing by 0.510!)

2. Find x.

(a)



(b)



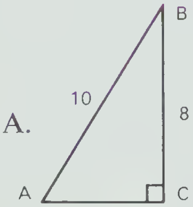
Summary of Steps in Solving Triangles

Step 1 Choose a known angle or a known fraction.
Step 2 Set up an equation determined by your choice in Step 1.
Step 3 Solve the equation you formed in Step 2.

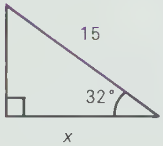
To "solve" a triangle is to find its missing angles and sides!

Example 3

(a) Find ∠A.



(b) Find x.



Solution

(a) Known fraction is $\frac{8}{10}$.

$\because \frac{8}{10}$ is $\frac{\text{OPP}}{\text{HYP}}$ for ∠A

$\therefore \sin A = \frac{8}{10} = 0.8$

$\therefore \angle A \doteq 53°$

(b) Known angle is 32°. *Step 1*

$\because \frac{x}{15}$ is $\frac{\text{ADJ}}{\text{HYP}}$ for 32°

$\therefore \frac{x}{15} = \cos 32°$ *Step 2*

and $x = 15 \times \cos 32°$

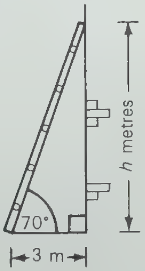
$\doteq 15 \times 0.848 \doteq 13$ *Step 3*

We may now solve problems involving right-angled triangles.

Example 4

A painter places a ladder 3 m from a vertical wall so that the ladder makes a 70° angle with the level ground. How far up the wall does the ladder reach?

Solution



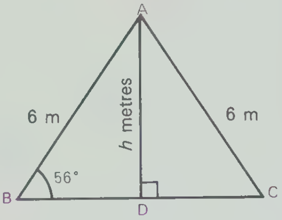
Let △XYZ represent the given information and let the unknown distance up the wall be h metres.

$\therefore \frac{h}{3} = \tan 70° \doteq 2.748$

$\therefore h \doteq 3(2.748) = 8.244$

∴ The ladder reaches approximately 8 m up the wall.

Remember? An algebraic solution involves a "let" statement and a final statement, with UNITS!



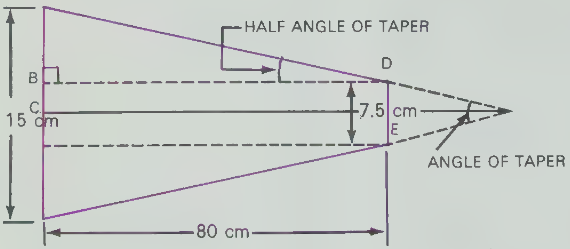
Example 5 The equal sides of an A-frame cottage are each 6 m in length and meet the ground at an angle of 56°. Find the height of the cottage.

Solution Let the height of the cottage be h metres.

In △ABD, $\frac{h}{6} = \sin 56° \doteq 0.829$

$\therefore h \doteq 6(0.829) = 4.974$

The cottage is approximately 5 m high.



Example 6 Find the angle of taper required to give diameters of 7.5 cm and 15 cm on an 80 cm piece of stock.

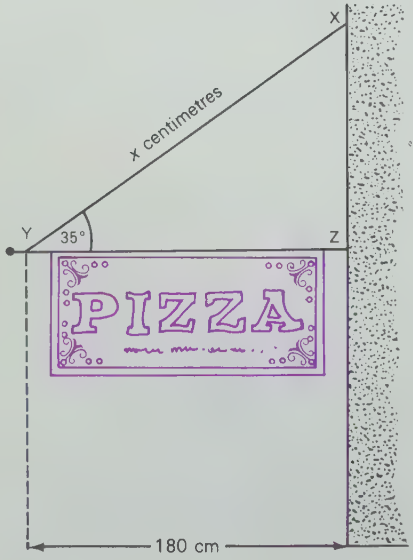
Solution In the diagram,

$$\begin{aligned} AC &= \tfrac{1}{2}(15 \text{ cm}) \\ &= 7.5 \text{ cm} \\ \therefore BC = DE &= \tfrac{1}{2}(7.5 \text{ cm}) \\ &= 3.75 \text{ cm} \\ \therefore AB &= AC - BC \\ &= 7.5 \text{ cm} - 3.75 \text{ cm} \\ &= 3.75 \text{ cm} \end{aligned}$$

In triangle ACB, $\tan \angle ADB = \frac{3.75}{80} \doteq 0.047$

$\therefore \quad \angle ADB \doteq 3°$

∴ Angle of taper is approximately 6°.



Example 7 An overhead sign hangs 180 cm out from a wall and is supported by a cable, as shown in the diagram. If the angle between the cable and the sign is 35°, find the length of cable required.

Solution

Let the required length of cable be x centimetres.

In △XYZ, $\frac{YZ}{x} = \cos 35°$

$\therefore \quad \frac{180}{x} \doteq \frac{0.819}{1}$

and $\quad 0.819x \doteq 180$

$\therefore \quad x \doteq \frac{180}{0.819} \doteq 219.78$

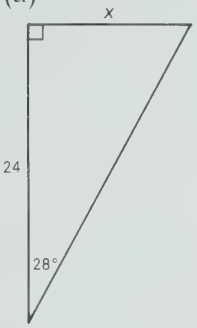
∴ The cable must be approximately 220 cm in length.

Exercise 18.3

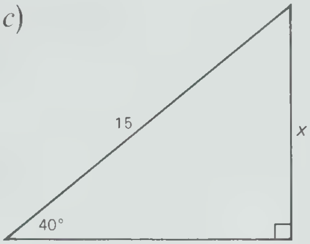
Discussion

1. State the equation you would write to find each unknown angle or side.

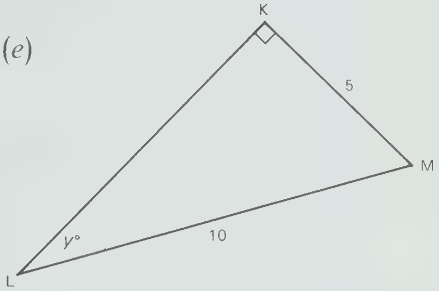
(*a*)



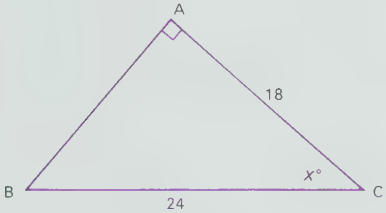
(*c*)



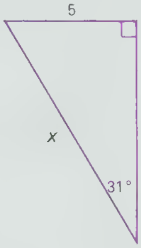
(*e*)



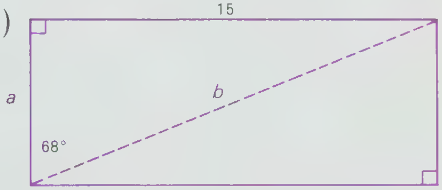
(*b*)



(*d*)

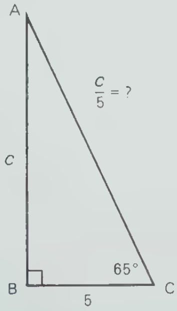


(*f*)



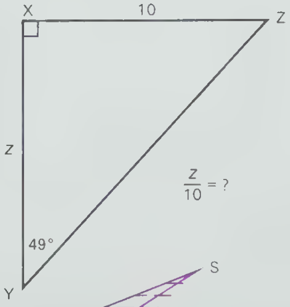
2. Express each fraction as a sine, cosine, or tangent of an acute angle.

(*a*)



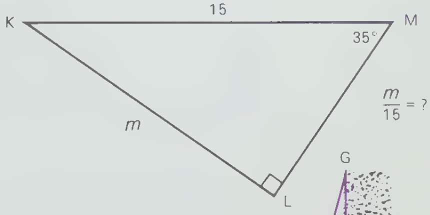
$\frac{c}{5} = ?$

(*c*)



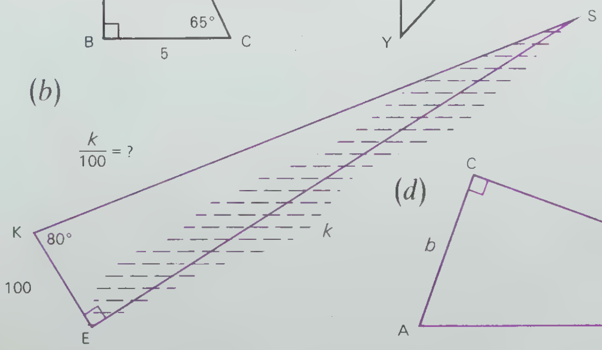
$\frac{z}{10} = ?$

(*e*)



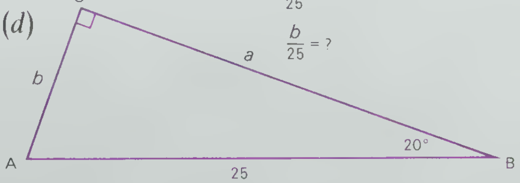
$\frac{m}{15} = ?$

(*b*)



$\frac{k}{100} = ?$

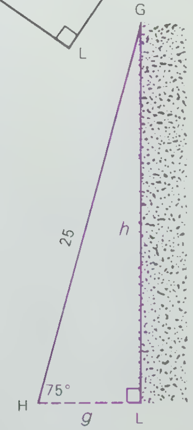
(*d*)



$\frac{a}{25} = ?$

$\frac{b}{25} = ?$

(*f*)

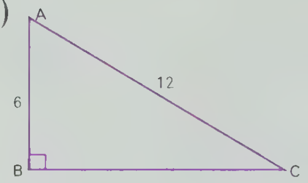


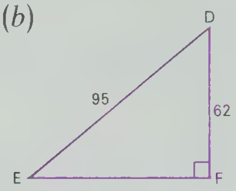
$\frac{h}{25} = ?$

$\frac{g}{25} = ?$

Written Solutions

3. Find the acute angles in each triangle.

(a)


(b)


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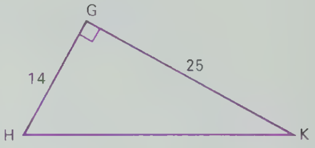
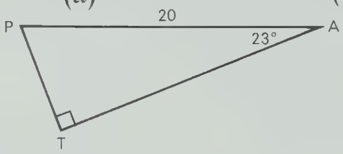
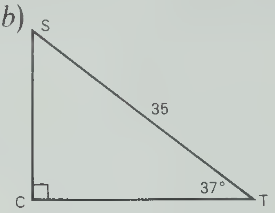
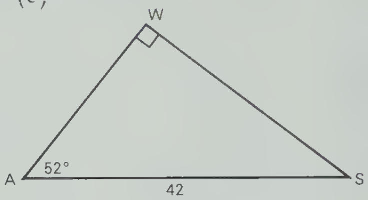


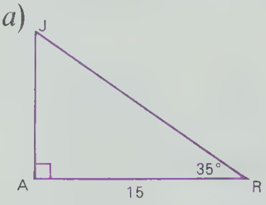
4. Find the remaining sides of each triangle.

(a)


(b)


(c)


5. Find the remaining sides of each triangle.

(a)


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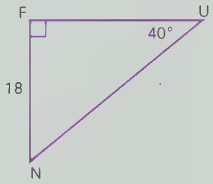

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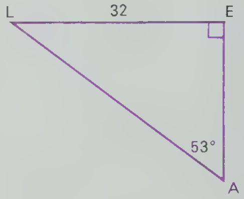
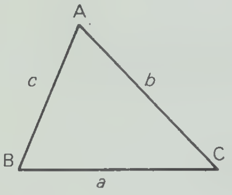


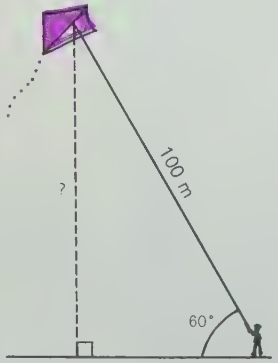
6. For each of the following triangles draw a representative diagram and find any side or angle not given.
   (a) In △ABC, $\angle B = 90°$, $b = 12$, $\angle C = 28°$.
   (b) In △POD, $\angle P = 90°$, $d = 25$, $p = 40$.
   (c) In △XYZ, $\angle Z = 90°$, $z = 15$, $y = 12$.
   (d) In △RST, $\angle R = 90°$, $t = 10$, $\angle T = 63°$.
   (e) In △KLM, $\angle M = 90°$, $l = 16$, $\angle K = 51°$.
   (f) In △PQR, $\angle Q = 90°$, $p = 18$, $r = 12$.

7. Kingsley Kiteflyer has let out 100 m of string while flying his kite and estimates that the angle the string makes with the ground is approximately 60°. Find how high his kite is flying.

8. Captain Jean Martin takes off in a Concorde Jumbo Jet that makes an angle of 20° to the horizontal. When the aircraft has attained an altitude of 15 km, what is its ground distance? (This means the distance measured along the ground.) Answer to the nearest kilometre.

3(a) $\angle A = 60°$, $\angle C = 30°$
(c) $\angle K = 29°$, $\angle H = 61°$
4(a) PT = 7.82 TA = 18.4
(c) WS = 33.1 WA = 25.9
5(a) JA = 10.5 JR = 18.3
(c) LA = 40.1 AE = 24.1
6(a) $\angle A = 62°$, $c = 5.64$, $a = 10.6$
(c) $x = 9$, $\angle X = 37°$, $\angle Y = 53°$
(e) $\angle L = 39°$, $k = 19.8$, $m = 25.4$
7. 86.6 m
8. 41 km



7.


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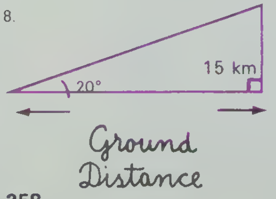


9. A metre stick is held at an angle of 40° to a horizontal table, and a light source is held directly above the raised end. Find the length of the shadow of the metre stick, to the nearest centimetre.

10. On a straight 100 m section of a ski run there is a drop of 30 m. What is the angle of incline on the hill?

11. A pendulum bob on a 25 cm string is pulled to one side until it is 5 cm above its original position. Find the angle the string makes with the vertical.

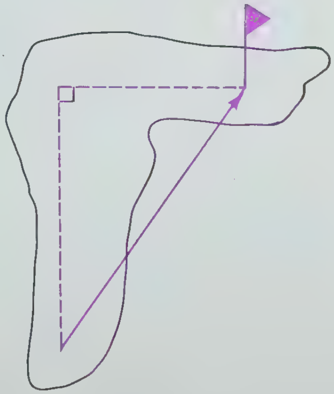
12. The diagonal of a rectangle is 35 cm long, and it meets 2 adjacent sides of the rectangle at angles of 56° and 34°. Find the length and width of the rectangle (to the nearest centimetre).

13. Find the height of a cone that has a vertical angle of 80° and a base with diameter 18 m (to 1 decimal place).

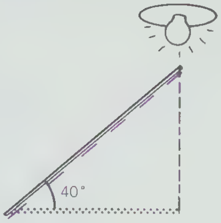
14. To find the length of a lake having a relatively straight shore, Sam Surveyor paces 300 m straight back from the shore and finds that his lines of observation to trees at either end of the lake make angles of 75° and 60° with his path back from the lake. Find the length of the lake, to the nearest metre.

15. On a 90° dogleg golf hole, Pat Parbuster decides to drive straight to the hole. If the first leg is 200 m and the second leg is 150 m, find
(*a*) the angle that her drive makes with the first part of the dogleg, and
(*b*) the distance she must drive to get a "hole-in-one."

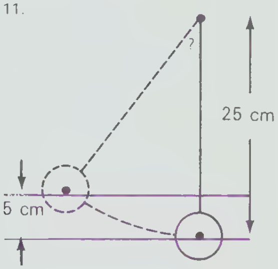
15.



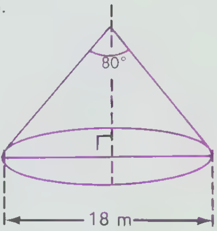
9.



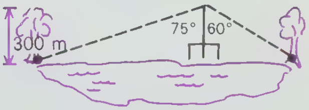
11.



13.



14.



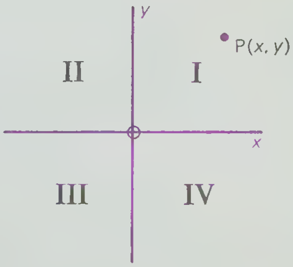
9. 77 cm
10. 17°
11. 37°
12. 29 cm, 20 cm
13. 10.7 m
14. 1639 m
15(a) 37° (b) 250 m

19

Unit 19
Coordinate Trigonometry

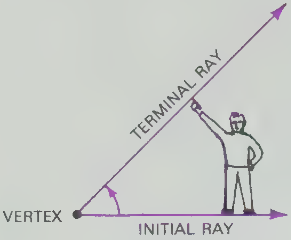
You will recall from your work on graphing in Units 9, 10, and 11 that the *x axis* and *y axis* divide a plane into 4 quarters, or *quadrants*. The axes intersect at the *origin*, whose *coordinates* are (0, 0). Any point in the plane may be located by means of an *ordered pair* (x, y).

In this unit, you will discover how the basic trig values may be defined in terms of coordinates.



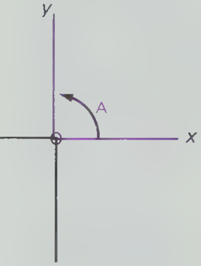
19.1 Angles in Standard Position

Angles can be considered as an *amount of rotation* from an initial ray to a terminal ray about a fixed point called the *vertex* of the angle.

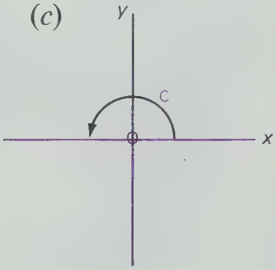


1. What is the measure of a right angle?
2. A cadet does an "about-turn." Through what angle does he turn in this rotation?
3. What angle is "swept out" by the second hand during one minute? How many hours pass when the hour hand rotates through a 180° angle?
4. In each diagram, state the measure of the angle named by the capital letter.

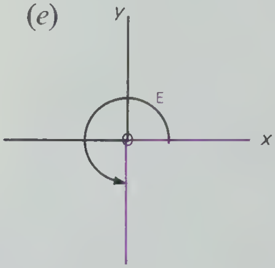
(*a*)



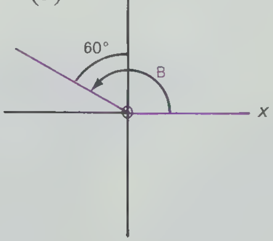
(*c*)



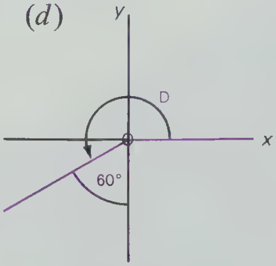
(*e*)



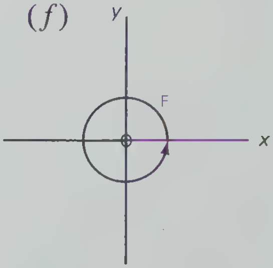
(*b*)



(*d*)



(*f*)



What do these 6 angles have in common?

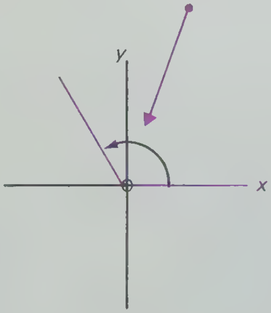
Each of the angles in Question 4 is in "standard position." Check, for each angle, that the following things are true.

(*a*) The vertex is at the origin.

(*b*) The initial ray lies along the positive x axis and rotation to a terminal ray is marked.

We shall consider only angles formed by counter-clockwise rotation in this section.

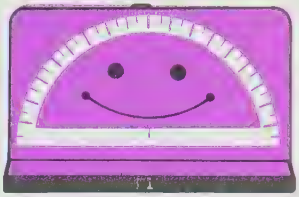
Angle in Standard Position



Example 1

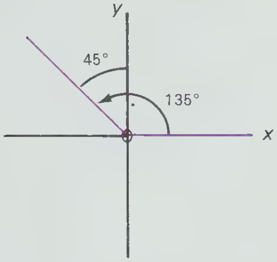
Construct the following angles in standard position.

(*a*) 135°   (*b*) 250°   (*c*) 320°


Remember me?

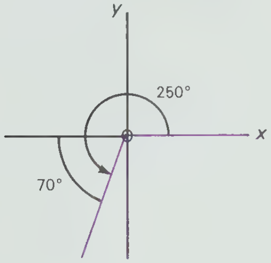
Solution

(*a*)



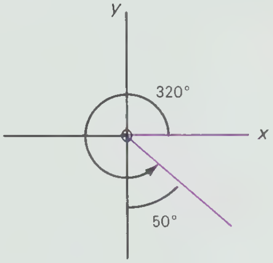
$135° = 90° + 45°$

(*b*)



$250° = 180° + 70°$

(*c*)



$320° = 270° + 50°$

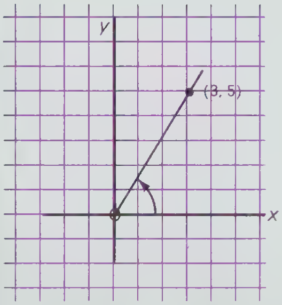
Example 2

Construct angles in standard position whose terminal rays pass through

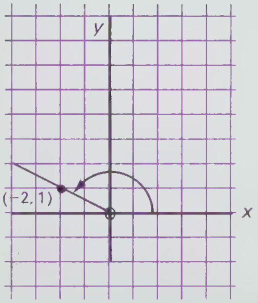
(*a*) the point (3, 5),
(*b*) the point (−2, 1),
(*c*) the point (4, −1).

Solution

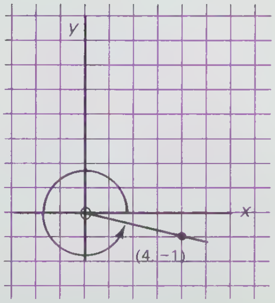
(*a*)

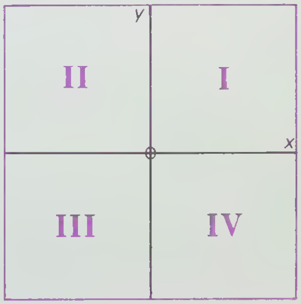


(*b*)



(*c*)




y
x
II
I
III
IV

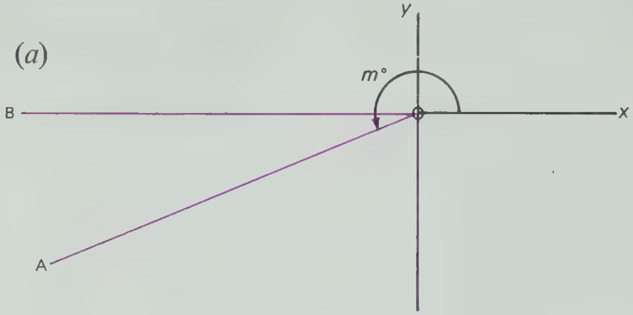
THE QUADRANTS

In part (*a*) of Example 2 the angle is a *1st quadrant angle.* Why? Using the same phrasing describe the angles in parts (*b*) and (*c*).

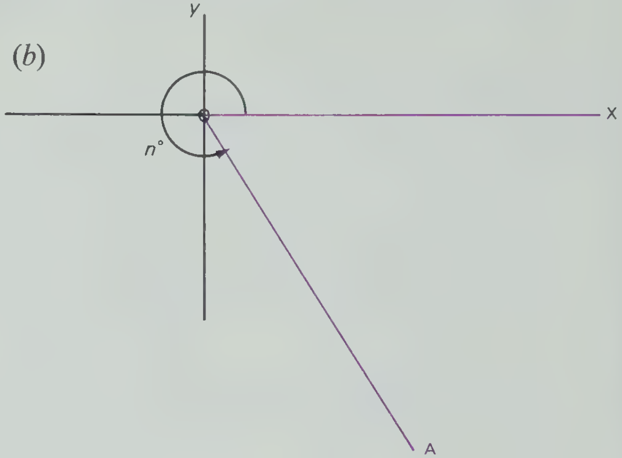
Example 3

Find by measurement the size of each of the following angles.

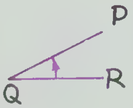
(*a*)



(*b*)



Recall



∠Q
or ∠PQR
or ∠RQP

Solution

Using a protractor,

(*a*) $\angle AOB = 22°$

$\therefore m° = 180° + 22°$

$= 202°$

(*b*) $\angle AOX = 58°$

$\therefore n° = 360° - 58°$

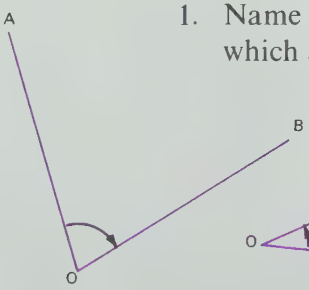
$= 302°$

Exercise 19.1

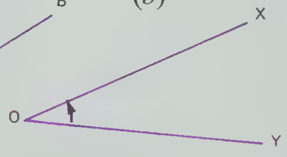
Discussion

1. Name the initial ray of each of the following angles. For which angles is the rotation counter-clockwise?

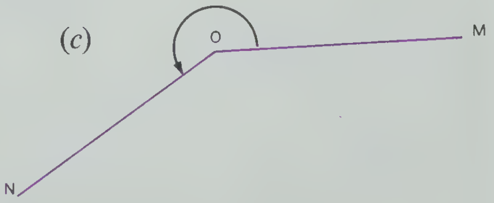
(*a*)



(*b*)

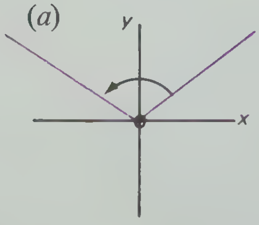


(*c*)

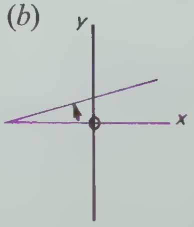


2. Which of the following angles are in standard position? Explain.

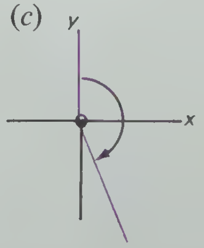
(*a*)



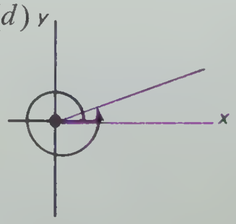
(*b*)



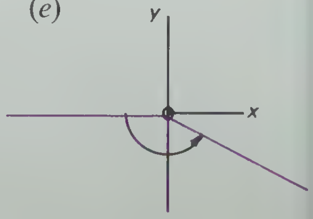
(*c*)



(*d*)

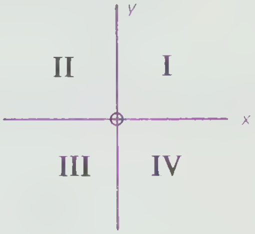


(*e*)

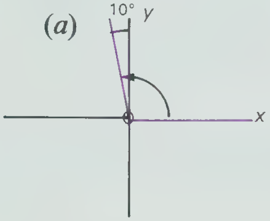


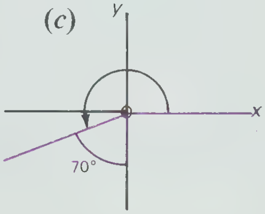
3. State the quadrant in which the terminal ray lies for each angle.
(*a*) 48° (*c*) 300° (*e*) 96° (*g*) 350°
(*b*) 160° (*d*) 220° (*f*) 185° (*h*) 75°

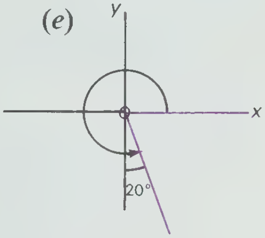
Unless specified otherwise, you may assume all angles are in Standard Position.

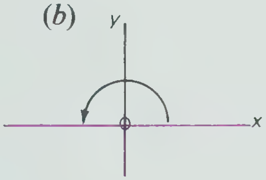


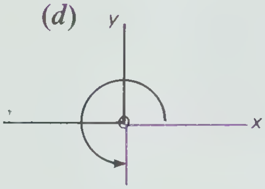
4. State the measure of each of the following angles.

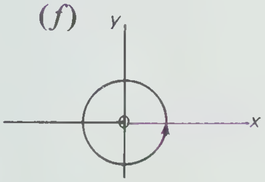
(*a*)


(*c*)


(*e*)


(*b*)


(*d*)


(*f*)


Written Solutions

5. Using graph paper, construct angles in standard position so that each terminal ray passes through the given point.
(*a*) (2, 6) (*c*) (1, −3) (*e*) (−4, 2) (*g*) (−5, −2)
(*b*) (0, 1) (*d*) (−1, 0) (*f*) (0, −1) (*h*) (−2, 1)
Measure each angle using your protractor.

6. Use a protractor to construct the following angles.
(*a*) 100° (*c*) 165° (*e*) 290° (*g*) 190°
(*b*) 95° (*d*) 210° (*f*) 340° (*h*) 200°

7. Without using a protractor find the measure of an angle in standard position whose terminal ray passes through:
(*a*) (1, 1), (*b*) (−1, 1), (*c*) (−1, −1), (*d*) (1, −1)

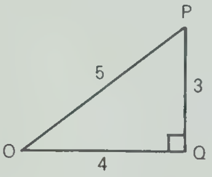
8. (*a*) Using graph paper construct an angle in standard position whose terminal ray passes through A (3, 4).
(*b*) On the same diagram construct an angle whose terminal ray passes through B (6, 8). What do you notice?
(*c*) Drop a perpendicular from A to meet the *x* axis at M, and a perpendicular from B to meet the *x* axis at N. Compare △OAM and △OBN.

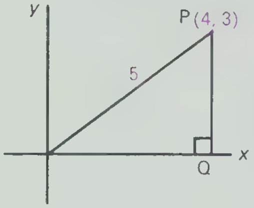
Remember similar triangles!

5(a) 72° **(e) 153°**
(c) 288° **(g) 202°**
7(a) 45° **(c) 225°**

19.2 Redefining the Basic Trig Values

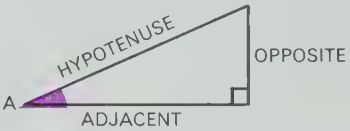
A Consider the 2 diagrams shown in the margin. Notice that the only difference between the 2 situations is that, in the second case, $\triangle POQ$ has been placed with vertex O at the origin.



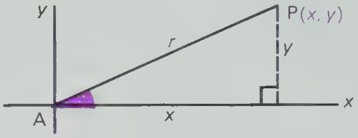


This means that we can redefine the basic trig values in terms of the coordinates (x and y) of a point P on the terminal arm of a standard angle and the distance r of P from the origin.

90° Triangle Definition



Coordinate Definition



$$\sin A = \frac{\text{OPPOSITE}}{\text{HYPOTENUSE}} \longleftrightarrow \sin A = \frac{y}{r}$$

$$\cos A = \frac{\text{ADJACENT}}{\text{HYPOTENUSE}} \longleftrightarrow \cos A = \frac{x}{r}$$

$$\tan A = \frac{\text{OPPOSITE}}{\text{ADJACENT}} \longleftrightarrow \tan A = \frac{y}{x}$$

r is always positive, since it is the distance from the origin to P(x,y).

The coordinate definition allows us to find trig values for angles greater than 90°, something that was not possible with the triangle definition.

Example 1

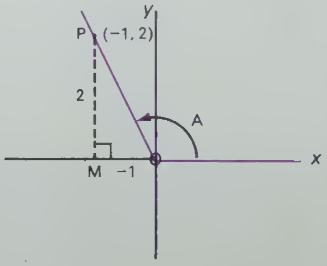
Find the sine, cosine, and tangent of the angle whose terminal ray passes through P(−1, 2).

Solution

Let A be the angle associated with the point P.

In $\triangle OMP$, $OP^2 = OM^2 + MP^2$

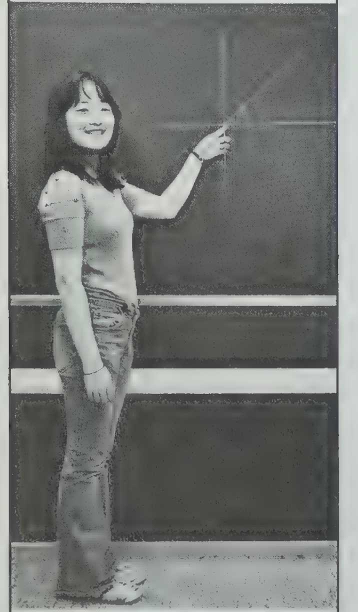
$$\begin{aligned} \text{or}\quad r^2 &= (-1)^2 + 2^2 \\ &= 1 + 4 = 5 \\ \therefore\quad r &= \sqrt{5} \end{aligned}$$



$$\sin A = \frac{y}{r} = \frac{2}{\sqrt{5}}$$

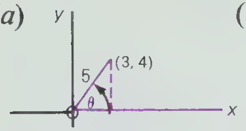
$$\cos A = \frac{x}{r} = \frac{-1}{\sqrt{5}}$$

$$\text{and}\quad \tan A = \frac{y}{x} = \frac{2}{-1} = -2$$

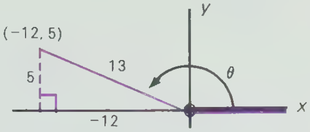


1. State the values of x, y, r, $\sin\theta$, $\cos\theta$, and $\tan\theta$ in each case.

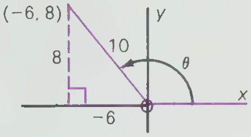
(*a*)



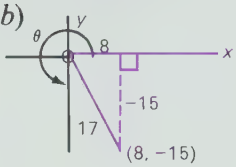
(*c*)


(−12, 5)
y
13
5
θ
x
−12

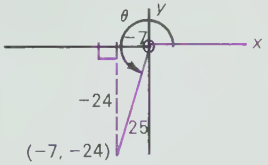
(*e*)



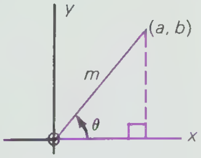
(*b*)


θ
y
8
x
−15
17
(8, −15)

(*d*)


θ
y
−7
x
−24
25
(−7, −24)

(*f*)



B The coordinate definition may also be used to determine trig values of angles whose terminal arms lie along the coordinate axes.

Example 2

Find the sine, cosine, and tangent of (*a*) 0°, (*b*) 270°.

Solution

Choose any point on the terminal arm of the angle.

(*a*) Consider the point (1, 0).

$$\therefore x = 1, y = 0, r = 1$$

$$\therefore \sin 0° = \frac{y}{r} = \frac{0}{1} = 0$$

$$\cos 0° = \frac{x}{r} = \frac{1}{1} = 1$$

$$\tan 0° = \frac{y}{x} = \frac{0}{1} = 0$$

(*b*) Consider the point (0, −3).

$$\therefore x = 0, y = -3, r = 3$$

$$\therefore \sin 270° = \frac{y}{r} = \frac{-3}{3} = -1$$

$$\cos 270° = \frac{x}{r} = \frac{0}{3} = 0$$

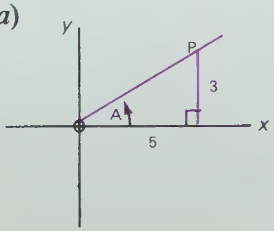
$$\tan 270° = \frac{y}{x} = \frac{-3}{0}$$

Exercise 19.2

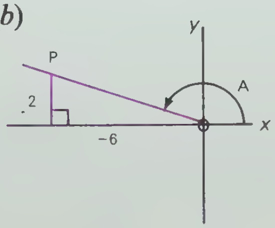
Discussion

1. State the coordinates of point P in each case.

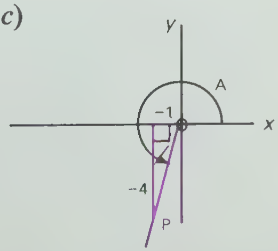
(*a*)



(*b*)

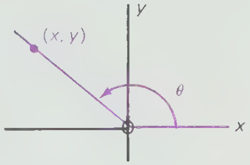


(*c*)



Remember!

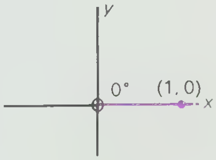
1) Standard angle

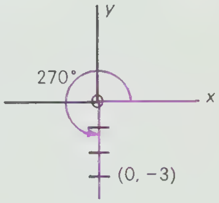


2) $\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$





Undefined;

∴ does not exist.

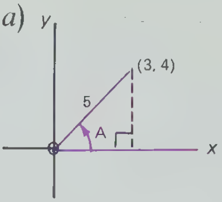
$$\sin\ \theta = \frac{y}{r}$$

$$\cos\ \theta = \frac{x}{r}$$

$$\tan\ \theta = \frac{y}{x}$$

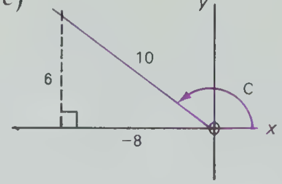
2. State the trig value indicated.

(a)



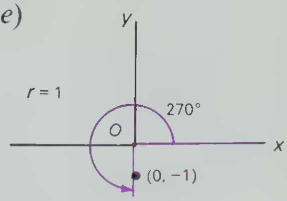
SIN A

(c)



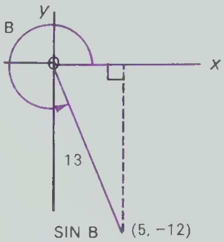
COS C

(e)



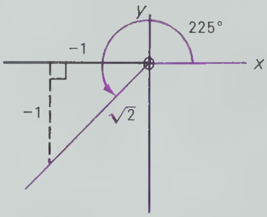
COS 270°

(b)



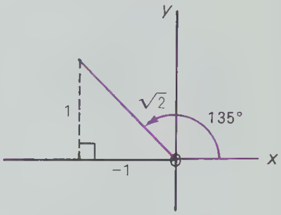
SIN B
TAN B

(d)



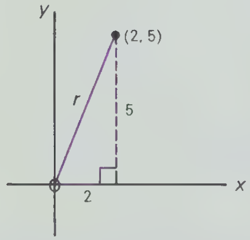
COS 225°
TAN 225°

(f)



SIN 135°
COS 135°

3.

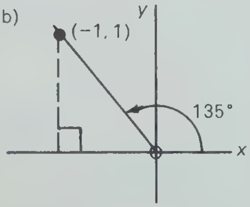


$x^2 + y^2 = r^2$

3. State the equation you would use to find r if the point on the terminal ray is as given.
(a) (2, 5) (b) (−3, 4) (c) (6, −8) (d) (−2, −3)

4. Select a point you might use on the terminal ray to find the trig values of each of the following angles.
(a) 0° (b) 45° (c) 90° (d) 135° (e) 180° (f) 225°

7(b)



Written Solutions

5. Find the distance from the origin to each of the following points.
(a) (6, 8) (c) (−8, 15) (e) (−5, −12) (g) (4, −3)
(b) (2, 3) (d) (−5, 5) (f) (2, −6) (h) $(1, \sqrt{3})$

6. Make a diagram of the angle whose terminal ray passes through each of the following points. Then find the sine, cosine and tangent of each angle.
(a) (1, 1) (b) (−3, 4) (c) (−2, −1) (d) (12, −5)

7. Use a diagram to find the sine, cosine and tangent of each of the following angles.
(a) 90° (b) 135° (c) 180° (d) 225° (e) 315° (f) 360°

5(a) 10 **(f) $2\sqrt{10}$**
(c) 17 **(h) 2**

6(a) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1$
(c) $-\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, \frac{1}{2}$

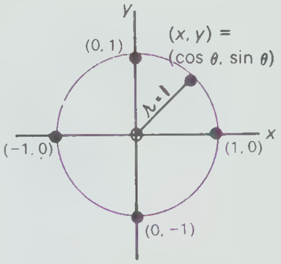
7(a) 1, 0, does not exist
(c) 0, −1, 0
(d) $-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 1$
(e) $-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -1$

8. If a circle has radius 1 unit, then the value of r for any point on the circle is 1.

Hence, $\sin\,\theta = \dfrac{y}{r} = \dfrac{y}{1} = y$

and $\cos\,\theta = \dfrac{x}{r} = \dfrac{x}{1} = x$

$\therefore$ The coordinates of any point on the circle will be $(x, y) = (\cos\,\theta, \sin\,\theta)$



(*a*) On a full sheet of graph paper, draw a circle whose radius is 10 units. Draw x and y axes with the center of the circle at the origin. Scale each axis so that 1 square on the graph paper = 0.1 units. The circle should meet the axes at the points (1, 0), (0, 1), (−1, 0), and (0, −1).

(*b*) At the origin, draw standard angles of 10°, 20°, 30°, . . . , 170°, 180°, using a protractor.

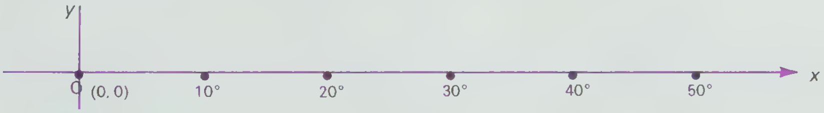
(*c*) Write the coordinates of each point where the rays of the angle meet the circle. Be accurate.

(*d*) Use these coordinates to make up a table of sines and cosines for angles 0°, 10°, 20°, . . . , 180°. Check your values against those in the trig table.

x	0°	10°	20°	30°	40°
sin x					
cos x					

9. Using the tables created above, draw the graphs of $y = \sin x$ and $y = \cos x$, for $x = 0°, 10°, \ldots, 180°$.

The axes would appear as follows.



10. (*a*) The table in the margin lists ordered pairs that satisfy the equation $y = \tan x$.

(*b*) Use a full sheet of graph paper to graph the relation given by $y = \tan x$. The x axis should be drawn at the center of the sheet and the y axis down the left side. Scale the x axis to provide for values of $x = 0°, 10°, 20°, \ldots, 90°$, as shown in the table. Use a scale for the y axis that is appropriate.

x	$y = \tan x$	x	$y = \tan x$
0°	0	50°	1.19
10°	0.18	60°	1.73
20°	0.36	70°	2.75
30°	0.58	80°	5.67
40°	0.84	90°	—

Maintaining Basic Skills

Exercise 4

1. Express in lowest terms.

(*a*) $\frac{15}{9}$ (*c*) $\frac{\$4.50}{\$3.00}$ (*e*) $\frac{28}{42}$

(*b*) $\frac{27}{12}$ (*d*) $\frac{3.2}{0.4}$ (*f*) $\frac{400}{250}$

2. Identify the equivalent ratios.

(*a*) 7:5, 9:6 (*b*) $\frac{8}{6}$, $\frac{28}{21}$ (*c*) 12:16, 3:4 (*d*) $\frac{1.4}{0.6}$, $\frac{35}{15}$

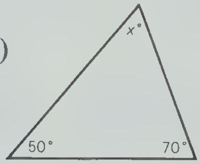
3. Find the missing proportional in each case.

(*a*) $\frac{9}{12} = \frac{m}{20} = \frac{n}{36}$ (*b*) $\frac{1.8}{9} = \frac{15}{p} = \frac{q}{5}$

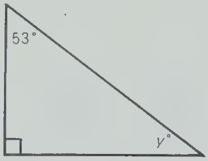
4. Determine the mean proportional of 72 and 2.

5. Determine the value of each variable.

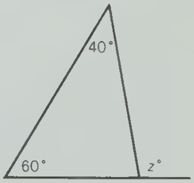
(*a*)



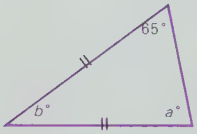
(*c*)



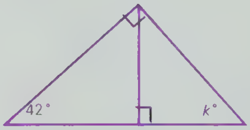
(*e*)



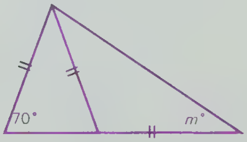
(*b*)



(*d*)

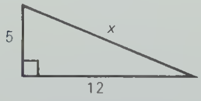


(*f*)

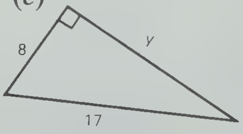


6. Find the length of the third side in each triangle.

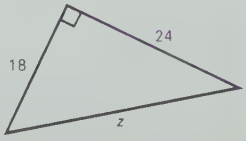
(*a*)



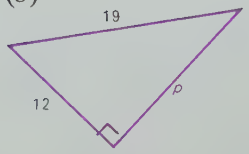
(*c*)



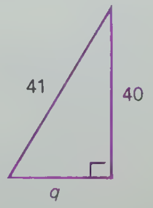
(*e*)



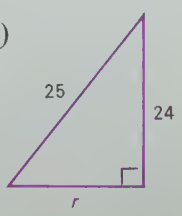
(*b*)



(*d*)



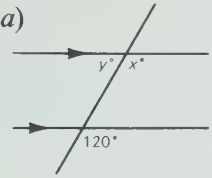
(*f*)



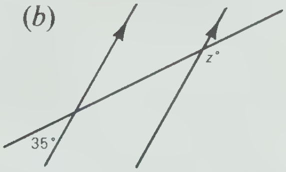
1(a) $\frac{5}{3}$ (d) $\frac{8}{1}$
3(a) $m = 15, n = 27$
5(a) $x = 60$
(b) $a = 65, b = 50$
6(a) $x = 13$ (d) $q = 9$

7. Determine the value of each variable.

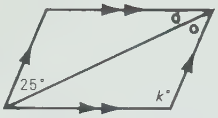
(*a*)



(*b*)

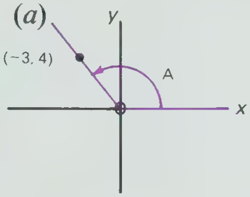


(*c*)

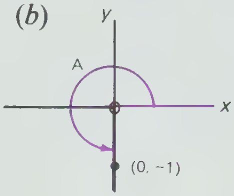


8. State the sine, cosine, and tangent of each angle A.

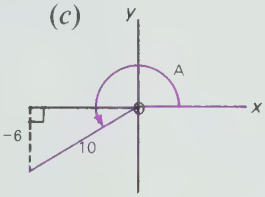
(*a*)



(*b*)

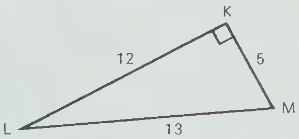


(*c*)

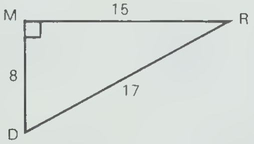


9. State the sine, cosine, and tangent of each acute angle.

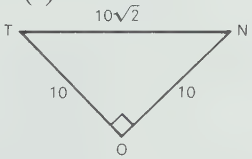
(*a*)



(*b*)



(*c*)


T
10√2
N
10
10
O

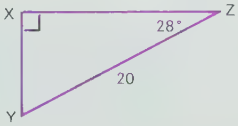
10. Using the trig table, find
(*a*) sin 32° (*b*) cos 68° (*c*) tan 73°

11. Find the angles.

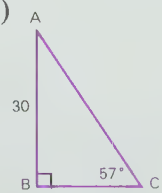
(*a*) cos A = 0.139 (*c*) tan C = 3.795
(*b*) sin B = 0.238 (*d*) cos D = 0.853

12. Find the length of the unknown sides.

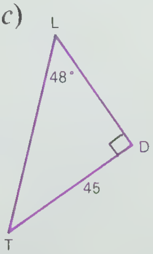
(*a*)



(*b*)

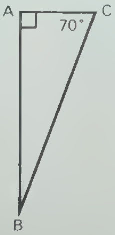


(*c*)

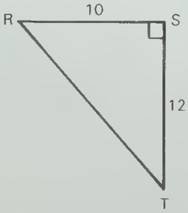


13. Find the unknown acute angles.

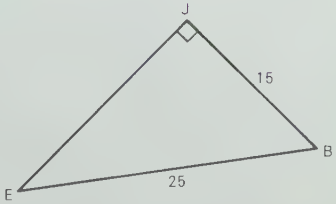
(*a*)



(*b*)



(*c*)



7(a) $x = 120, y = 60$
8(a) $\frac{4}{5}, -\frac{3}{5}, -\frac{4}{3}$
9(a) $\sin L = \frac{5}{13}$ $\sin M = \frac{12}{13}$
$\cos L = \frac{12}{13}$ $\cos M = \frac{5}{13}$
$\tan L = \frac{5}{12}$ $\tan M = \frac{12}{5}$
10(a) 0.530
11(a) A = 82° (c) C = 75°
12(a) XZ = 17.7, XY = 9.38
13(b) ∠R = 50°, ∠T = 40°

Summary of Units 9 to 19

	Do you understand . . . ?	*Can you . . . ?*
Unit 9 **Relations and Their Graphs**		
9.1 Graphing Ordered Pairs	ordered pairs, components quadrants I, II, III, IV origin, coordinates of a point	graph ordered pairs
9.2 Graphing a Relation	a relation shown by a statement a formula a set of ordered pairs a table of values its graph	list ordered pairs for a relation and draw its graph Express the statement of a relation using a formula
9.3 The Possible Values of a Relation	the difference between graphing a relation using integers and using real numbers continuous and discrete values	draw the graph of a relation involving real numbers
Unit 10 **Graphing Lines and Curves**		
10.1 Graphing Straight Lines	linear equation x- and y-intercepts of an equation	graph a line using its intercepts
10.2 Graphing Lines Using Slope	rise, run slope of a line positive and negative slope	find the slope and y-intercept of a line, given its equation use slope to help graph a line
10.3 Graphing the Circle, Ellipse, and Parabola	first- and second-degree equations the difference between linear and quadratic equations and their graphs ellipse major axis, minor axis parabola	tell when an equation will have a curved graph identify a circle, ellipse, or parabola from its equation graph a circle, ellipse, or parabola, given its equation find the intersection of 2 curves graphically

	Do you understand . . . ?	*Can you . . . ?*
Unit 11 **Graphing Regions**		
11.1 Graphing Simple Regions	inequation solid or dotted boundary of a region	draw the graph of an equation like $x = 2$ or $y = -3$ tell when the graph should be a region
11.2 Graphing Linear Inequations	test point for graphing a region	test whether an ordered pair satisfies an equation or inequation write the equation of the boundary of a region, given the inequation of the region graph a linear inequation
Unit 12 **Quadratic and Simultaneous Equations**		
12.1 Solving Quadratic Equations	quadratic equation the quadratic formula	find values of a, b, and c from a given quadratic equation solve a given quadratic equation by factoring, or by formula
12.2 Solving Equations Graphically	simultaneous equations solving equations graphically	form a table of values for an equation solve a pair of equations graphically check the solution of 2 simultaneous equations
12.3 Solving Equations by Elimination	solving equations algebraically eliminating a variable	add or subtract to eliminate one variable check a solution by substitution

Unit 13
Word Problems

	Do you understand . . . ?	*Can you . . . ?*
13.1 Number Problems	why you must check a word problem in the original statement, *not* in your equation	list the 5 steps in an algebraic solution convert a verbal statement into a mathematical expression
13.2 Measurement Problems		recognize a measurement-type word problem
13.3 Age and Mass Problems	an age or mass line	solve age and mass problems
13.4 Number-Value Problems	the difference between number and worth, or value	set up a number-value "chart" and solve number-value problems check a number-value problem

Unit 14
Ratio, Proportion, and Variation

14.1 Ratio and Proportion	ratio, proportion equivalent ratios first, second, third, fourth, and mean proportionals	test for equivalent ratios find any particular proportional, given the others
14.2 Direct Variation	direct variation	state 2 conditions for direct variation give an example of direct variation
14.3 Inverse Variation	inverse variation	state 2 conditions for inverse variation give an example of inverse variation
14.4 Partial Variation	partial variation	identify the 2 parts involved in partial variation give an example of partial variation solve problems involving the 3 types of variation

	Do you understand . . . ?	*Can you . . . ?*
Unit 15 **Basic Ideas of Geometry**		
15.1 Triangles	why a triangle is a rigid figure	state facts about the angle sum of a triangle an isosceles triangle an equilateral triangle
15.2 The Right-Angled Triangle	hypotenuse Law of Pythagoras	find one side of a right-angled triangle, given the other sides
15.3 Parallel Lines and Parallelograms	parallel lines parallelogram	name pairs of alternate and corresponding angles when a transversal cuts across 2 parallel lines state facts about the angles of a parallelogram the sides of a parallelogram
Unit 16 **The Circle**		
16.1 Chord Property of a Circle	radius, chord, diameter, center, sector, segment, semicircle, arc, tangent perpendicular bisector of a line segment circumscribed circle, inscribed triangle, circumcenter	draw the perpendicular bisector of a given line draw the circumscribed circle of a given triangle locate the center of a given circle
16.2 Angle Properties of a Circle	inscribed angle subtended angle sector angle inscribed quadrilateral	distinguish between an inscribed angle and a sector angle inscribe a quadrilateral in a circle state a relation among the angles of an inscribed quadrilateral construct a regular hexagon
16.3 Tangents	tangent to a circle	construct a tangent to a circle from a point outside the circle construct the inscribed circle of a given triangle
Unit 17 **Constructing Solids**		
17.1 Cylinder and Prisms	cylinder prisms—square, rectangular, and triangular	construct a cylinder, and square, rectangular, and triangular prisms
17.2 Pyramids	pyramids—square and triangular	construct square and triangular pyramids
17.3 Cones	cone sector of a circle	construct a cone
17.4 Polyhedra	regular polyhedron semi-regular polyhedron space filling	construct the 5 Platonic solids

	Do you understand. . . ?	*Can you. . . ?*
Unit 18 **Introduction to Trigonometry**		
18.1 The Basic Trigonometric Values	using a 90° triangle to define sine, cosine, and tangent of an acute angle SOHCAHTOA	find values for sine, cosine, and tangent of an acute angle, given at least 2 sides of a right-angled triangle
18.2 Trig Values of Any Angle	that the trig values of an angle do not depend on the length of the arms of the angle	state the trig values for 30°, 45°, and 60° find the trig values for any acute angle, either by construction or from the table find an angle, given one of its trig values
18.3 Applications of Trigonometry	the steps in "solving" triangles	use the trig table to "solve" triangles solve word problems involving right-angled triangles
Unit 19 **Coordinate Trigonometry**		
19.1 Angles in Standard Position	amount of rotation initial ray, terminal ray standard position of an angle	identify a standard angle draw a standard angle, given a point for its terminal arm
19.2 Redefining the Basic Trig Values	the coordinate definition of the sine, cosine, and tangent of any angle	find the trig values of any standard angle, given the coordinates of a point on its terminal arm find the trig values of a standard angle whose terminal arm falls along one of the coordinate axes

Review Exercise, Units 9 to 19

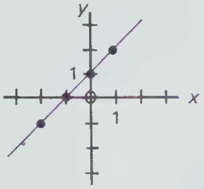
1. Name 5 ways used to express a relation.

2. Express each of the following relations as a set of at least 4 ordered pairs.

(*a*) $\{(x, y) \mid y = 2x + 1, \quad x, y \in R\}$

(*b*) The sum of the components of the ordered pairs of a relation is 6.

(*c*)



(*d*)

d	50	150	200	500
t	1	3	4	10

3. Draw the graph of each of the following relations.

(*a*) $\{(x, y) \mid y = 2x - 3, \quad x, y \in R\}$

(*b*) The components of the ordered pairs differ by 3.

(*c*)

x	5	2	-1	-4
y	3	-1	-5	-9

4. Find, by graphing on the same set of axes, the points of intersection of the graphs of each of the following pairs of equations.

(*a*) $x + 2y = 3$
$3x - y = -5$

(*b*) $2x + 5y = 3$
$5x - y = 21$

5. Construct the graphs of each of the following equations.

(*a*) $y = -x$

(*b*) $x^2 + y^2 = 36$

(*c*) $9x^2 + 4y^2 = 36$

(*d*) $y = x^2 + 1$

6. Find where each of the following graphs crosses the x axis and the y axis.

(*a*) $2x - 3y = 6$

(*b*) $y = x + 1$

(*c*) $y = x^2 - 9$

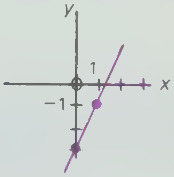
(*d*) $x^2 + y^2 = 25$

7. State the equation of the boundary of the region that is the graph of each of the following inequations.

(*a*) $x \geq 3$ (*b*) $y \leq -4$ (*c*) $x + 3y > 1$

2(a) $\{(0, 1), (1, 3), (-2, -3), (4, 9), \ldots\}$
(c) $\{(-2, -1), (0, 1), (1, 2), (-1, 0), \ldots\}$

3(a)



4(a) $(-1, 2)$

6(a) $(3, 0), (0, -2)$
(c) $(3, 0), (-3, 0), (0, -9)$

7(a) $x = 3$

9(a) $y = -\frac{1}{3}x + \frac{5}{3}; -\frac{1}{3}, \frac{5}{3}$

10(a) (3, 1) (c) (1, −2)

11. 3 s

12(a) (5, −2) (c) (2, 3)

13(a) $x = 0$ or $x = -3$

(c) $x = \frac{-3 \pm \sqrt{17}}{2}$

14. 13 cm by 10 cm

15. 7, 10

8. Construct the graphs of the following inequations.

(*a*) $x < 5$ (*c*) $3x - y \leq 6$

(*b*) $y \geq -1$ (*d*) $-x + 2y > 4$

9. Express each equation in the form $y = mx + b$, and state the slope and the y-intercept.

(*a*) $x + 3y = 5$ (*c*) $2x + y = 7$

(*b*) $x - y = -3$ (*d*) $x - 3y = 7$

10. Solve graphically, and check by substitution.

(*a*) $x - y = 2$, $4x + y = 13$ (*b*) $x - y = -5$, $2x - y = -10$ (*c*) $2x + y = 0$, $3x + 2y = -1$

11. The height of water in 2 containers is given by the equations

$$t - h = 1 \text{ (container A)}$$
$$2t + h = 8 \text{ (container B)}$$

where h is the height (centimetres) after time t (seconds). Graphically, find the time at which the 2 containers contain the same height of water.

Try to solve (a)
in your head!

12. Solve for x and y.

(*a*) $x - y = 7$, $x + y = 3$ (*c*) $3x + y = 9$, $-3x + 4y = 6$

(*b*) $3x + y = 13$, $4x - 3y = -13$ (*d*) $5x = 4y + 1$, $8y = 3x - 2$

13. Solve by factoring, or by formula.

(*a*) $x^2 + 3x = 0$ (*c*) $x^2 + 3x - 2 = 0$

(*b*) $x^2 + 2x - 35 = 0$ (*d*) $2x^2 - 2x - 1 = 0$

14. The length of a rectangle is 3 cm greater than the width. Find the dimensions of the rectangle if its area is 130 cm².

15. One number is 3 larger than another. If the smaller number is subtracted from twice the larger, the result is 13. Find the numbers.

16. The sides of a right-angled triangle are 2 units and 9 units shorter than the hypotenuse. Find the dimensions of the triangle.

17. At a bake sale, Nick O'Teen bought 3 more tarts than cookies. If he doubled the number of tarts, he would have 20 more tarts than cookies. How many of each did Nick buy?

18. An iron-nickel alloy casting with mass 42 kg contains 4 kg more iron than nickel. What mass of each metal is there in the casting?

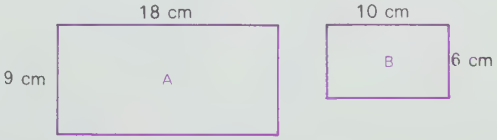
19. Dee Frost is 5 years older than her brother, Jack. Six years ago, Dee was twice as old as Jack. Find their present ages.

20. Douglas Firr's mass is 36 kg greater than his dog's. If Doug's mass were 2 kg more, his mass would be 3 times as much as his dog's. What is the mass of Doug's dog?

21. On her paper route, Reddi Cash collected $8.05 in quarters and dimes only. If the number of dimes exceeded the number of quarters by 21, how many dimes did Reddi collect?

22. Phil Atelist purchases some 10¢ and 25¢ postage stamps for $10.80. If there are 25 more 25¢ stamps than there are 10¢ stamps, how many of each kind did Phil purchase?

23. From the two rectangles shown, calculate the ratio of the
    (*a*) length
    (*b*) width
    (*c*) perimeter
    (*d*) area
    of rectangle A to rectangle B.



24. Find the fourth proportional to
    (*a*) 3, 7, and 12, (*b*) 5, 3, and 11.

25. Find the mean proportional of 36 and 4.

16. 8, 15, 17
17. 17 tarts, 14 cookies
18. 23 kg iron, 19 kg nickel
19. Dee, 16 years Jack, 11 years
20. 19 kg
21. 38 dimes
22. 13 10¢ stamps 38 25¢ stamps
23(a) 9 : 5 (c) 27 : 16
24(a) 28

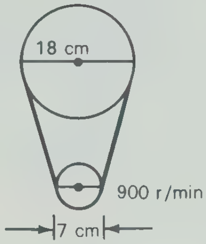
26. Determine the missing proportionals.

(*a*) $\frac{5}{4} = \frac{x}{8} = \frac{19}{y}$ (*b*) $\frac{m}{1.8} = \frac{5}{9} = \frac{n}{7.2}$

27. The surface area of a sphere varies directly as the square of its radius. If a sphere of radius 5 cm has a surface area of 314 cm^2, determine the surface area of a sphere of radius 15 cm.

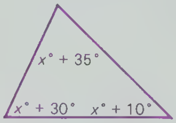
28. Determine the speed of the larger pulley in the diagram shown in the margin.

28.



29. A sales representative earns a salary of \$135/week and also receives a commission of 40% on sales. In one month, her weekly sales were \$315, \$290, \$345, and \$408. Determine her pay cheque (before deductions) each week.

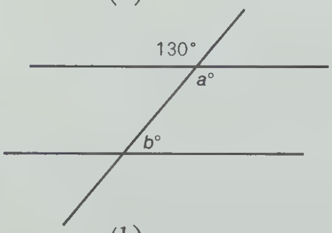
30. Find x, and the measure of each angle of the triangle.



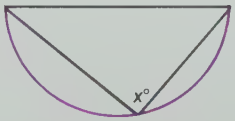
31. Draw the circumscribed circle of $\triangle$DEF with DE = 5 cm, EF = 8 cm, and DF = 6 cm.

32. Determine the value of each variable.

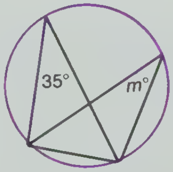
(*a*)



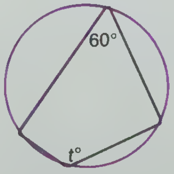
(*b*)



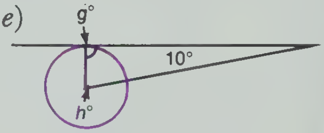
(*c*)



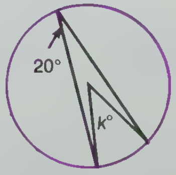
(*d*)



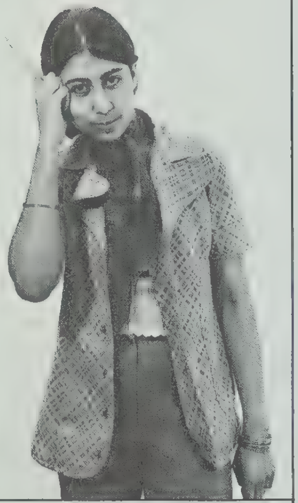
(*e*)



(*f*)



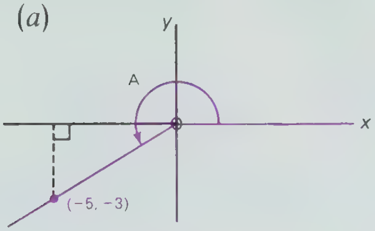
33. Draw a circle with radius 4 cm and center O. Locate a point P, 8 cm from O. Construct 2 tangents from P to the circle.



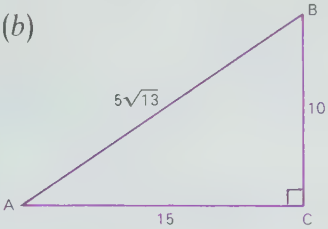
26(a) $x = 10, y = 15.2$
27. **2826 cm^2**
28. **350 r/min**
29. **1st week, \$261**
30. $x = 35$
32(a) $a = 130, b = 50$
(b) $x = 90$
(c) $m = 35$
(d) $t = 120$
(e) $g = 90, h = 80$
(f) $k = 40$

34. State the sine, cosine, and tangent of ∠A in each diagram.

(a)



(b)



35. Using a diagram for each, find
(a) sin 60° *(b)* cos 90° *(c)* tan 180° *(d)* sin 225°

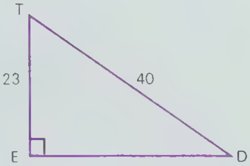
36. Evaluate the following expressions without using tables.

(a) $\sin 0° + \sin 90°$ *(c)* $\dfrac{\sin 0°}{\cos 0°}$

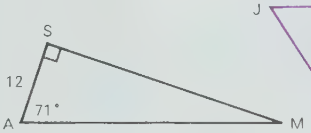
(b) $2 \sin 60° \cos 60°$ *(d)* $\sin^2 45° + \cos^2 45°$

37. Find the unknown sides and angles of the following triangles.

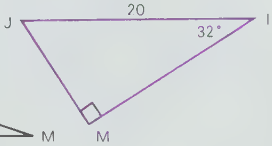
(a)



(b)



(c)



38. The shadow cast by a tree is 30 m long when the elevation of the sun is 60° above the horizon. Find the height of the tree, to the nearest metre.

39. A trench is to be constructed as shown. Find the angle of slope for the sides of the cut.

40. The angles of elevation of the top of Mount Oktoberfest from points on either side of it are 35° and 47°. If the top of the mountain is 1510 m above the level of the 2 points, find the length of a tunnel through the mountain between the 2 points (to the nearest metre).

41. Thelma Theodolite is at a window in her apartment building when she notices that the angle of elevation of the top of the building next door is 17° and the angle of depression of its base is 32°. If the buildings are 100 m apart, find the height of the building next door (to the nearest metre).

34(a) $\sin A = -\frac{3}{\sqrt{34}}$
(b) $\sin A = \frac{2}{\sqrt{13}}$

35(a) $\sin 60° = \frac{\sqrt{3}}{2}$
(d) $\sin 225° = -\frac{1}{\sqrt{2}}$

36(a) 1 (c) 0
(b) $\frac{\sqrt{3}}{2}$ (d) 1

37(a) ∠D = 35°
∠T = 55°
ED = 32.8

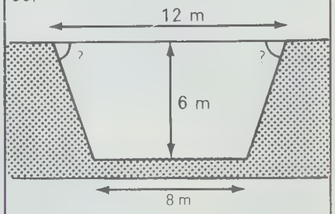
38. 52 m

39. 72°

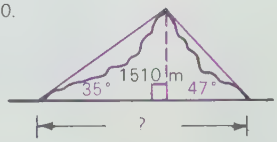
40. 3565

41. 93 m

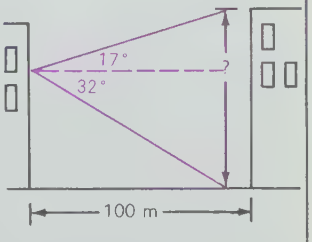
39.



40.



41.



20

Unit 20
Introduction to Vectors

Most of us have pushed a lawnmower, swung a baseball bat, walked into a brisk wind, or felt the pull of a water current. These experiences involve quantities such as force, velocity, and acceleration, known as *vector quantities.* Without these vector quantities, our technology could not have advanced to its present space-age level.

20.1 Experimenting with Forces

Great ! I go for experiments!

A For the following experiment, you will need

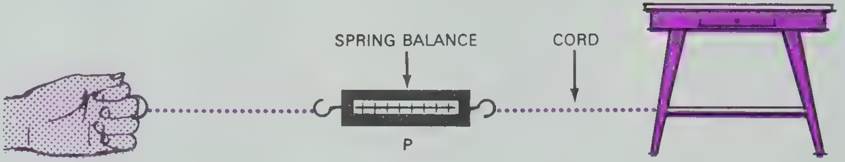
2 spring balances,
some stout cord, and
1 small metal ring.

Your spring balance should read up to at least 100 N.

N means "newtons"

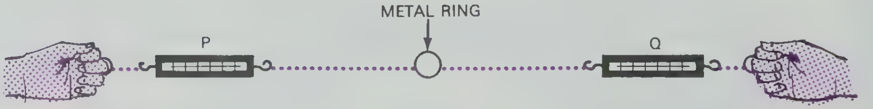
Experiment

1. Place the equipment on the floor or some other flat surface, as shown in the diagram.



2. Attach balance P to some fixed object (a table leg, for example) and pull on the balance. What happens to the reading on the balance as you pull harder? How is the direction of the force or pull related to the direction of the cord?

3. Arrange the 2 spring balances, P and Q, as shown.



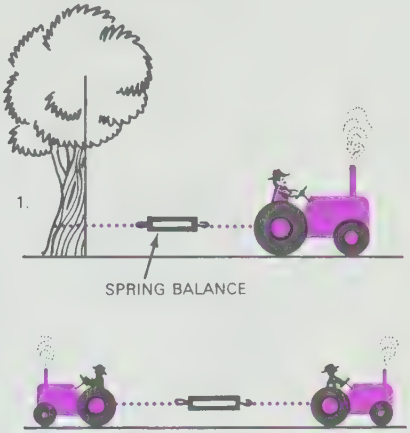
4. With balance P held steady in a fixed position, apply a force (pull) on balance Q. What do you notice about the readings on the 2 balances? Increase the pull on balance Q and again compare the 2 balance readings.

5. Apply a force to *both* balances and note the scale readings. Increase the forces and note the readings. Can you obtain appreciably different readings on the 2 scales?

Discussion

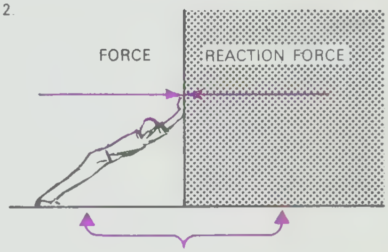
1. Imagine 3 identical tractors, each capable of exerting a pull of 10 000 N, set up as illustrated with spring balances used to record the forces involved. With the tractors exerting "full power," what will the reading be on each balance? Is there any difference between just holding, and pulling? Put yourself in the place of the spring balance: would you feel differently in each case?

1.



2. Larry Leaner pushes against a wall with a force of 400 N. Much to Larry's surprise, the wall does not move. The wall resists Larry with a force that is the same size as his push, but acting in the opposite direction. Consider the following situations involving a force and a reaction force.
   (*a*) Joyce and Patti step onto a table top to tile the ceiling of a room. If the table is unable to counteract their weight of 1300 N with an equal and opposite force, what will happen?
   (*b*) Snow Whyte tries to pull a snowmobile out of a snowdrift with a rope. Discuss the problems Snow might encounter and the conditions under which she might be successful in freeing the machine from the drift.

2.

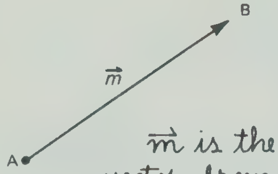


Equilibrium (no motion) exists under the action of these 2 opposing forces.

B In the preceding experiment, you were dealing with forces. (The reading on the spring balance gave you the size of the force.) A force is an example of a *vector quantity*.

> A *vector quantity* is a quantity involving both *magnitude* (size) and *direction*.

1. Explain why each of the following represents a vector quantity.
   (*a*) the flight of a bullet
   (*b*) a car travelling along the highway
   (*c*) a punch in the nose
   (*d*) a hammer blow
   (*e*) an operating punch-press



$\vec{m}$ is the vector from point A to point B.

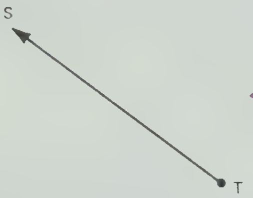
A directed line segment has length and direction and may be used to represent a vector quantity. It is called a *geometric vector* (or simply a *vector*).

The vector from point A to point B, shown in the margin, may be written 2 ways.

(*a*) $\overrightarrow{AB}$ (read "vector AB") where A is the *tail* of the vector and B is the *head* of the vector.

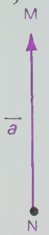
(*b*) $\vec{m}$ (read "vector *m*").

Notice that the use of 2 capital letters automatically indicates the direction of the vector. How would you name the vector shown here?

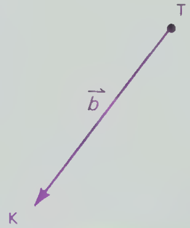


2. Name each of the vectors shown below in 2 ways.

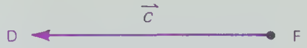
(*a*)



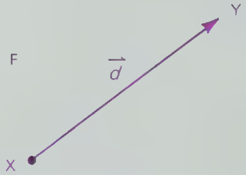
(*b*)



(*c*)



(*d*)

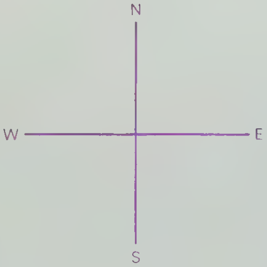


A *scalar quantity* is one involving *magnitude* only.

But not direction!

3. Determine the scalar quantities in each of the following expressions.
   (*a*) I watched TV for 3.5 h.
   (*b*) Bill is 16 years old.
   (*c*) There are 28 students in our class.
   (*d*) The width of the hem is 5 cm.

C Scale diagrams play an important part in working with vectors.



Example

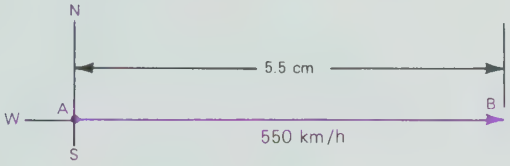
Use a scale diagram to represent

(*a*) the velocity of an aircraft flying due east at a speed of 550 km/h (Scale: 1 cm = 100 km/h),

(*b*) a force of 63 N to the right (Scale: 1 cm = 10 N).

Solution

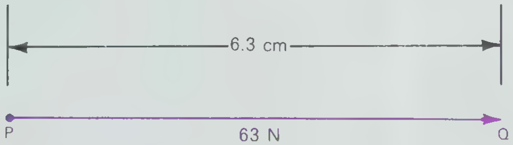
(*a*) If 100 km/h is represented by a length of 1 cm, then 550 km/h will be represented by a length of 5.5 cm.



Scale: 1 cm = 100 km/h

∴ $\overrightarrow{AB}$ represents the required velocity.

(*b*) If 10 N is represented by a length of 1 cm, then 63 N will be represented by a length of 6.3 cm.



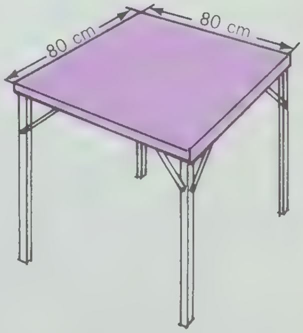
Scale: 1 cm = 10 N

∴ $\overrightarrow{PQ}$ represents the required force.

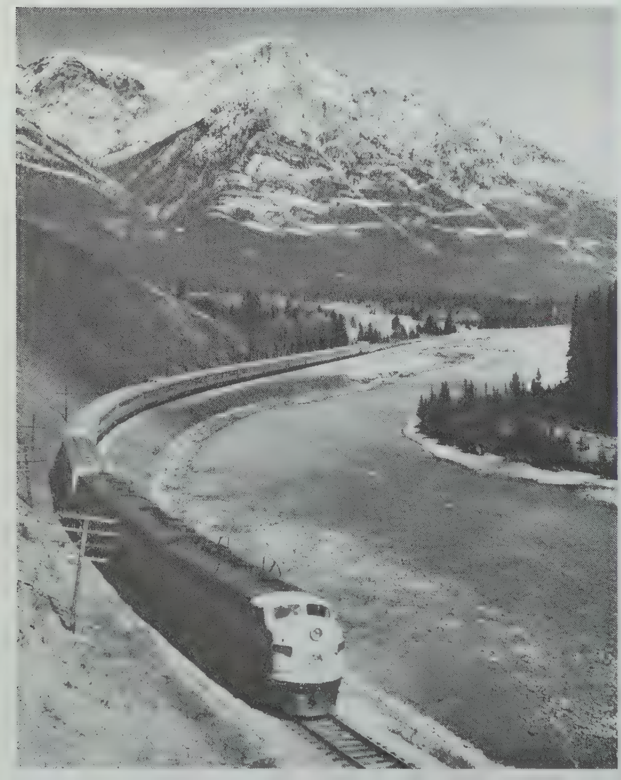
Exercise 20.1

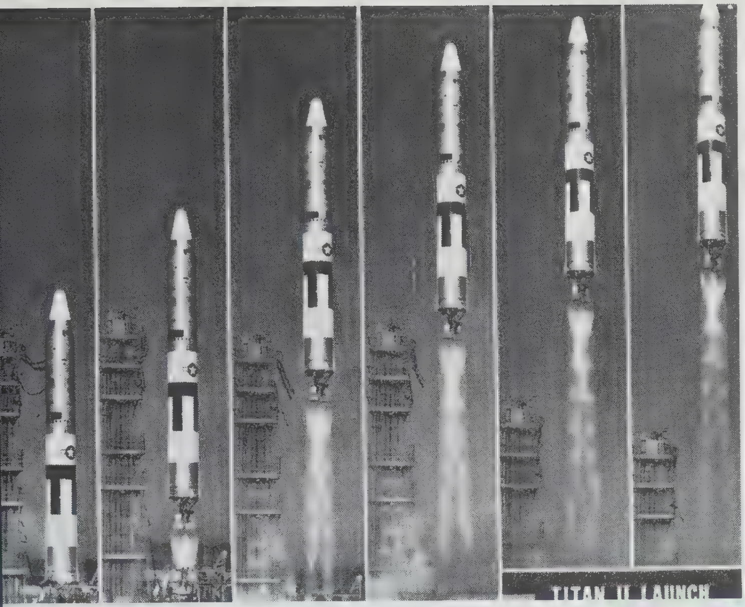
Discussion

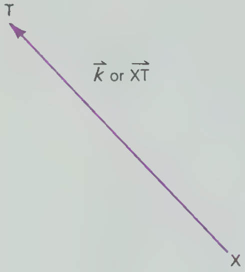
1. Distinguish between scalar and vector quantities.
2. How is each situation best represented (scalar or vector)?
   (*a*) a boy determining his mass on a scale
   (*b*) a tennis ball leaving a racket
   (*c*) a woman celebrating her 100th birthday
   (*d*) a card table 80 cm square
   (*e*) drilling a hole using a drill press
   (*f*) cutting a dress pattern from a piece of material with pair of scissors



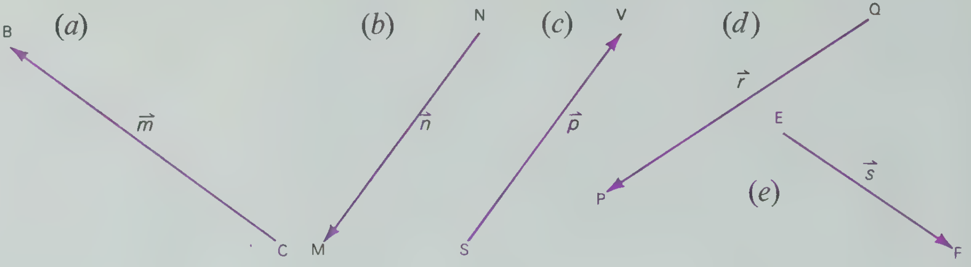
3. Name as many scalar and vector quantities as you can in each of the following pictures.




TITAN II LAUNCH

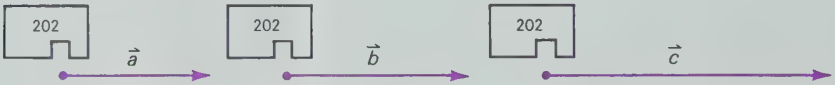

T
$\vec{k}$ or $\overrightarrow{XT}$
X

4. Name each vector in 2 ways.


(a)
B
$\vec{m}$
C
(b)
N
$\vec{n}$
M
(c)
V
$\vec{p}$
S
(d)
Q
$\vec{r}$
P
(e)
E
$\vec{s}$
F

5. Is the sentence $\overrightarrow{AB} = \overrightarrow{BA}$ true or false? Explain.

6. Each vector represents Joe's direction and speed as he leaves classroom 202.


202
$\vec{a}$
202
$\vec{b}$
202
$\vec{c}$

Decide which vector represents going to gym class, going to lunch, and going to math class.

7. State one scalar quantity and one vector quantity associated with each of the following sports.

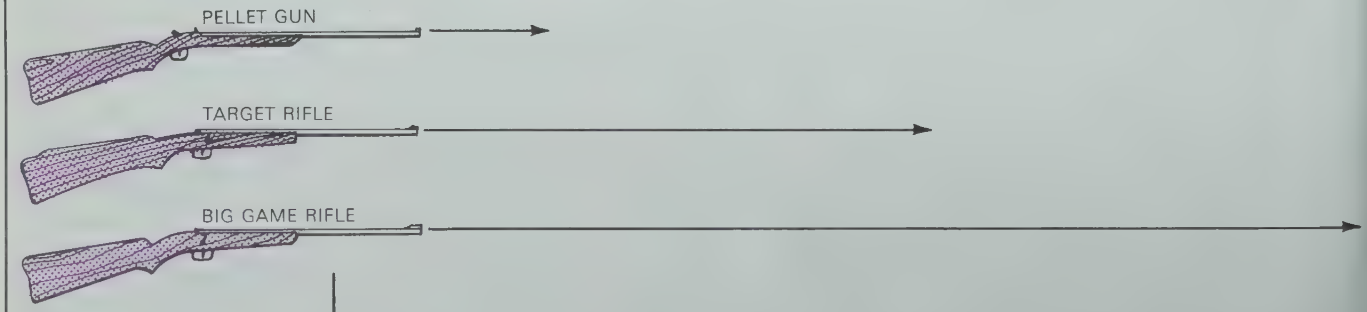
(*a*)	baseball	(*c*)	hockey	(*e*)	golf
(*b*)	football	(*d*)	volleyball	(*f*)	badminton

2 over par?
serve? home run?
score? kickoff?
drive? 3 out?

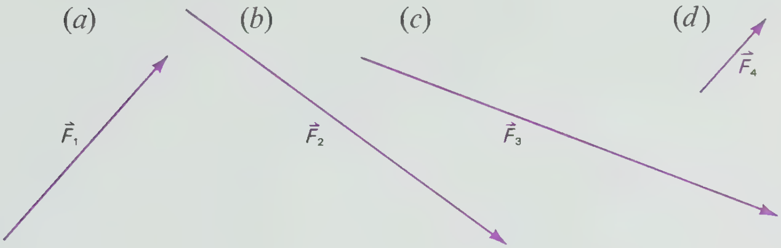
Written Solutions

You'll need a ruler and a sharp pencil here!

8. Using a scale of 1 cm = 60 m/s, determine the muzzle velocities represented by the given vectors for the 3 firearms shown.


PELLET GUN
TARGET RIFLE
BIG GAME RIFLE

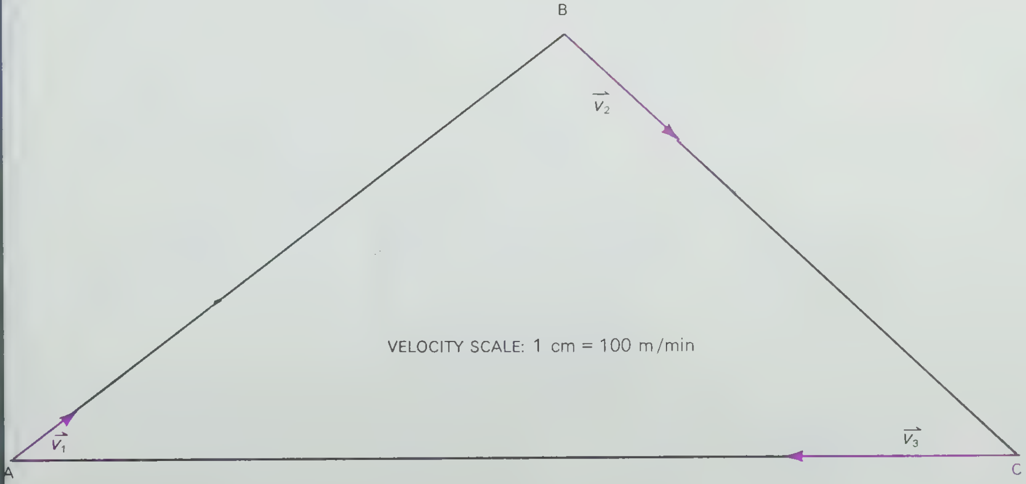
9. Using a scale of 1 cm = 20 N, determine the magnitude of each force shown.


(a)
(b)
(c)
(d)
$\vec{F}_1$
$\vec{F}_2$
$\vec{F}_3$
$\vec{F}_4$

10. Use a suitable scale to draw vectors representing each of the following quantities.

(a) a speed of 500 km/h due north
(b) an acceleration of 12 m/s^2 straight up
(c) a wind out of the west at 40 km/h
(d) a force of 1200 N to the right

11. A sailboat travels a triangular course from A to B to C and back to A, as shown. The vectors $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$ represent the respective velocities for the three legs of the course. Using the given scale, find the time required to complete the course.

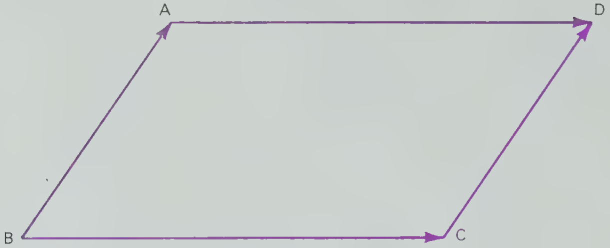

B
$\vec{V}_2$
VELOCITY SCALE: 1 cm = 100 m/min
$\vec{V}_1$
A
$\vec{V}_3$
C

9. Magnitude of
$\vec{F}_1 \doteq 60$ N
$\vec{F}_2 \doteq 116$ N
11. $17\frac{1}{3}$ min

m/s^2 means "metres per second per second"

20.2 Working with Parallel Vectors

A Consider parallelogram ABCD.

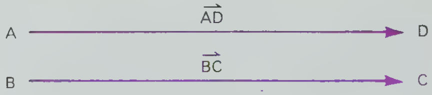


From the properties of a parallelogram, recall that opposite sides are of the same length and are parallel to each other. If we use side AD to represent $\overrightarrow{AD}$ and side BC to represent $\overrightarrow{BC}$, then we *must* agree that $\overrightarrow{AD}$ and $\overrightarrow{BC}$ have the same magnitude and direction. Such vectors are called *equal vectors*.

Equal vectors have the same magnitude and the same direction.

Can you identify another pair of equal vectors in ||gm ABCD?

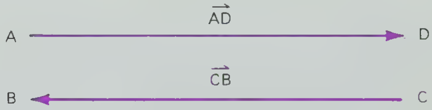
$$\overrightarrow{AD} = \overrightarrow{BC}$$



Notice also that $\overrightarrow{CB}$ has the same length as $\overrightarrow{AD}$, but is *opposite* in direction. We say that $\overrightarrow{CB}$ is the *opposite vector* of $\overrightarrow{AD}$, and we write this,

State the opposite of $\overrightarrow{AB}$

$$\overrightarrow{CB} = -\overrightarrow{AD} \quad \text{or} \quad \overrightarrow{AD} = -\overrightarrow{CB}$$



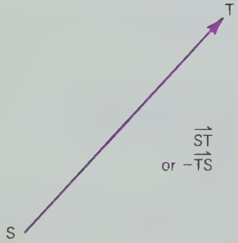
If we now compare the above two results, namely

$$\overrightarrow{AD} = \overrightarrow{BC}$$
$$\text{and} \quad \overrightarrow{AD} = -\overrightarrow{CB},$$

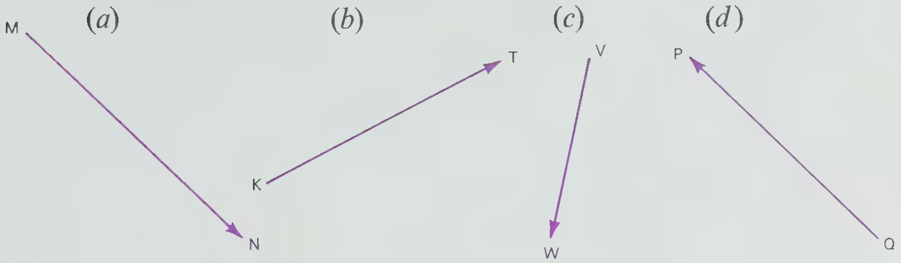
we see that

$$\boxed{\overrightarrow{BC} = -\overrightarrow{CB}}$$

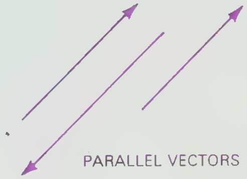
Thus, we can name any given vector in two ways using the end points. For example, the vector in the margin could be designated as either $\overrightarrow{ST}$ or $-\overrightarrow{TS}$



1. Name each of the following vectors in 2 ways, using the end point notation.

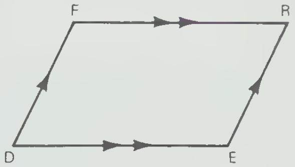

(a)
(b)
(c)
(d)
M
N
K
T
V
W
P
Q

B Vectors that are parallel are called *parallel vectors*. Such vectors may have either the same direction or opposite directions, but do not have to be the same length.

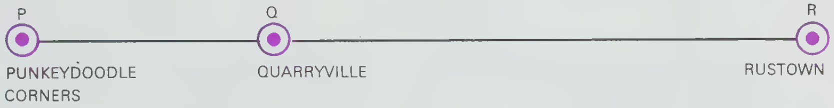


PARALLEL VECTORS

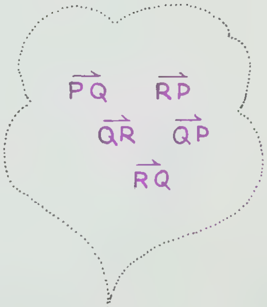
Referring to parallelogram FRED in the margin, $\overrightarrow{FR}$ and $\overrightarrow{RF}$, $\overrightarrow{DE}$ and $\overrightarrow{ED}$ are *parallel vectors*.



Consider the situation in which 3 towns, P, Q, and R lie along a straight stretch of highway, as shown.


P
Q
R
PUNKEYDOODLE CORNERS
QUARRYVILLE
RUSTOWN

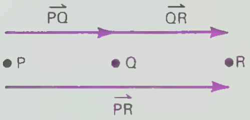
1. Name all the possible vectors that represent trips between the three towns.
2. What vector could represent the trip from
   (a) P to Q? (b) Q to R?
3. What single vector could represent the net result of the 2 trips in Question 2?


$\overrightarrow{PQ}$
$\overrightarrow{RP}$
$\overrightarrow{QR}$
$\overrightarrow{QP}$
$\overrightarrow{RQ}$

I think I missed one!

It seems reasonable to agree that a trip from P to Q followed by a trip from Q to R gives the same result as a single trip from P to R; that is,

$$\boxed{\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}}$$

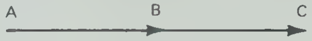

$\overrightarrow{PQ}$
$\overrightarrow{QR}$
P
Q
R
$\overrightarrow{PR}$

Here, vector $\overrightarrow{PR}$ is called the *resultant* vector of the parallel vectors $\overrightarrow{PQ}$ and $\overrightarrow{QR}$, and $\overrightarrow{PQ} + \overrightarrow{QR}$ is called a *vector sum*.

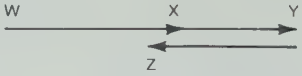
Example

State the vector sum and the resultant of the parallel vectors shown in each case.

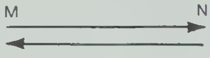
(*a*)



(*c*)

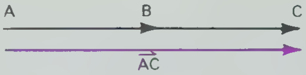


(*b*)



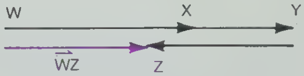
Solution

(*a*)



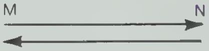
$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

(*c*)



$\overrightarrow{WX} + \overrightarrow{XY} + \overrightarrow{YZ} = \overrightarrow{WZ}$

(*b*)



$\overrightarrow{MN} + \overrightarrow{NM} = \vec{0}$

Note that we "tack on" vectors in the order tail-head-tail-head.

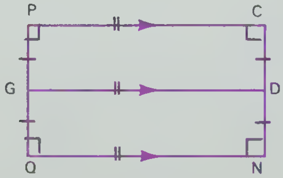
Notice the result in part (*c*) of the example. $\vec{0}$ is called the *zero vector*.

Exercise 20.2

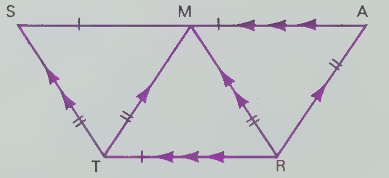
Discussion

1. On each of the diagrams below, locate and name directed segments that represent
   (*i*) equal vectors,
   (*ii*) opposite vectors,
   (*iii*) parallel vectors.

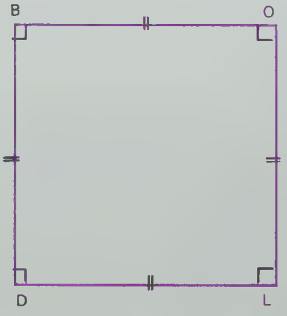
(*a*)



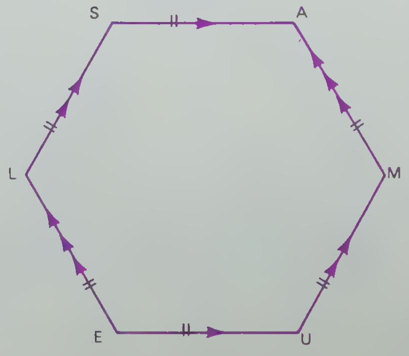
(*c*)



(*b*)



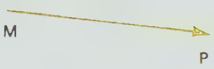
(*d*)



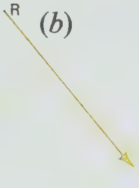
2. Are equal vectors also parallel vectors? Are parallel vectors also equal vectors? Explain.

3. Using the idea of opposite vectors, name each vector shown in 2 ways.

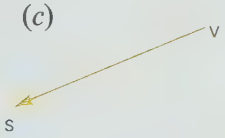
(a)



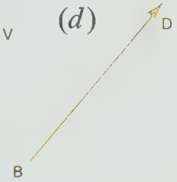
(b)



(c)



(d)



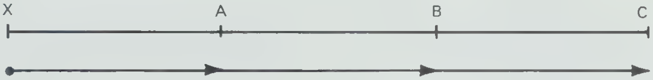
4. What is the resultant of the vector sum $\vec{a} + (-\vec{a})$?

5. What is the magnitude and direction of the zero vector $\vec{0}$?

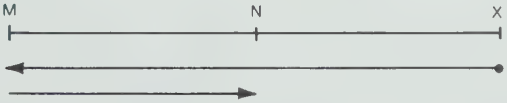
Written Solutions

6. In each of the following systems of parallel vectors,
   (i) copy the vector diagram and draw on it the resultant vector from the initial point X to the head of the final vector,
   (ii) express the resultant as a sum of the given vectors.

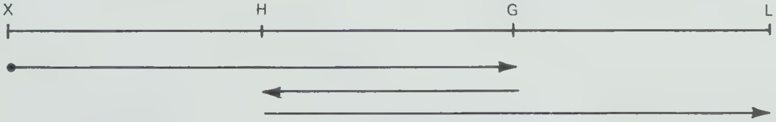
(a)



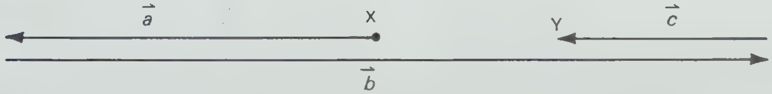
(b)



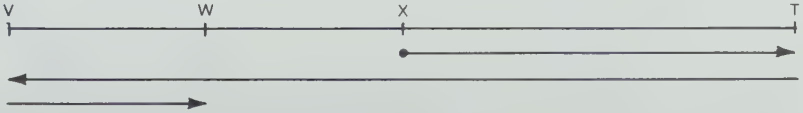
(c)



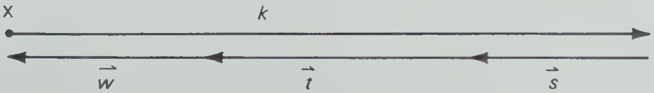
(d)


$\vec{a}$
X
Y
$\vec{c}$
$\vec{b}$

(e)



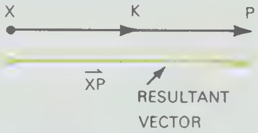
(f)


X
$\vec{k}$
$\vec{w}$
$\vec{t}$
$\vec{s}$

Remember:

$\overrightarrow{MP} = -\overrightarrow{PM}$

VECTOR DIAGRAM

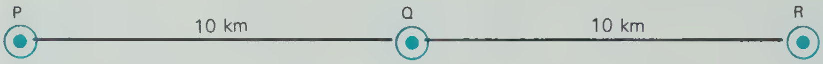

X
K
P
$\overrightarrow{XP}$
RESULTANT VECTOR

VECTOR EQUATION
$\therefore \overrightarrow{XK} + \overrightarrow{KP} = \overrightarrow{XP}$

6(a) $\overrightarrow{XA} + \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{XC}$
(c) $\overrightarrow{XG} + \overrightarrow{GH} + \overrightarrow{HL} = \overrightarrow{XL}$
(e) $\overrightarrow{XT} + \overrightarrow{TV} + \overrightarrow{VW} = \overrightarrow{XW}$

20.3 Multiplying Vectors by Scalars

Consider 3 towns P, Q, and R situated so that town Q is midway between towns P and R.



From work of the previous section,

$$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$$

Recall equal vectors.

But $\overrightarrow{PQ} = \overrightarrow{QR}$

$\therefore\ \overrightarrow{PQ} + \overrightarrow{PQ} = \overrightarrow{PR}$

or $2\,\overrightarrow{PQ} = \overrightarrow{PR}$

Here, $2\,\overrightarrow{PQ}$ is called a *scalar multiple* of $\overrightarrow{PQ}$.

$2\ \overrightarrow{PQ}$

SCALAR VECTOR

Notice that $2\,\overrightarrow{PQ}$ is still a vector quantity, with the *same* direction as $\overrightarrow{PQ}$ or $\overrightarrow{PR}$, but with a magnitude *twice* as great as $\overrightarrow{PQ}$.

Recall opposite vectors.

Moreover, since $\overrightarrow{PQ} = -\overrightarrow{QP}$

$\therefore\ 2\,\overrightarrow{PQ} = 2(-\overrightarrow{QP}) = -2\,\overrightarrow{QP}$

$\therefore\ -2\,\overrightarrow{QP} = \overrightarrow{PR}$

In this case, $\overrightarrow{QP}$ and $\overrightarrow{PR}$ have *opposite* directions.

Multiplication of a vector by a scalar results in a vector parallel to the original vector.

Example

Express each of the vectors shown as a scalar multiple of vector $\vec{a}$.

Solution

$\vec{b} = 3\vec{a}$

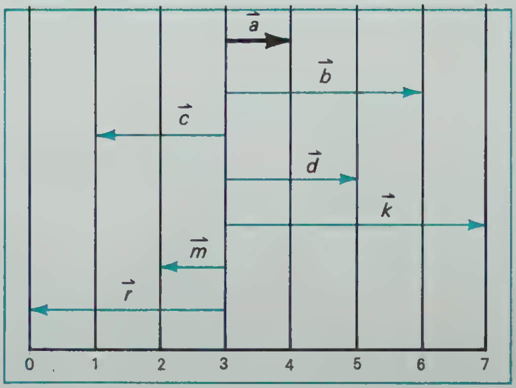
$\vec{c} = -2\vec{a}$

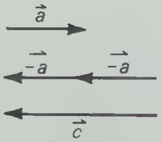
$\vec{d} = 2\vec{a}$

$\vec{k} = 4\vec{a}$

$\vec{m} = -\vec{a}$

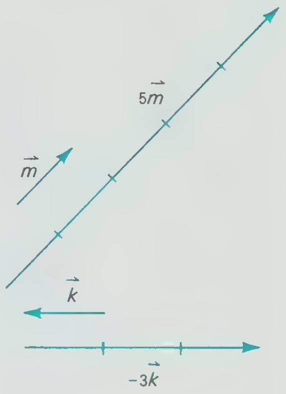
$\vec{r} = -3\vec{a}$





$\therefore\ \vec{c} = -2\vec{a}$

Exercise 20.3


$\vec{m}$
$5\vec{m}$
$\vec{k}$
$-3\vec{k}$

Discussion

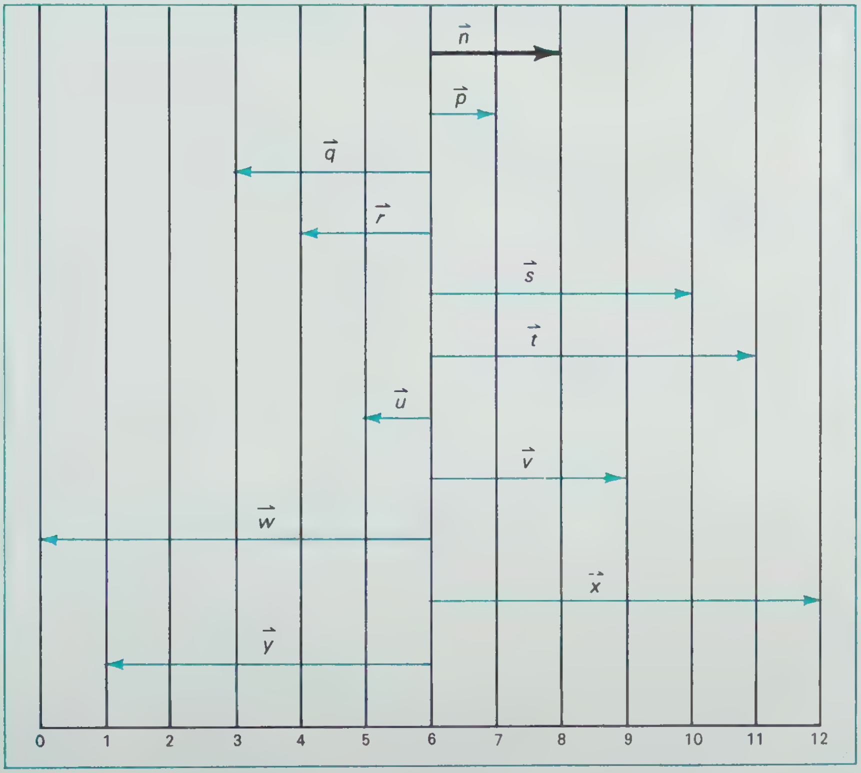
1. How is the vector $5\vec{m}$ related to the vector $\vec{m}$?
2. How is the vector $-3\vec{k}$ related to the vector $\vec{k}$?
3. How are each of the following vectors related to vector $\vec{r}$?

(*a*)	$2\vec{r}$	(*d*)	$4.5\,\vec{r}$
(*b*)	$-\vec{r}$	(*e*)	$1000\,\vec{r}$
(*c*)	$\frac{1}{2}\vec{r}$	(*f*)	$-92.63\,\vec{r}$

4. How is the vector $0\vec{t}$ related to the vector $\vec{t}$?

Written Solutions

5. Express each of the vectors shown as a scalar multiple of $\vec{n}$.

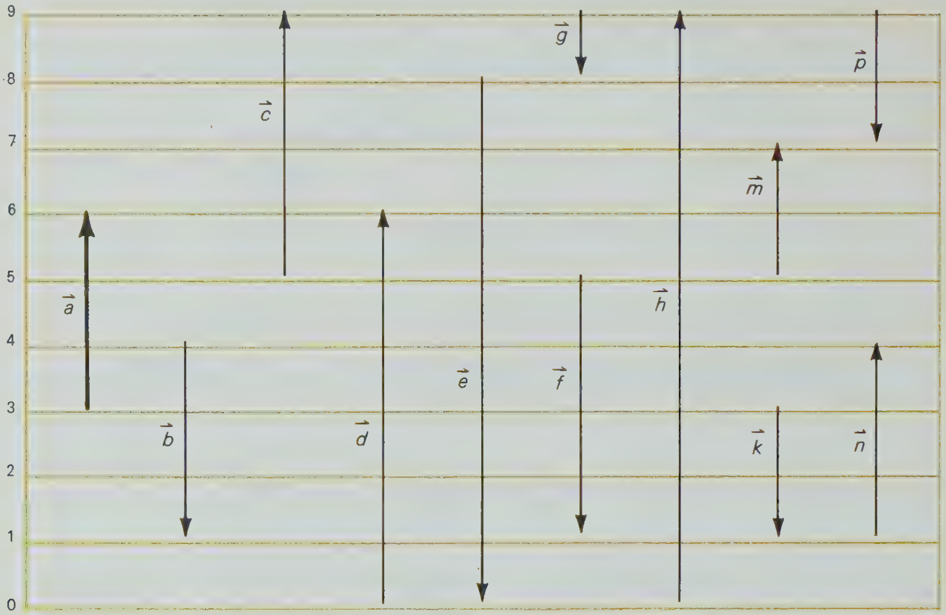

$\vec{n}$
$\vec{p}$
$\vec{q}$
$\vec{r}$
$\vec{s}$
$\vec{t}$
$\vec{u}$
$\vec{v}$
$\vec{w}$
$\vec{x}$
$\vec{y}$
0
1
2
3
4
5
6
7
8
9
10
11
12

5. $\vec{p} = \frac{1}{2}\vec{n}$ $\quad \vec{v} = 1\frac{1}{2}\vec{n}$

$\vec{r} = -\vec{n}$ $\quad \vec{x} = 3\vec{n}$

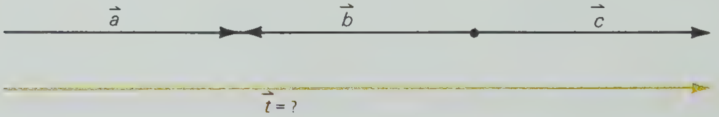
$\vec{t} = 2\frac{1}{2}\vec{n}$

6. Express each of the vectors shown as a scalar multiple of the vector $\vec{a}$.

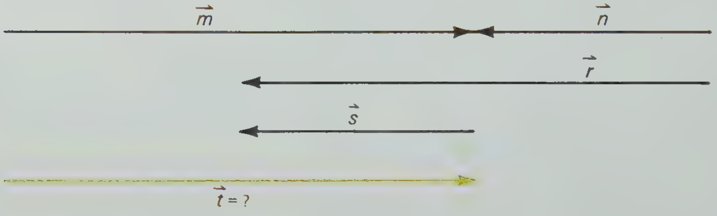

9
8
7
6
5
4
3
2
1
0
$\vec{a}$
$\vec{b}$
$\vec{c}$
$\vec{d}$
$\vec{e}$
$\vec{f}$
$\vec{g}$
$\vec{h}$
$\vec{k}$
$\vec{m}$
$\vec{n}$
$\vec{p}$

7. Express each resultant, $\vec{t}$, as a sum of the given vectors.

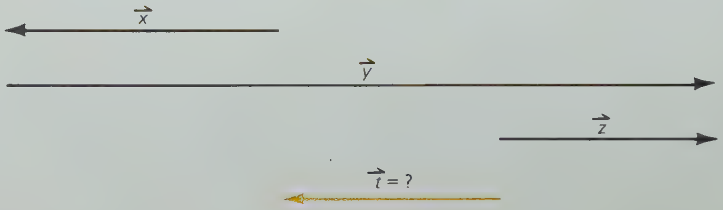
(*a*)


$\vec{a}$
$\vec{b}$
$\vec{c}$
$\vec{t} = ?$

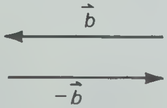
(*b*)


$\vec{m}$
$\vec{n}$
$\vec{r}$
$\vec{s}$
$\vec{t} = ?$

(*c*)


$\vec{x}$
$\vec{y}$
$\vec{z}$
$\vec{t} = ?$

Remember
opposite vectors.

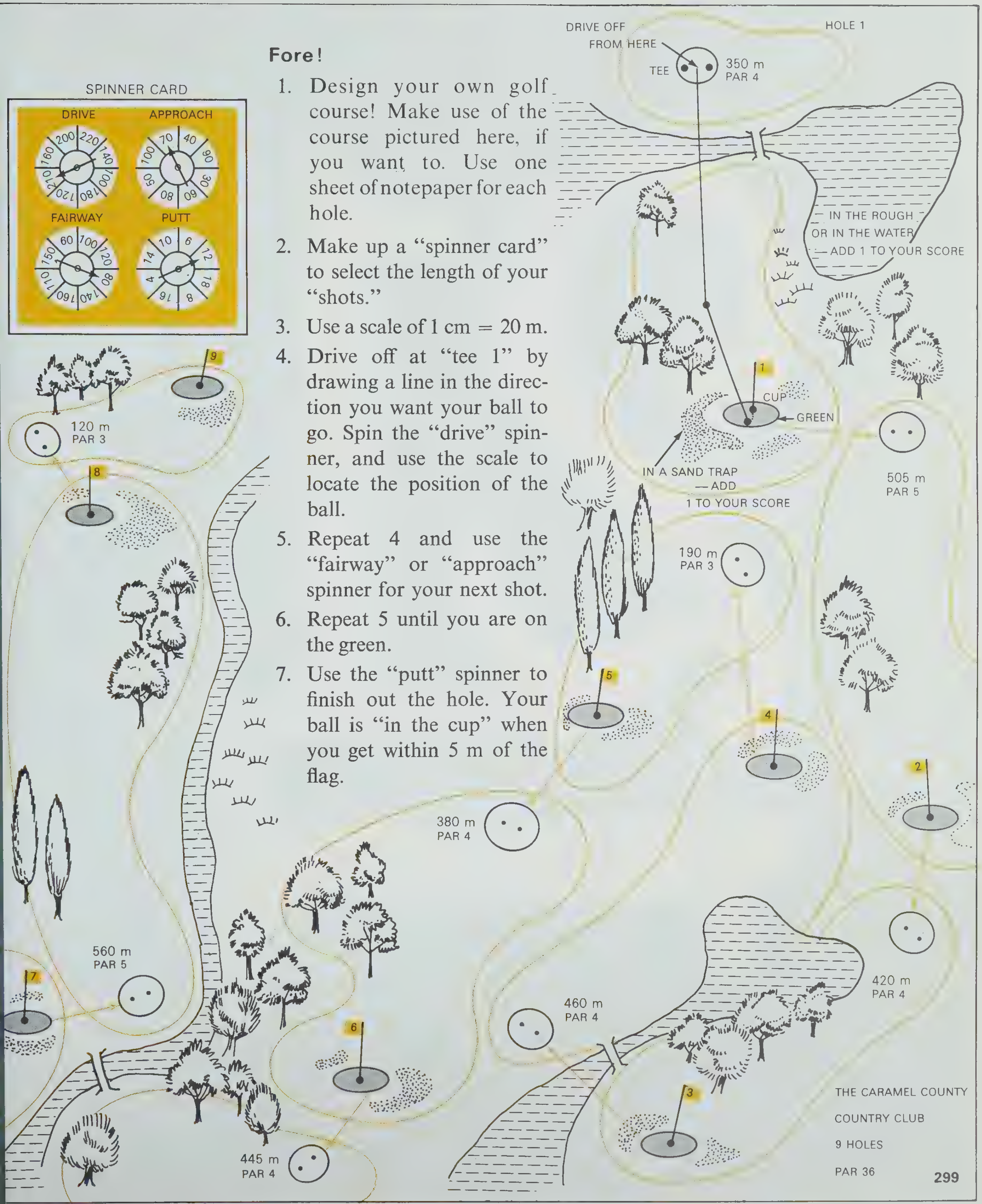


6. $\vec{b} = -\vec{a}$ $\quad\vec{h} = 3\vec{a}$
$\vec{d} = 2\vec{a}$ $\quad\vec{m} = \frac{2}{3}\vec{a}$
$\vec{f} = -1\frac{1}{3}\vec{a}$ $\quad\vec{p} = -\frac{2}{3}\vec{a}$

7(a) $\vec{t} = \vec{a} + (-\vec{b}) + \vec{c}$
(c) $\vec{t} = \vec{z} + (-\vec{y}) + (-\vec{x})$

Fore!

1. Design your own golf course! Make use of the course pictured here, if you want to. Use one sheet of notepaper for each hole.
2. Make up a "spinner card" to select the length of your "shots."
3. Use a scale of 1 cm = 20 m.
4. Drive off at "tee 1" by drawing a line in the direction you want your ball to go. Spin the "drive" spinner, and use the scale to locate the position of the ball.
5. Repeat 4 and use the "fairway" or "approach" spinner for your next shot.
6. Repeat 5 until you are on the green.
7. Use the "putt" spinner to finish out the hole. Your ball is "in the cup" when you get within 5 m of the flag.

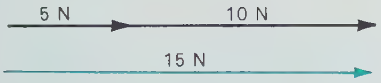

SPINNER CARD
DRIVE
200
220
140
100
140
180
120
170
160
APPROACH
70
40
90
30
60
80
50
100
FAIRWAY
60
100
120
80
140
160
110
150
PUTT
10
6
12
18
8
16
4
14
DRIVE OFF FROM HERE
TEE
HOLE 1
350 m PAR 4
IN THE ROUGH OR IN THE WATER — ADD 1 TO YOUR SCORE
1
CUP
GREEN
IN A SAND TRAP — ADD 1 TO YOUR SCORE
505 m PAR 5
2
190 m PAR 3
4
5
380 m PAR 4
420 m PAR 4
460 m PAR 4
3
6
445 m PAR 4
560 m PAR 5
7
8
9
120 m PAR 3
THE CARAMEL COUNTY COUNTRY CLUB
9 HOLES
PAR 36

21

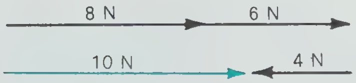
Unit 21
Adding Non-Parallel Vectors

21.1 Resultant of Two Vectors

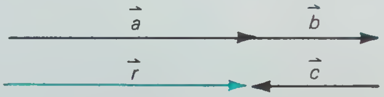
A Recall that to find the resultant of 2 or more parallel vectors, we "tacked on" one vector after another, and the resultant was the single vector joining the tail of the first vector to the head of the last vector.



∴ The resultant force is 15 N.



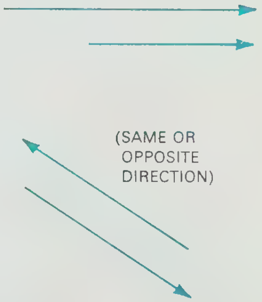
∴ The resultant force is 10 N.



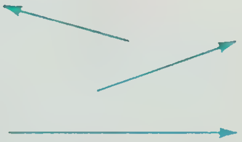
$\therefore \vec{r} = \vec{a} + \vec{b} + \vec{c}$

Will this same method work for finding the resultant of 2 non-parallel vectors?

PARALLEL VECTORS



(SAME OR OPPOSITE DIRECTION)



NON-PARALLEL VECTORS

Lay everything out flat on the floor!

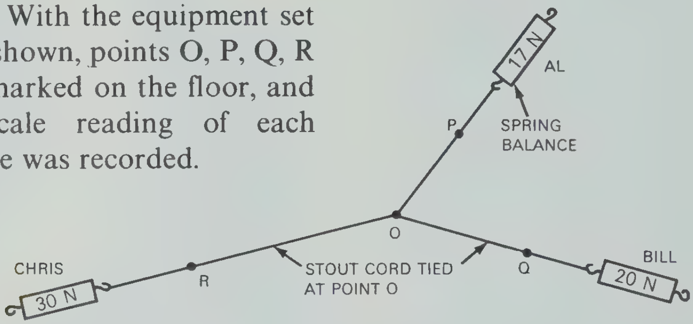
N means "newtons"

Experiment

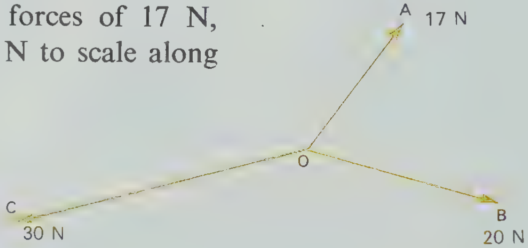
Using 3 spring balances and some stout cord, Al, Bill, and Chris did the experiment outlined below.

Method

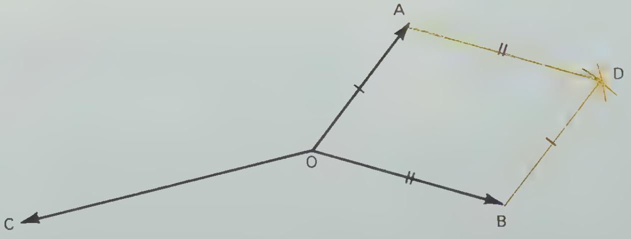
Step 1 With the equipment set up as shown, points O, P, Q, R were marked on the floor, and the scale reading of each balance was recorded.


17 N
AL
SPRING BALANCE
P
O
CHRIS
30 N
R
STOUT CORD TIED AT POINT O
Q
BILL
20 N

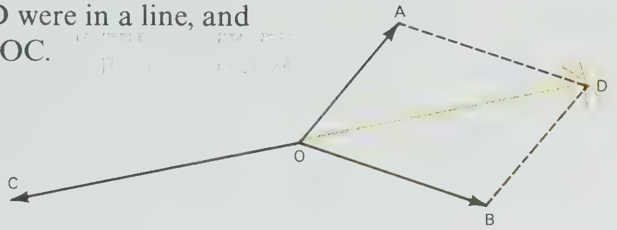
Step 2. The boys removed the equipment, drew lines from O through P, Q, and R, and represented the forces of 17 N, 20 N, and 30 N to scale along these lines.


A
17 N
O
C
30 N
B
20 N

Step 3 Parallelogram AOBD was drawn, as accurately as possible.



Step 4 The boys drew in the diagonal OD and found that C, O, and D were in a line, and that OD = OC.

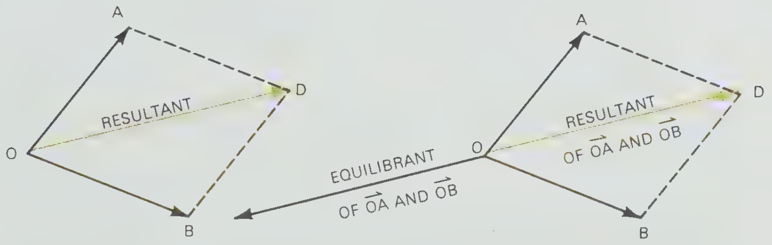


1. Try the experiment yourself, with 3 different forces, and see if you get the same results; namely,
   (*a*) D, O, and C are in a line, and
   (*b*) OD = OC.

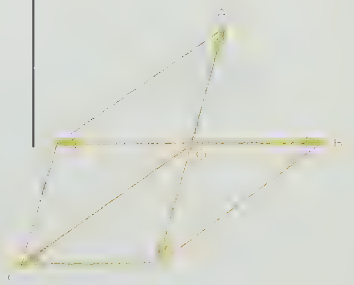
2. We can say that force $\overrightarrow{OC}$ balances the 2 forces $\overrightarrow{OA}$ and $\overrightarrow{OB}$. What forces does $\overrightarrow{OA}$ balance? What forces does $\overrightarrow{OB}$ balance? In this situation, we say that any 1 of the forces balances the other 2.

Why ?

   $\therefore$ $\overrightarrow{OC}$ balances $\overrightarrow{OA}$ and $\overrightarrow{OB}$.
   $\therefore$ $\overrightarrow{OC}$ balances the total effect of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.
   $\therefore$ $\overrightarrow{OC}$ balances the *resultant* of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.
   But, $\overrightarrow{OC}$ balances $\overrightarrow{OD}$!
   $\therefore$ $\overrightarrow{OD}$ is the *resultant* of $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

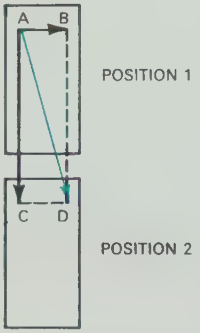

A
D
RESULTANT
O
B
A
D
RESULTANT
OF $\overrightarrow{OA}$ AND $\overrightarrow{OB}$
O
EQUILIBRANT
OF $\overrightarrow{OA}$ AND $\overrightarrow{OB}$
B

3. Redraw the force diagram for your experiment and complete the parallelogram with OA and OC as sides. Is the diagonal in the same line as OB and equal to OB? Repeat with the parallelogram formed using OB and OC as sides.



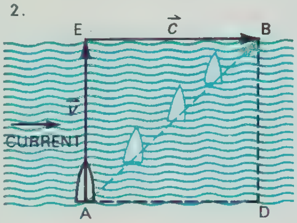
B Consider the following situations.

1.

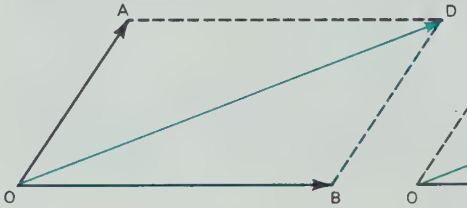


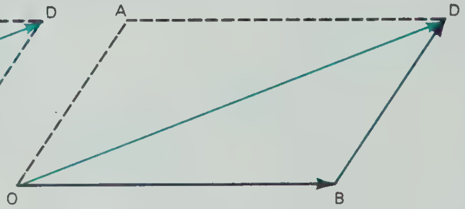
1. Mitzi walks across the platform of a railway flatcar from A to B and, at the same time, the flatcar travels from position 1 to position 2. As she moves from A to B, the flatcar moves from A to C. To an observer overhead, Mitzi appears to move from A to D. AD is the diagonal of the parallelogram ACDB and $\overrightarrow{AD}$ is the resultant of $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

2.



2. A boat heads across a stream with a velocity $\vec{v}$, as shown. If there is a current represented by $\vec{c}$, then $\overrightarrow{AB}$ is the sum of 2 velocities — velocity $\vec{v}$ *across* the stream and velocity $\vec{c}$ *down* the stream. AB is the diagonal of the parallelogram AEBD and $\overrightarrow{AB}$ is the resultant of these 2 velocities. Note that $\vec{v}$ and $\vec{c}$ are non-parallel vectors and, as the boat heads *across* the stream, the current pushes it *down* the stream.



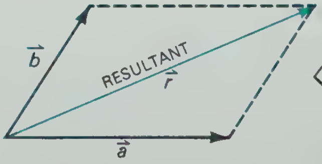


Notice that in parallelogram OADB,

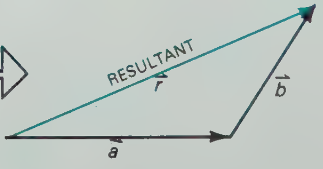
$$OA = BD \text{ and } \therefore \overrightarrow{OA} = \overrightarrow{BD}$$

$\overrightarrow{OD}$ is the *resultant* of $\overrightarrow{OA}$ and $\overrightarrow{OB}$, and may be obtained by

(i) drawing $\overrightarrow{OB}$,
(ii) tacking $\overrightarrow{BD}$ (which is equal to $\overrightarrow{OA}$) on to $\overrightarrow{OB}$,
(iii) joining O to D.



Parallelogram Law of Vector Addition

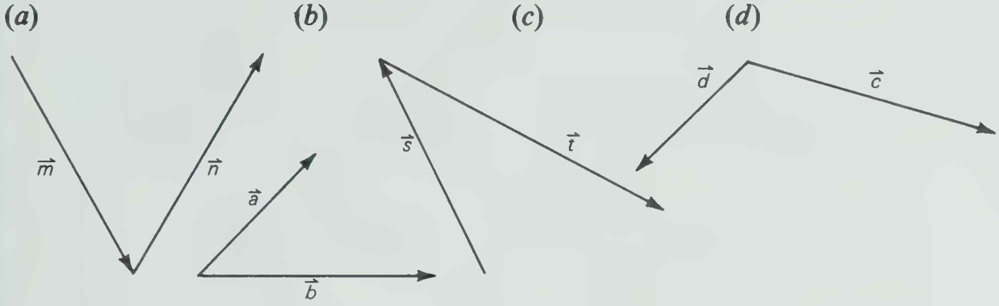


Triangle Law of Vector Addition

$$\vec{r} = \vec{a} + \vec{b}$$

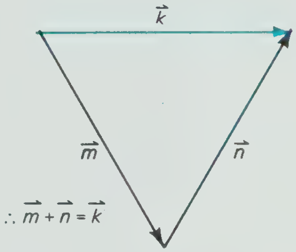
Example

Show the resultant for each pair of vectors.


(a)
(b)
(c)
(d)
$\vec{m}$
$\vec{n}$
$\vec{a}$
$\vec{b}$
$\vec{s}$
$\vec{t}$
$\vec{d}$
$\vec{c}$

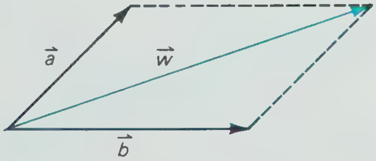
Solution

(a)


$\vec{k}$
$\vec{m}$
$\vec{n}$

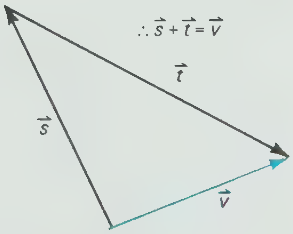
$\therefore \vec{m} + \vec{n} = \vec{k}$

(b)


$\vec{a}$
$\vec{w}$
$\vec{b}$

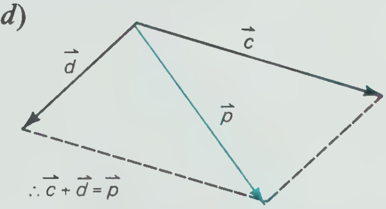
$\therefore \vec{a} + \vec{b} = \vec{w}$

(c)


$\vec{t}$
$\vec{s}$
$\vec{v}$

$\therefore \vec{s} + \vec{t} = \vec{v}$

(d)

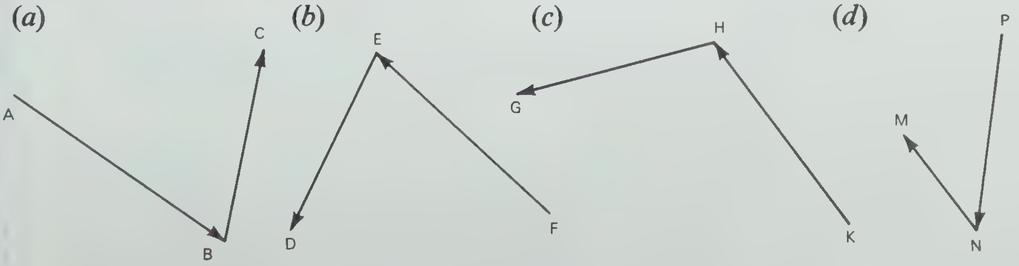

$\vec{d}$
$\vec{c}$
$\vec{p}$

$\therefore \vec{c} + \vec{d} = \vec{p}$

Use the Triangle Law!

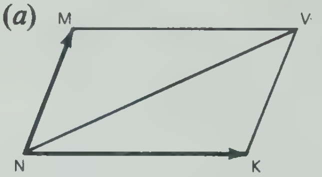
Use the Parallelogram Law!

1. Name the resultant vector for each pair of vectors shown.

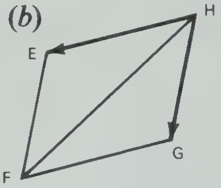

(a)
C
A
B
(b)
E
D
F
(c)
H
G
K
(d)
P
M
N

2. State the resultant of the indicated vectors in each case.

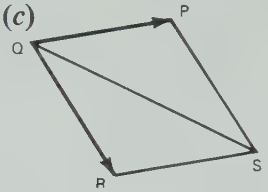
(*a*)



(*b*)

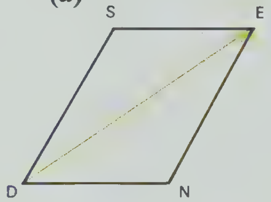


(*c*)

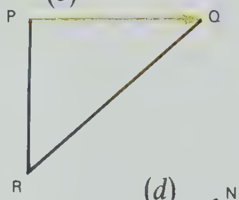


3. In each case, state 2 vectors for which the given vector(s) is a resultant.

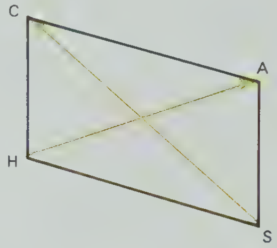
(*a*)



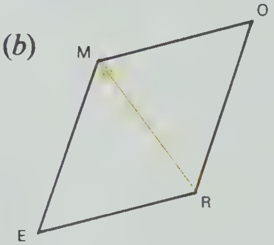
(*c*)



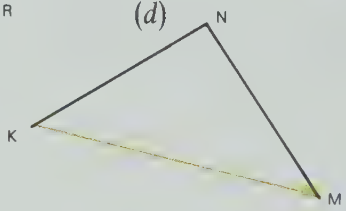
(*e*)

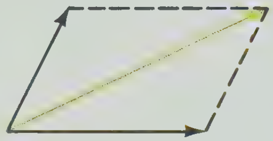


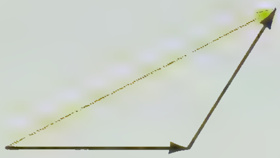
(*b*)



(*d*)



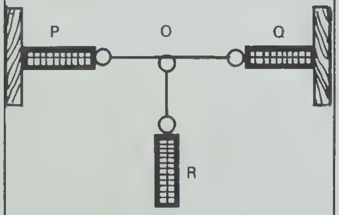




Exercise 21.1

Discussion

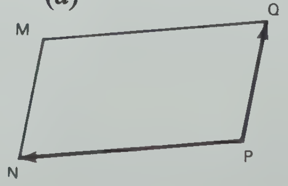
1. An experiment is set up as shown, with the cord joining balances P and Q passing *through* the ring O, but *not* tied to it. A force is applied at R until the readings on all 3 balances are the same. What do you expect the angles between the strings to be? Check by performing the experiment yourself.



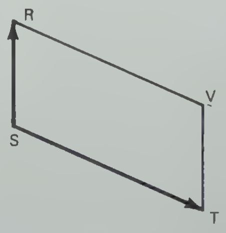
2. In your own words, state the Parallelogram Law of Vector Addition.

3. In each parallelogram, name the resultant vector for each pair of vectors shown.

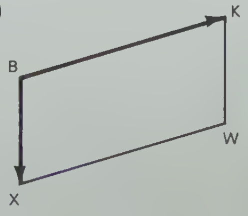
(*a*)



(*b*)



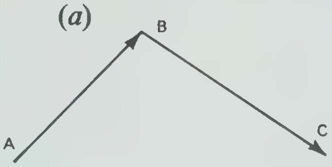
(*c*)



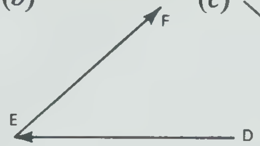
4. In your own words, state the Triangle Law of Vector Addition.

5. Name the resultant vector for each pair of vectors shown.

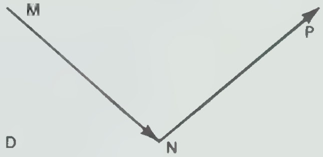
(*a*)



(*b*)



(*c*)



6. If the vectors representing 2 forces acting at a point have $\vec{0}$ as their resultant, describe the relationship between the 2 forces.

7. If 4 forces acting at a point have no effect upon it, what is the vector sum of the forces?

Written Solutions

You will need protractor, ruler, and compasses here!

8. In parallelogram PQRS, express each of the following in terms of $\vec{a}$ and $\vec{b}$.
(*a*) $\overrightarrow{RS}$ (*b*) $\overrightarrow{PS}$ (*c*) $\overrightarrow{QS}$ (*d*) $\overrightarrow{QT}$ (*e*) $\overrightarrow{PR}$

9. Use a scale diagram to find the resultant of each pair of forces acting at a point in each of the following cases.

	Forces	*Angle between forces*
(*a*)	10 N, 15 N	45°
(*b*)	10 N, 10 N	90°
(*c*)	20 N, 25 N	70°
(*d*)	15 N, 20 N	120°

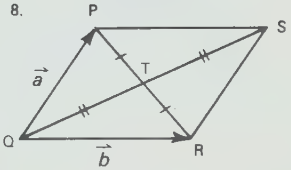
10. An aircraft flying 300 km/h is aided by a tail wind of 100 km/h making an angle of 40° with the flight path. Use a scale diagram to find the resultant speed and direction.

11. A ship heads northwest at a speed of 20 knots, in a current flowing south at 5 knots. By means of a scale diagram, determine the actual speed and direction of the ship.

12. I. Brain and U. Brawn attempt to move a large stone by pulling on 2 ropes attached to it. If the forces they exert are 40 N and 60 N, respectively, and the angle between the ropes is 30°, find the resultant force on the stone and the angle between the resultant and the 60 N force.

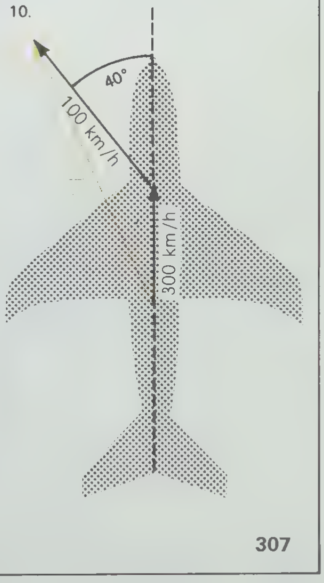
8(a) $\overrightarrow{RS} = \vec{a}$
(c) $\overrightarrow{QS} = \vec{a} + \vec{b}$
(e) $\overrightarrow{PR} = \vec{b} - \vec{a}$
9(a) **23.2 N at 18° to the 15 N force**
(b) **14.1 N at 45° to either 10 N force**
(c) **37 N at 31° to the 25 N force**
(d) **18 N at 46° to the 20 N force**
10. **382 km/h at 10° to heading of aircraft**
11. **16.8 knots in a direction 57° west of north**
12. **97 N, 12°**

8.



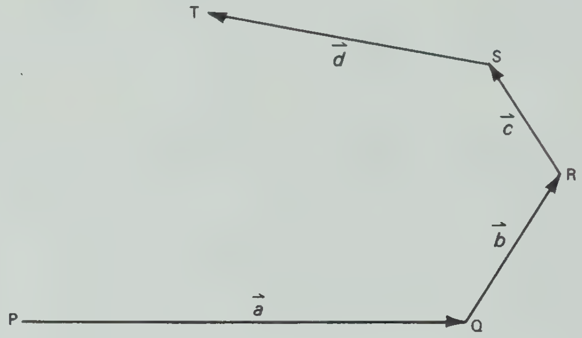
Use a convenient scale !

10.



21.2 Vector Polygons

Quite often it is necessary to find the vector sum of more than 2 vectors. Consider the 4 vectors shown here.

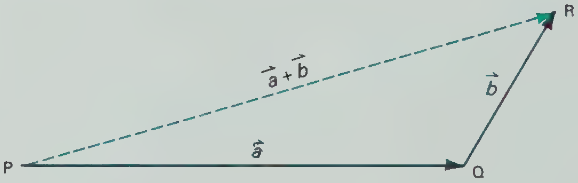


T
$\vec{d}$
S
$\vec{c}$
R
$\vec{b}$
P
$\vec{a}$
Q

Can we still apply our basic method of finding the resultant by joining the tail of the first vector to the head of the last vector? Study the following sequence of diagrams.

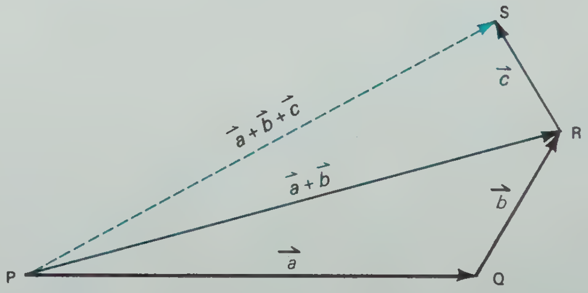
Remember the Triangle Law of Vector Addition?

Step 1



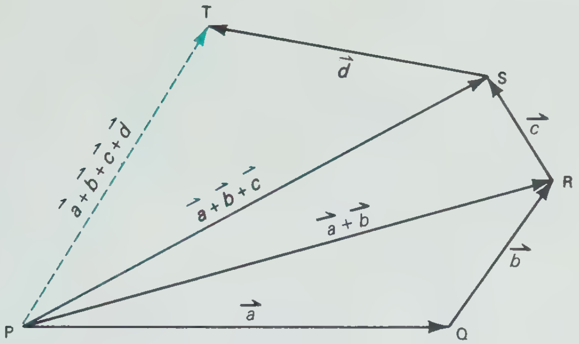
R
$\vec{a}+\vec{b}$
$\vec{b}$
P
$\vec{a}$
Q

Step 2

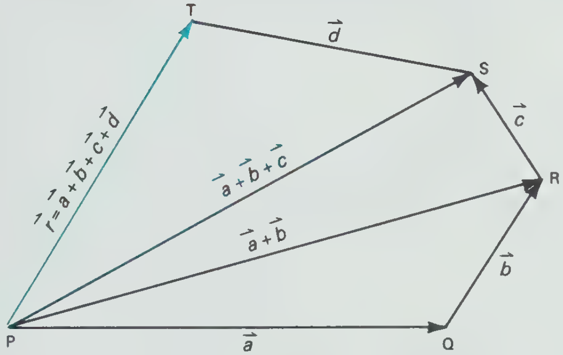


S
$\vec{c}$
$\vec{a}+\vec{b}+\vec{c}$
R
$\vec{a}+\vec{b}$
$\vec{b}$
P
$\vec{a}$
Q

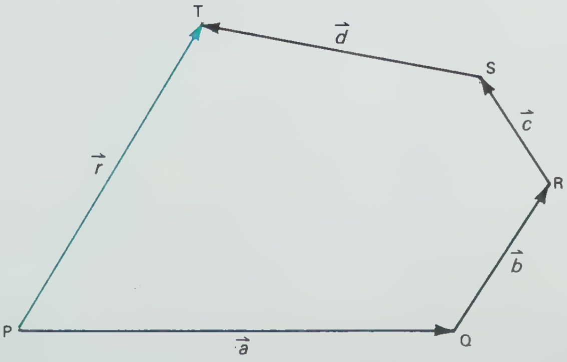
Step 3


T
$\vec{d}$
S
$\vec{c}$
$\vec{a}+\vec{b}+\vec{c}+\vec{d}$
$\vec{a}+\vec{b}+\vec{c}$
R
$\vec{a}+\vec{b}$
$\vec{b}$
$\vec{a}$
P
Q

Step 4


T
$\vec{d}$
S
$\vec{c}$
$\vec{r}=\vec{a}+\vec{b}+\vec{c}+\vec{d}$
$\vec{a}+\vec{b}+\vec{c}$
R
$\vec{a}+\vec{b}$
$\vec{b}$
P
$\vec{a}$
Q

Step 5

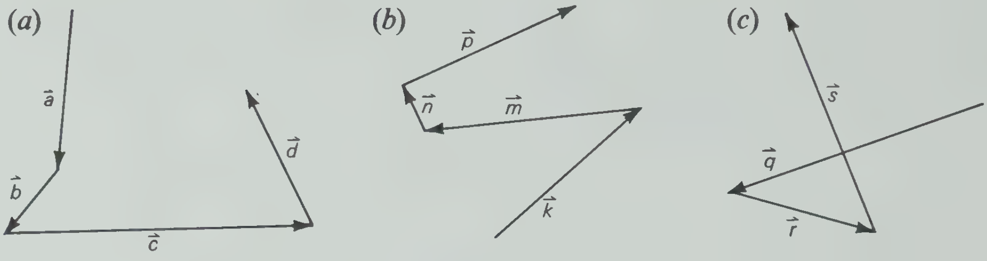

T
$\vec{d}$
S
$\vec{c}$
$\vec{r}$
R
$\vec{b}$
P
Q
$\vec{a}$

Hence, to find the resultant of several vectors, join the tail of the first vector to the head of the last vector.
A figure such as PQRST is called a *vector polygon.*

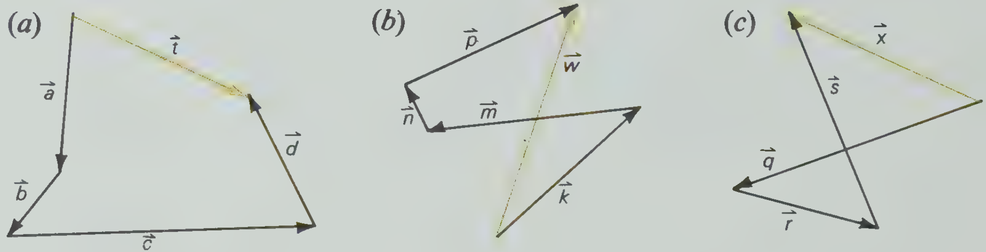
Great! Our basic method still works! (Guess we could call this the Polygon Law of Vector Addition.)

Example

Show the resultant of each system of vectors and state the vector equation in each case.


(a)
$\vec{a}$
$\vec{b}$
$\vec{c}$
$\vec{d}$
(b)
$\vec{p}$
$\vec{n}$
$\vec{m}$
$\vec{k}$
(c)
$\vec{s}$
$\vec{q}$
$\vec{r}$

Solution


(a)
$\vec{t}$
$\vec{a}$
$\vec{b}$
$\vec{c}$
$\vec{d}$
(b)
$\vec{p}$
$\vec{w}$
$\vec{n}$
$\vec{m}$
$\vec{k}$
(c)
$\vec{x}$
$\vec{s}$
$\vec{q}$
$\vec{r}$

$\therefore \vec{t} = \vec{a} + \vec{b} + \vec{c} + \vec{d}$

$\therefore \vec{w} = \vec{k} + \vec{m} + \vec{n} + \vec{p}$

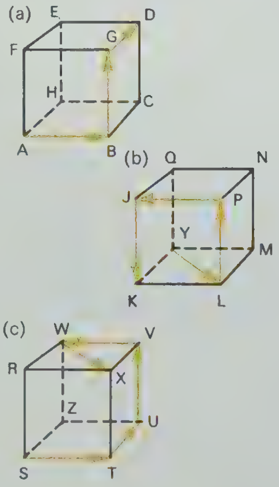
$\therefore \vec{x} = \vec{q} + \vec{r} + \vec{s}$

Exercise 21.2

Discussion

1. Is there any limit to the number of vectors that can be added in a vector polygon?
2. A vector polygon is drawn to find the resultant of 5 forces. How many sides does the polygon have?
3. The resultant of a system of 10 forces is the zero vector $\vec{0}$. How many sides are there to the vector polygon?
4. Vector polygons may be applied to 3-dimensional objects as well as 2-dimensional ones. Consider the cubes shown in the margin. Name the resultant of the vectors shown in each case.

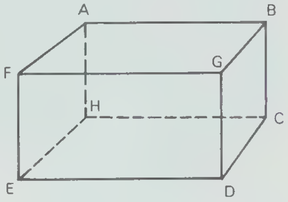
4.


(a)
E
D
G
F
H
C
A
B
(b)
Q
N
J
P
Y
M
K
L
(c)
W
V
R
X
Z
U
S
T

5. Using the diagram in the margin, state a vector equal to each of the following.

(*a*) $\overrightarrow{FG}$
(*b*) $\overrightarrow{BC}$
(*c*) $\overrightarrow{EH}$
(*d*) $\overrightarrow{FD}$
(*e*) $\overrightarrow{DB}$
(*f*) $\overrightarrow{EC}$
(*g*) $-\overrightarrow{DE}$
(*h*) $-\overrightarrow{EG}$

5. AND 6.



This is a "rectangular solid".

Written Solutions

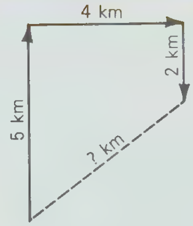
6. Using the diagram in the margin, express each of the following as a single vector.

(*a*) $\overrightarrow{FG} + \overrightarrow{GD} + \overrightarrow{DC}$
(*b*) $\overrightarrow{ED} + \overrightarrow{DH} + \overrightarrow{HB}$
(*c*) $\overrightarrow{FA} + \overrightarrow{BC}$
(*d*) $\overrightarrow{GH} + \overrightarrow{HD} + \overrightarrow{DA}$
(*e*) $\overrightarrow{FH} + \overrightarrow{CG}$
(*f*) $\overrightarrow{DB} - \overrightarrow{HC}$

Remember!
opposite vectors
equal vectors

7. Flora Forestwalker hikes 5 km due north, 4 km due east, and then 2 km due south before she realizes that she is lost. Use a vector polygon drawn to scale to determine how far Flora is from her starting point and the direction she should walk to return to her starting point.

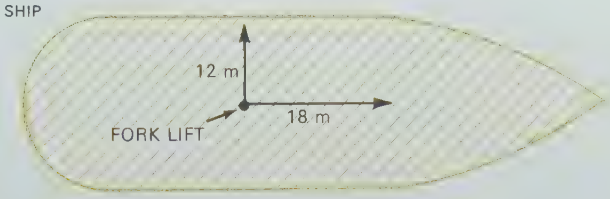
7.



8. Moving at a steady rate of 10 km/h a ship sails due west for 4 h, due south for 2 h, due east for 3 h, and then due north for 8 h. Determine the distance and direction of the ship from its starting point.

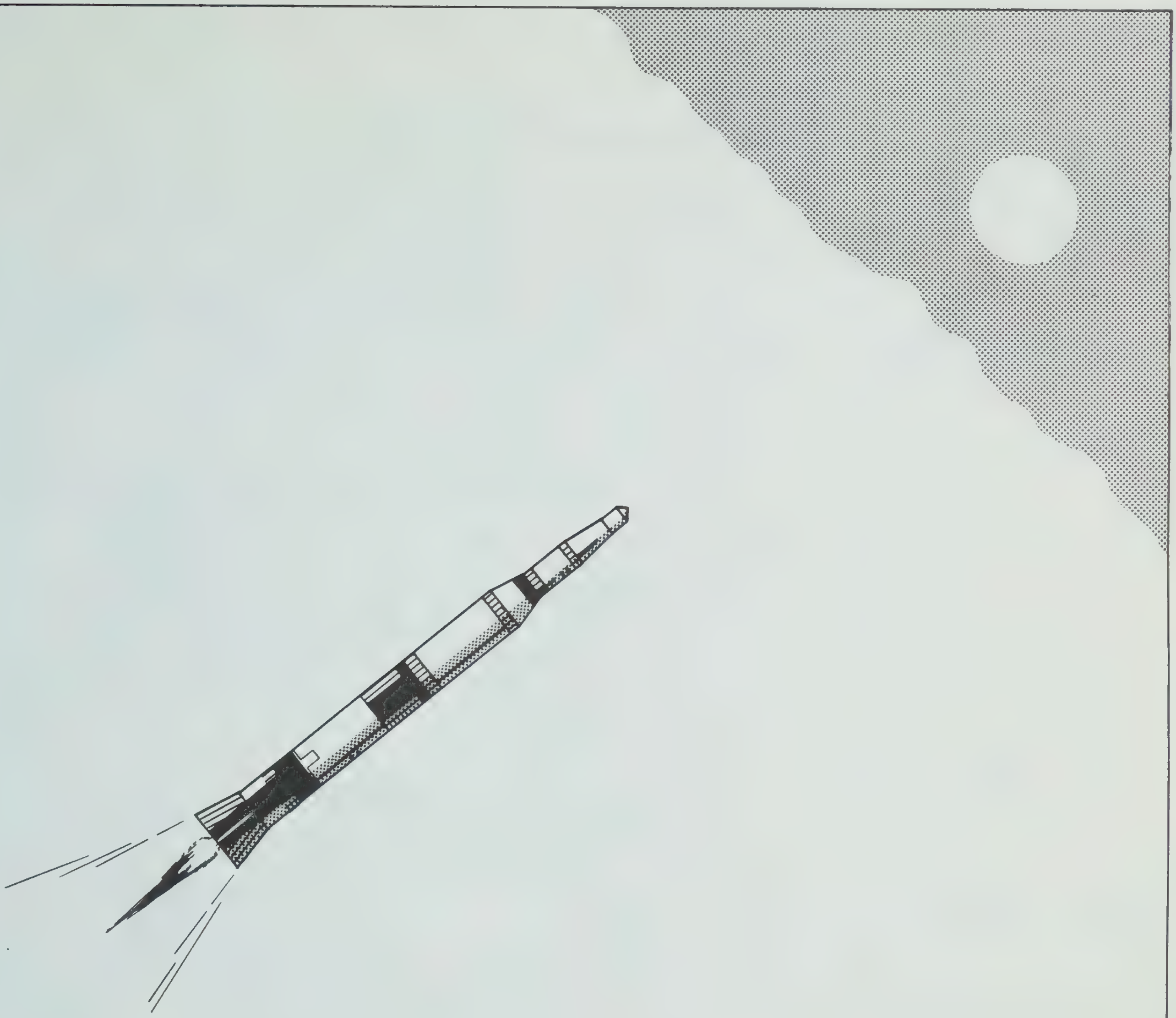
9. A fork lift on the deck of a lake freighter moves 12 m across the deck as the freighter itself moves 18 m. During this time, a box on the platform of the fork lift is raised 3 m.

(*a*) Sketch a vector diagram to illustrate the motion of the box.
(*b*) Using scale diagrams, determine the total distance the box moved.



6(a) $\overrightarrow{FC}$ **(c)** $\overrightarrow{FH}$ **(e)** $\vec{0}$
7. 5 km S53°W
8. 60.8 km N9°W
9. 21.8 m

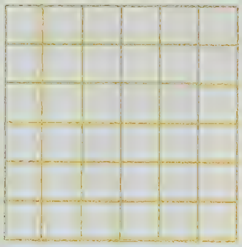
22

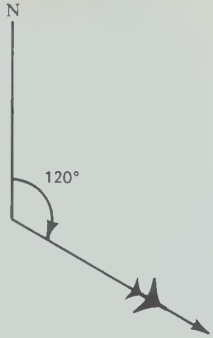


Unit 22
Application of Vectors

Vectors are more than just a mathematical curiosity. As mentioned earlier, they play an important part in practical problems related to force and motion. Vectors are used to illustrate and explain basic principles of electricity; vectors appear in problems related to meteorology and astronomy; they help solve problems in navigation and space travel.

Get some squared paper like this.





A BEARING OF 120°

22.1 Navigation Problems

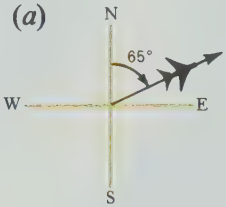
The basic method of dealing with navigation problems is that of scale diagrams. The larger and more accurate the diagram, the more accurate the results.

A common term used in many types of navigation is *bearing*. Bearing is an *angle* (expressed in *degrees*) measured *clockwise* from the *north*.

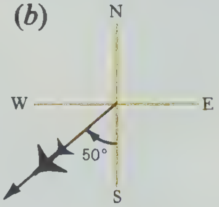
Example 1

State the bearing of the aircraft in each of the following.

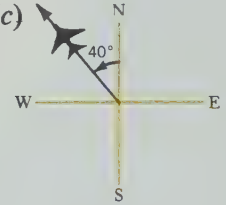
(*a*)



(*b*)



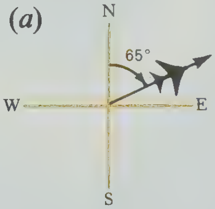
(*c*)



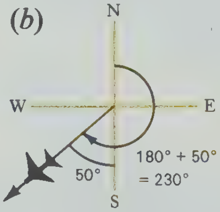
Solution

Since bearing is always measured from the north, the bearing of each aircraft is as shown.

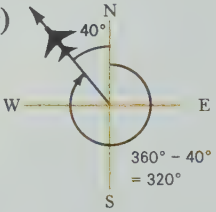
(*a*)



(*b*)



(*c*)

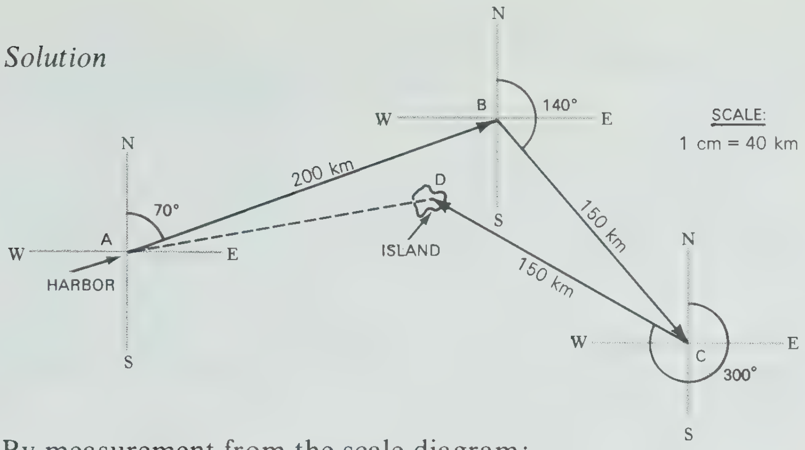


Example 2

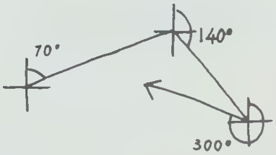
A ship sails 200 km from a harbor on a bearing of 70°, changes to a bearing of 140° and sails 150 km, then sails another 150 km on a bearing of 300°, and anchors at an island. By means of a scale diagram, determine the following.

(*a*) the distance from the harbor to the island
(*b*) the greatest distance between the harbor and the ship during its cruise
(*c*) the bearing of the island from the harbor
(*d*) the bearing of the harbor from the island

Solution


N
W
E
S
A
HARBOR
70°
200 km
D
ISLAND
B
140°
150 km
150 km
C
300°
SCALE:
1 cm = 40 km

A rough sketch helps!



Recall Vector Polygons?

By measurement from the scale diagram:

(*a*) The distance from the harbor to the island is approximately 160 km.

∵ AD ≑ 4 cm

(*b*) The greatest distance between the harbor and the ship during its cruise is about 295 km.

∵ AC ≑ 7.4 cm

(*c*) The bearing of the island from the harbor is 79°.*

(*d*) The bearing of the harbor from the island is 259°.

* Notice that 79° + 180° = 259°

Exercise 22.1

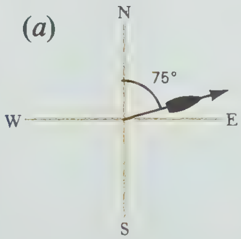
Discussion

1. From your own experience, give an example of a situation involving bearing.

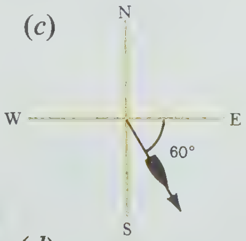
Scouts and Guides should find this easy!

2. State the bearing of the ship in each situation shown.

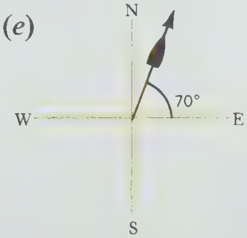
(*a*)



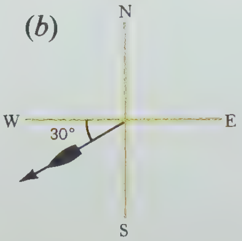
(*c*)



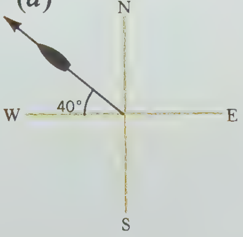
(*e*)



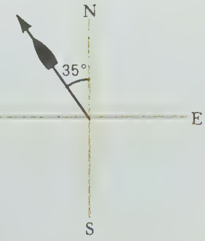
(*b*)



(*d*)



(*f*)



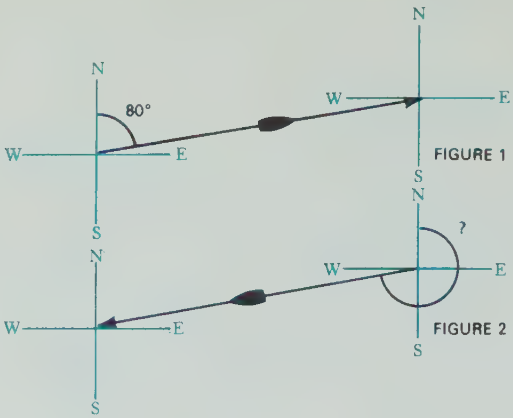

N
80°
W
E
S
N
W
E
S
N
?
W
E
S
N
W
E
S

FIGURE 1

FIGURE 2

3. Consider a ship on a bearing of 80° as shown in Figure 1.

   If the ship reverses direction to retrace its path (Figure 2), what is its new bearing?

Notice that
80° + 180° = 260°

We say that 260° is the *opposite bearing* of 80°. State the opposite bearing for each of the following.

(*a*) 60° (*c*) 12° (*e*) 200°
(*b*) 100° (*d*) 160° (*f*) 350°

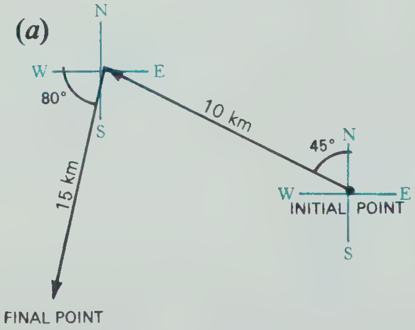
Careful in (e) and (f).
(Remember the meaning of bearing.)

Written Solutions

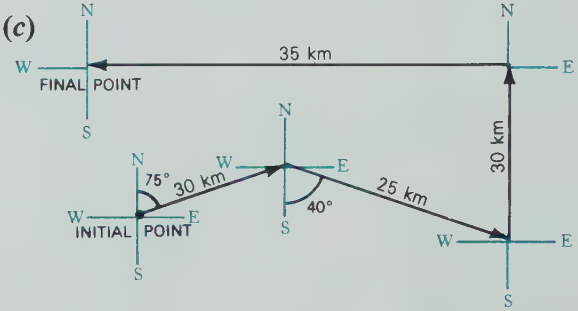
4. The following are sketches (*not* to scale). Make an accurate scale diagram in each case and determine
   (i) the distance from the initial point to the final point,
   (ii) the bearing of the initial point from the final point,
   (iii) the bearing of the final point from the initial point.

Recall the
Z rule!

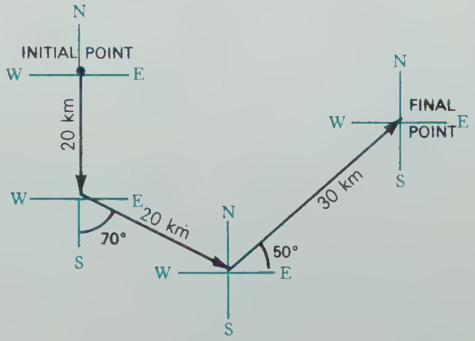
(*a*)


N
W
E
S
80°
10 km
45°
N
W
E
S
INITIAL POINT
15 km
FINAL POINT

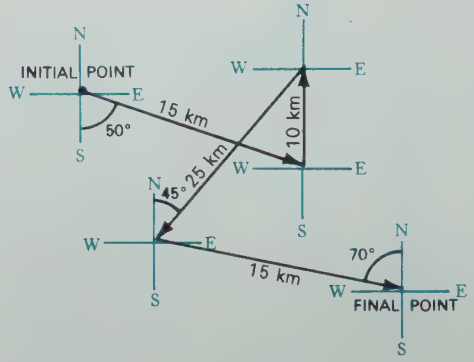
(*c*)


N
W
FINAL POINT
S
35 km
N
E
30 km
N
75°
30 km
W
INITIAL POINT
E
S
N
W
E
40°
S
25 km
W
E
S

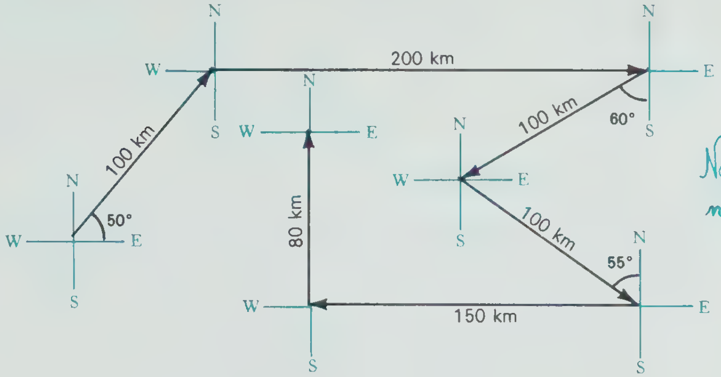
(*b*)


N
INITIAL POINT
W
E
20 km
W
E
S
70°
20 km
N
W
E
S
50°
30 km
N
W
FINAL POINT
E
S

(*d*)


N
INITIAL POINT
W
E
S
50°
15 km
N
W
E
10 km
W
E
S
N
45°
25 km
W
E
S
15 km
N
70°
W
FINAL POINT
E
S

5. By means of a scale diagram, determine the magnitude and bearing of the resultant vector.



Note: Drawing not to scale.

6. Samuel Steeremstrate leaves his house and drives 3 blocks north, 2 blocks west, 5 blocks south, and 4 blocks east. If each block is 150 m long, how far is Sam from home? What is the bearing of his house from his present position?

7. Girl Guide Frieda Fearless starts out through the bush from the Guide Main Camp at noon on a compass reading of due west. After 1 h she turns 40° to her right, and 2 h later she turns 30° to her left. At 15:45, she pauses 15 min for a snack, and then turns 120° to the right of her previous course and travels for 1 h before making camp for the night. Determine her bearing at 16:30.

8. A Search and Rescue ship leaves port on a bearing of 200° and at a steady speed of 30 knots. After 3 h at sea, the ship alters bearing to 80° and maintains speed; 2 h later it turns and heads directly for port but at a reduced speed of 20 knots. Find
(*a*) the bearing the ship should follow to reach port, and
(*b*) how long it will take to reach port.

9. The aircraft carrier *Botomzup* is travelling due west at 40 km/h. A reconnaissance plane takes off from its deck on a bearing of 120° and at a constant speed of 200 km/h. The plane changes course 60° to its right after flying for 100 km and then a further 100° right after flying for another 50 km. Use a scale diagram to determine the distance from the carrier to the plane 1.75 h after the plane takes off.

4(a) 12.4 km, 52°, 232°
(b) 38.3 km, 276°, 96°
(c) 21.2 km, 208°, 28°
(d) 23.8 km, 341°, 161°

5. 120 km, 66°

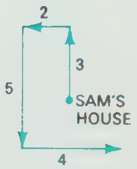
6. 424 m, 315°

7. 40°

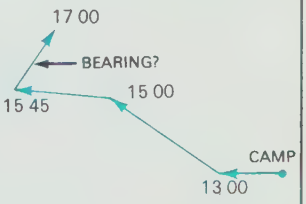
8(a) 339°
(b) About 4 h

9. About 77 km

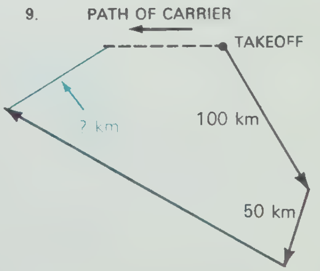
6.

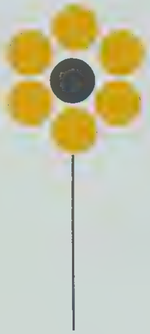


7.



9.





10. An aircraft leaves the airport at 10:00 on a bearing of 230° and at a constant speed of 200 km/h. The flight plan calls for course changes as indicated below.

Time	*Course Change*
12:00	30° left
13:00	115° right
15:00	40° left

At 17:00, the aircraft reaches its destination, and an hour later heads directly back for its original airport at a speed of 250 km/h. Calculate

(*a*) the bearing required for the return flight, and

(*b*) the time of arrival at the original airport.

10(a) 83°
(b) About 22 15

22.2 Physics and Shop Problems

Recall resultant vector.

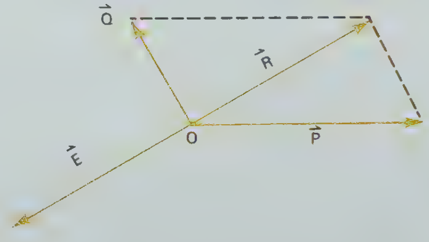
In this section, we apply the Parallelogram Law of Vector Addition to physics and shop problems. Again, make your scale diagrams large to ensure accurate results.

In physics, the *resultant* of a system of forces acting at a point is the single force that will replace the system and have the same effect on the point.

The *equilibrant* is the force that can be added to the system to produce *equilibrium*. (A system of forces is in equilibrium if their vector sum is the zero vector $\vec{0}$.)

Consider two forces $\vec{P}$ and $\vec{Q}$ acting at a point O as shown.

$\vec{R}+\vec{E}=\vec{0}$
BUT $\vec{R}=\vec{P}+\vec{Q}$
$\therefore \vec{P}+\vec{Q}+\vec{E}=\vec{0}$



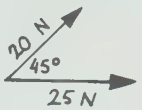
$\vec{E}=-\vec{R}$

From the Parallelogram Law, the resultant of $\vec{P}$ and $\vec{Q}$ is $\vec{R}$, and the equilibrant is $\vec{E}$. Notice that $\vec{E}$ has the *same* magnitude as $\vec{R}$, but the *opposite* direction.

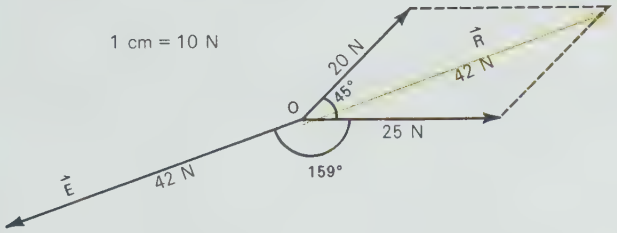
Example 1

Two forces of 20 N and 25 N act at a point O, and the angle between them is 45°. Find the equilibrant of the 2 forces and the angle it makes with the 25 N force.

Sketch



Solution

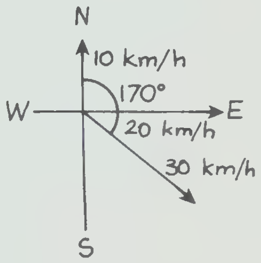

1 cm = 10 N
20 N
45°
O
$\vec{R}$
42 N
25 N
159°
$\vec{E}$
42 N

By measurement, $\overrightarrow{E}$ has magnitude of 42 N and acts at an angle of 159° to the 25 N force.

Example 2

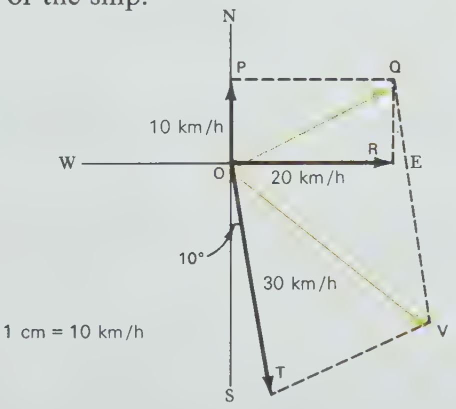
A ship heading on a bearing of 170° at a speed of 30 km/h encounters a current flowing due east at 20 km/h and a wind blowing from the south at 10 km/h. Determine the actual speed and direction of the ship.

Sketch


N
10 km/h
170°
W
E
20 km/h
30 km/h
S

Do you see how this could be extended to a case involving 4 or more vectors?

Solution

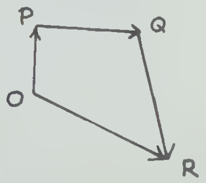

N
P
Q
10 km/h
W
R
E
O
20 km/h
10°
30 km/h
1 cm = 10 km/h
V
T
S

Step 1 Form parallelogram OPQR to find the resultant $\overrightarrow{OQ}$ of the wind and current velocities.

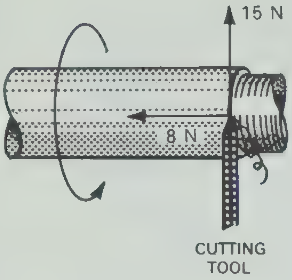
Step 2 Form parallelogram OQVT to determine the final resultant $\overrightarrow{OV}$.

By measurement, the actual speed of the ship is 32 km/h on a bearing of 128°.

Could we use the Polygon Law?



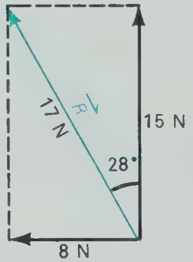
Example 3


15 N
8 N
CUTTING TOOL

The tip of the lathe cutting tool shown produces a vertical circumferential force (due to rotation) of 15 N, and a horizontal force of 8 N as it feeds past the work. (See sketch in margin). Determine the magnitude and direction of the resultant force of the tip of the cutting tool.

Solution

Due to the nature of the problem, it should be obvious that the angle between the two forces is 90°.


$\vec{R}$
17 N
15 N
28°
8 N

1 cm = 5 N

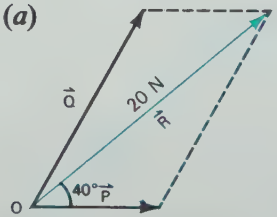
By measurement, the resultant force of the tip of the cutting tool is a 17 N force acting at 28° to the vertical 15 N force.

Exercise 22.2

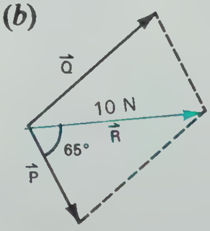
Discussion

1. Distinguish between the resultant and the equilibrant of a system of forces.
2. State the magnitude and direction of the equilibrant in each case.

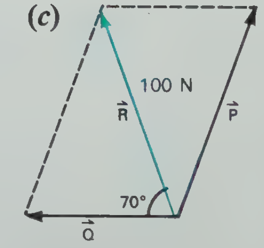
(a)


$\vec{Q}$
20 N
$\vec{R}$
40°
$\vec{P}$
O

(b)

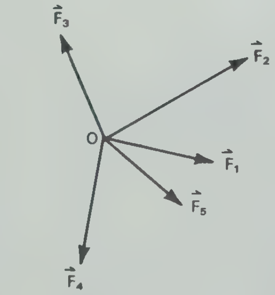

$\vec{Q}$
10 N
$\vec{R}$
65°
$\vec{P}$

(c)


100 N
$\vec{R}$
$\vec{P}$
70°
$\vec{Q}$

3. In your own words, describe how you would determine graphically the resultant of a system of 5 forces acting at a point.

3.


$\vec{F}_3$
$\vec{F}_2$
O
$\vec{F}_1$
$\vec{F}_5$
$\vec{F}_4$

Written Solutions

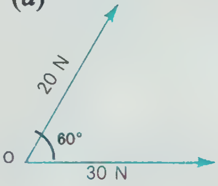
4. A cam and follower are in contact as shown in the sketch. Find the resultant force on the tip of the follower, and the angle between the resultant and the axis of the follower.

5. An airplane flying on a bearing of 60° at an air speed of 400 km/h encounters a wind blowing from the west at 100 km/h. Determine the ground speed and track (direction relative to the ground) of the airplane.

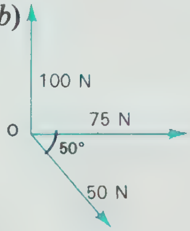
6. A worm gear drives an involute spur gear, and the forces acting on the face of one tooth of the spur gear are as indicated in the sketch. Determine the resultant force (magnitude and direction) on the face of the spur gear tooth.

7. Determine the magnitude and direction of the *equilibrant* for each system of forces shown. (Diagrams are *not* to scale!)

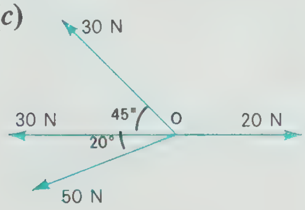
(*a*)



(*b*)



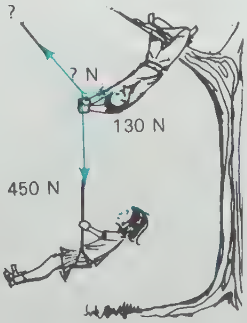
(*c*)



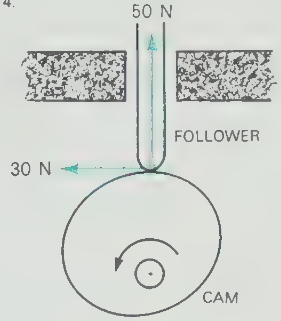
8. The forces acting on a point on a drill bit are as shown in the margin. What is the magnitude and direction of the force exerted by the drill bit on the piece of work being drilled? (Careful here!)

9. A ship travelling on a bearing of 300° at 40 km/h encounters a current flowing due east at 15 km/h and a wind blowing from the south at 30 km/h. What is the actual speed and direction of motion of the ship?

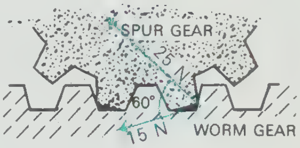
10. Dorothy Dogood sits on a swing that is pulled to one side by her cousin Marvin Mischiefmaker. If Dorothy's mass exerts a downward force of 450 N in the rope, and Marvin exerts a horizontal force of 130 N, determine the tensile force in the rope, and the angle the rope makes with the vertical.



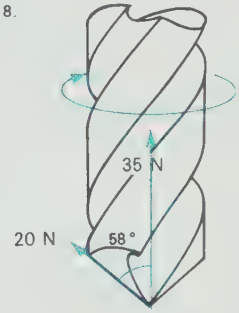
4.



6.



8.



4. 58 N, 31°

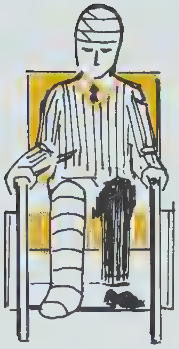
6. 35 N at 22° to the 25 N force

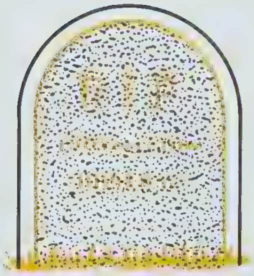
7(a) 43.6 N at 157° to the 30 N force

9. 54 km/h, bearing 339°

10. 468 N, 16°

23





Unit 23
Insurance

The purpose of insurance is to provide protection against an unexpected financial loss.

In a few short years you may find yourself responsible for the upkeep of a home or apartment and a family. If you are the wage earner, you will probably wish to provide protection for your dependants, should anything happen to you. For this reason, you should be aware of the various forms of insurance available.

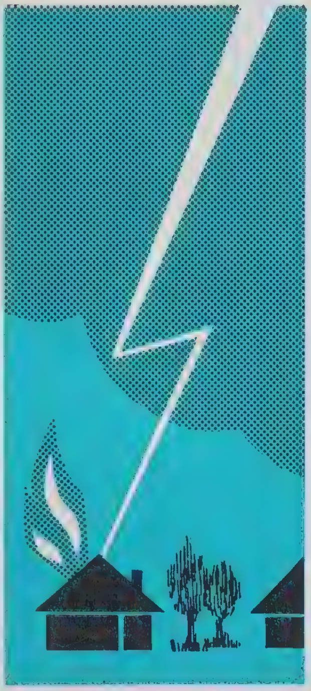
There are 2 main principles upon which all insurance is based.

1. Uncertainty as to who will suffer the loss.
2. Certainty that the loss will occur to *someone* in a large group.

Do you see how insurance companies make money?

For example, consider 1000 homes, each valued at $50 000 and insured against fire loss at an annual cost of $100 each. If 2 of these homes were completely destroyed by fire during the year, the total loss would be $100 000. However, a moderate payment of $100 from each of the 1000 policyholders saves each of the 2 unfortunate ones a loss of $50 000.

You should understand the meaning of several terms used in the language of insurance. The insurance *policy* is the written contract or agreement between the insured person (the policyholder) and the insurance company. The *face* (also called *risk*) of the policy is the amount of insurance stated in the policy. The *premium* is the sum paid by the insured person to the insurance company for the protection stated in the policy. The amount paid by the insurance company in the event of a loss is called the *indemnity*.



The greater the risk, the greater the premium.

23.1 Insuring Your Property

A The most common type of property insurance is *fire insurance*. However, many fire insurance companies offer an extended coverage policy that gives protection from various other hazards such as smoke, wind, hail, lightning, explosions, falling objects, vandalism, and so on. Such a policy is known as a *Homeowner's Policy*.

The *premium*, or cost of your insurance, is always based on the risk that the insurance company is taking. Premiums for fire insurance depend upon 2 main factors.

1. The material used in construction (brick or frame).
2. The location of the dwelling (degree of fire protection available).

Where's the nearest hydrant?

Hazards for which coverage is given in a "standard" Homeowner's Policy include

fire
smoke and water damage
lightning
hail
wind
explosion
vandalism
damage from aircraft
theft
damage by thieves
glass breakage
falling objects
water damage from plumbing system
injury to guests
breakage of neighbor's window
loss due to stolen credit cards

A "deluxe" Homeowner's Policy is also available to cover additional items such as

collapse of building due to weight of ice or snow
damage of wall by rain
counter top scorched by hot appliance
damage from your own vehicle

Coverage is not
included for
losses due to
termites
wear and tear
settling
flood
earthquake

B Insurance companies use tables of insurance rates to calculate the premium in a given situation. A portion of such a table is shown below.

HOMEOWNER'S PREMIUM RATES (FULL COVERAGE)

AMOUNT OF INSURANCE	BRICK		FRAME	
	STANDARD	DELUXE	STANDARD	DELUXE
$20 000	$50	$65	$55	$70
21 000	51	68	57	73
22 000	52	70	58	76
23 000	55	72	61	78
24 000	58	77	64	84

The complete table
is on page 449.

If you cancel your policy during the year, the insurance company will keep a portion of the annual premium.

IF YOU CANCEL WITHIN	PREMIUM PERCENT RETAINED BY COMPANY
15 days	13%
1 month	19%
45 days	22%
2 months	25%
75 days	31%
3 months	35%

See page 448
for the complete
table.

See the Tables on page 325!

The use of these tables is illustrated by the following example.

Example 1

Mr. F. N. Brimstone purchases a $22 000 Homeowner's Policy from the N. E. Daynow Insurance Company. Mr. Brimstone decides on the "deluxe" coverage for his brick home, but cancels his policy after 80 days, when he moves out of town. Determine the amount of the premium returned to Mr. Brimstone.

Solution

The amount of insurance required is $22 000.
From the table of premium rates:
The annual premium is $70.
From the table for cancelled policies:
Since Mr. Brimstone cancelled his policy within 3 months, the insurance company would keep 35% of the premium, or $24.50.

∴ Mr. Brimstone will have $70 – $24.50 or $45.50 returned to him.

C *Co-Insurance* Insurance policies offered by most insurance companies contain a *co-insurance* clause. Such a clause in the policy states that the insured agrees to keep the property insured for at least a certain stated percent of its value (usually 80%).

The insurance company will never pay more than the face of the policy.

For instance, if your property is insured for at least 80% of its value at the time of a loss, the insurer will pay all your loss up to the face of your policy.

If the condition of the 80% clause has *not* been met by the policyholder, then the insurance is paid according to the formula:

$$\boxed{\text{Loss paid} = \frac{\text{Face of policy}}{80\% \text{ of the property value}} \times \text{Loss}}$$

Example 2

A building valued at \$80 000 is damaged by fire to the extent of \$40 000. Calculate the loss paid by the insurance company if the building is insured under an 80% co-insurance clause for

(*a*) \$65 000 (*b*) \$48 000

Solution

(*a*) 80% of the property value is \$64 000. Since the face of the policy is \$65 000, the insured is adequately protected and the insurance company will pay the full loss, up to the face of the policy.

∴ The full loss of \$40 000 is paid.

80% of \$80 000
= 0.80 × \$80 000
= \$64 000

(*b*) The insurance carried (\$48 000) is less than the 80% (\$64 000) required.

$$\therefore \text{Loss paid} = \frac{\text{Face of policy}}{80\% \text{ of property value}} \times \text{Loss}$$

$$= \frac{\$48\ 000}{\$64\ 000} \times \$40\ 000$$

$$= \$30\ 000$$

∴ Only \$30 000 of the \$40 000 loss is paid.

Example 3

A building valued at \$50 000 is insured for \$36 000 under an 80% co-insurance clause. If a fire loss of \$45 000 occurs, how much of the loss must the policyholder absorb?

Solution

80% of the property value is \$40 000.

$$\therefore \text{Loss paid} = \frac{\text{Face of policy}}{80\% \text{ of the property value}} \times \text{Loss}$$

$$= \frac{\$36\ 000}{\$40\ 000} \times \$45\ 000$$

$$= \$40\ 500$$

80% of \$50000
= 0.80 × \$50 000
= \$40 000

However, the insurance company will *never* pay more than the face of the policy. Therefore, the amount paid by the company is \$36 000, and the policyholder must absorb the remaining \$9000.

Exercise 23.1

Discussion

1. Explain how the factors listed below might affect fire insurance premium rates.
   - (*a*) location of dwelling
   - (*b*) value of property
   - (*c*) type of construction
   - (*d*) degree of available fire protection
   - (*e*) risk to the insurance company

2. A building and a piece of land are valued together at $60 000. Why would it be a waste of money to buy a $60 000 fire insurance policy on the property?

3. If a person takes out a fire insurance policy containing an 80% co-insurance clause, why should the face of the policy be adjusted periodically?

4. Determine what is meant by a "$50-Deductible" clause in an insurance policy. Why would the premium for a policy containing such a clause be somewhat lower than the premium for a non-deductible policy of the same amount?

Written Solutions

5. Find the annual premium in each case. Refer to the table of premium rates on page 449.

	Face	*Construction*	*Type of Policy*
(*a*)	$40 000	Frame	Standard
(*b*)	6 000	Brick	Standard
(*c*)	32 000	Frame	Deluxe
(*d*)	19 000	Frame	Standard
(*e*)	27 000	Brick	Deluxe
(*f*)	14 000	Brick	Standard
(*g*)	38 000	Frame	Deluxe
(*h*)	55 000	Brick	Deluxe

Careful here! →

6. Calculate the amount of the premium kept by the company in each case. Use the tables on pages 448 and 449.

	Face	*Construction*	*Type of Policy*	*Policy cancelled after*
(*a*)	$11 000	Brick	Deluxe	5 weeks
(*b*)	48 000	Frame	Deluxe	280 days
(*c*)	21 000	Brick	Standard	11 months
(*d*)	30 000	Brick	Standard	137 days
(*e*)	41 000	Frame	Standard	48 days
(*f*)	29 000	Frame	Deluxe	7 months

7. Cecilia Sellmore insures her condominium under a standard $34 000 Homeowner's Policy on October 6. Cecilia moves to another city on December 15, however, cancelling her policy. If her condominium has a "brick" rating, calculate the amount of the premium returned to her.

8. How much less is the premium for a deluxe brick $29 000 Homeowner's Policy than for a standard frame $36 000 policy?

9. Samuel Sparks purchases a $33 000 standard Homeowner's Policy on his brick home and renews the policy each year. At the end of 3 years, he switches to a deluxe policy. If he had purchased the deluxe policy initially, how much more would he have paid for his insurance over the 3-year period?

10. Find the indemnity paid if each Homeowner's Policy contains an 80% co-insurance clause.

	Property Value	*Face*	*Loss*
(*a*)	$30 000	$20 000	$21 600
(*b*)	50 000	36 000	35 000
(*c*)	60 000	52 000	39 000
(*d*)	42 000	36 000	25 000
(*e*)	36 000	24 000	30 000

Indemnity is the amount paid by the insurance company.

6(a) $12.54 (e) $34
(c) $47.43

7. $66.24

8. $15

9. $90

10(a) $18 000 (e) $24 000
(c) $39 000

11. $933.33
12(a) $36 000 (c) $7111.11
(b) $41

11. Although his home was worth $30 000, C. D. Embers insured it for only $20 000 against fire loss. If fire damaged his house to the extent of $5600, how much of this loss would he have to absorb himself if his policy contained an 80% clause?

PRICE OF HOMES

12. Shirley U. Jestt carries a $28 000 deluxe Homeowner's Policy on her brick home, whose value is set at $35 000. In the next 5 years, however, property values soar, and the new value of her home is set at $45 000.
    (*a*) In order to comply with the 80% co-insurance clause in her policy, what amount of insurance should Shirley now carry on the house?
    (*b*) Calculate the increase in her annual premium if she complies with the 80% clause.
    (*c*) If a fire loss of $32 000 should occur near the end of the 5-year period, and just before she planned to increase her coverage, what amount of the loss would Shirley have to pay herself?

23.2 Insuring Your Life

A The main purpose of *life insurance* is to provide financial protection for your dependants in case of your death. If you die, the insurance company will provide income to your survivors according to the terms of the policy. The person who receives the benefits is called the *beneficiary*.

Life insurance is purchased by making regular premium payments to the insuring company. Premiums are paid in advance, and are usually set at a fixed annual amount. This system of payment is known as the *level premium system*. Under this system, a premium surplus builds up in the early years of the contract so that the premium in later years may be kept down. Premium rates will vary with the age and health of the insured and with the type of policy.

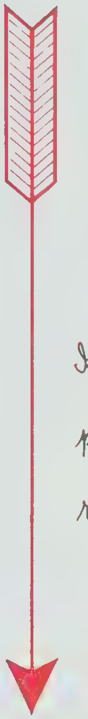
There are four main types of life insurance.

1. *Term Insurance.* This type provides protection only for the period of time stated in the policy, and not for as long as you live. Premiums must be paid throughout the period of the policy. Such policies may be called 10-Year Term, 15-Year Term, and so on.

2. *Whole Life Insurance (or Straight Life Insurance).* Under this type of policy, the insured person pays premiums for life or until age 85, whichever comes first. At the time of death of the insured, the beneficiary receives the face of the policy.

3. *Limited-Payment Life Insurance.* The insured pays premiums for a fixed period (usually to age 60 or 65) or until death, whichever comes first. If still alive at the end of the stated period, the insured no longer pays any premiums, but remains insured for life.

4. *Endowment Insurance.* Again, the insured person agrees to pay premiums for a fixed period or until death, whichever comes first. If the insured dies within this period, the beneficiary receives the face of the policy. If still alive at the end of the period, however, the insured would receive the face of the policy and would no longer be insured. This type of insurance is designed to have a savings or investment aspect, along with protection.



Increasing premium rates!

The policies described above may be either *participating* or *non-participating* policies. Premiums on participating policies are greater than the expected cost of insurance to the insurance company. The company invests this surplus money and may then pay a *dividend* annually to the insured person. The insured person may then apply the dividend to reduce the annual premium, use it to purchase more insurance, leave it to accumulate interest, or decide to take it as cash.

"Par" and "Non-par" policies.

Non-participating policies do not pay dividends, and thus have lower premiums. Most term insurance is of this type.

B *Calculation of Premiums.* Life insurance companies also use tables in calculating premiums. The table on page 450 shows typical premium rates for the various types of insurance. Part of the table is shown below.

LIFE INSURANCE: ANNUAL PREMIUMS (PER $1000 WORTH OF INSURANCE)

AGE AT ISSUE	TERM		WHOLE LIFE (TO AGE 85)	LIMITED-PAYMENT LIFE TO AGE 65	ENDOWMENT	
	10-YEAR	15-YEAR			TO AGE 60	TO AGE 65
15	$3.58	$4.08	$11.14	$11.82	$14.35	$14.98
16	3.64	4.10	11.48	12.21	14.85	15.86
17	3.70	4.13	11.82	12.60	15.34	16.74
18	3.76	4.15	12.16	12.99	15.84	17.62
19	3.83	4.17	12.50	13.38	16.34	18.50

For the convenience of the insured person, premiums may be paid semi-annually, quarterly, or even monthly. Of course, the cost will rise with the frequency of payment, due to greater recording and calculating done by the insurance company. The following table is used to determine periodic premiums.

TO COMPUTE PERIODIC PREMIUM

IF PERIOD IS:	MULTIPLY ANNUAL PREMIUM BY:
Semi-annual	0.51
Quarterly	0.26
Monthly	0.087

life insurance

Example 1

(*a*) Find the annual premium on a \$10 000 whole life policy purchased at age 19.
(*b*) Determine the monthly premium for this policy.
(*c*) Calculate the saving by making annual payments rather than monthly payments.

Solution

(*a*) From the table of premiums, the annual premium per \$1000 of insurance for this policy at age 19 is \$12.50.
∴ The annual premium for a \$10 000 policy would be 10 × \$12.50 or \$125.
(*b*) If the premium is paid monthly, the periodic premium table on page 332 indicates that we must multiply the annual premium by 0.087 to obtain the monthly premium.
∴ The monthly premium is 0.087 × \$125 or \$10.88.
(*c*) When premiums are paid monthly, the total premium paid each year would be 12 × \$10.88 or \$130.56. Hence, the saving by making one annual payment would be \$130.56 − \$125.00 or \$5.56.

Example 2

At age 15, Ima Millyunair's parents purchase for her a \$20 000 endowment policy to age 65. If premium payments are made semi-annually rather than quarterly, how much money would be saved over the life of the policy?

Solution

From the table of premiums, the annual premium on a \$20 000 endowment policy to age 65 issued at age 15 is 20 × \$14.98 or \$299.60.

From the table for periodic premiums, the quarterly premium is 0.26 × \$299.60 or \$77.90, and the semi-annual premium is 0.51 × \$299.60 or \$152.80.

The total premium paid each year
on a quarterly basis is \$ 77.90 × 4 or \$311.60
on a semi-annual basis is \$152.80 × 2 or \$305.60
∴ The total annual saving would be \$ 6.00

∴ The total saving over the 50 years the policy is in force would be 50 × \$6 or \$300.

C *Non-Forfeiture Benefits.* Under the level premium system, there is an accumulation of money that the insurance company sets aside to pay future claims. If a person decides to cancel his policy, he is entitled to a share of this money, according to a *non-forfeiture benefit.*

Term policies seldom have non-forfeiture benefits.

The table below illustrates the 4 ways in which non-forfeiture benefits may be paid on a whole life policy issued at age 20.

Similar tables exist for limited-pay life and endowment policies.

NON-FORFEITURE BENEFITS PER $1000 OF INSURANCE ON A WHOLE-LIFE POLICY ISSUED AT AGE 20.

END OF POLICY YEAR	CASH VALUE	PAID-UP INSURANCE	EXTENDED TERM INSURANCE	
			YEARS	MONTHS
1	$ 0	$ 0	0	0
2	0	0	0	0
3	1	5	0	2
4	8	37	1	4
5	14	64	2	4
10	52	216	10	0
15	102	342	15	8
20	165	444	21	3

The cash (*or surrender*) *value* is stated in the policy. This amount may be taken in cash when the policy is cancelled, or it may be used as the basis for a loan from the insurance company at any time the policy is in effect.

The cash value may also be applied to the purchase of a small amount of additional *paid-up insurance.* From the table, for instance, we see that a $1000 policy cancelled after 10 years has a cash value of $52, which may be used to purchase $216 worth of paid-up whole life insurance.

Whole life insurance, that is.

The same $52 will also purchase $1000 worth of *extended term insurance* for 10 years.

Exercise 23.2

Discussion

1. Which type of life insurance provides the greatest protection at the lowest cost?

2. State as many factors as possible that might affect the premium paid on a life insurance policy.

3. What is a beneficiary?

4. At what stage of a person's life might he or she require the most protection through life insurance?

5. What might be meant by the statement, "He's insurance poor"?

6. Joe Anyman is 28 years old and supports a wife and 2 young children. What type of life insurance should Joe purchase? Give reasons for your answer.

7. Why should premium rates increase with more frequent premium payments?

8. Why are some figures missing from the table of life insurance premiums on page 450?

9. Employees of a single company often buy life insurance under one policy covering the whole group; hence the term *group insurance*. Why is the premium usually less than that for a policy of a similar amount taken out by an individual member of the group?

Term? Whole-Life? Endowment?

Health? Age?

In just 10 years you could have a wife, (or husband) and a family!

Monthly payments cost more than semi-annual payments.

Written Solutions

10. Use the tables on page 450 to find the annual premium for each policy.

	POLICY	AGE AT ISSUE	FACE
(*a*)	15-year term	20	$10 000
(*b*)	whole life	35	15 000
(*c*)	endowment to age 60	50	15 000
(*d*)	limited-payment life	25	20 000
(*e*)	whole life	30	40 000

10(a) $42 (e) $699.20
(c) $1056.15

11. Calculate the periodic premium for each policy.

	POLICY	AGE AT ISSUE	FACE	PERIOD
(*a*)	whole life	40	$15 000	semi-annually
(*b*)	endowment to age 65	25	40 000	quarterly
(*c*)	10-year term	20	10 000	monthly
(*d*)	limited-payment life	35	25 000	quarterly
(*e*)	endowment to age 60	25	15 000	monthly

12. How much less will a person pay each year if he or she purchases a $20 000 whole life policy at age 20, rather than an endowment to age 60 policy with the same coverage?

13. How much money would be saved over the life of a $10 000 limited-payment life insurance policy purchased at age 35, if premium payments are made annually rather than semi-annually?

14. At age 25, Nathan Neverdie purchased a $20 000 limited-payment policy and, after making 5 annual payments, he died.
    (*a*) How much had he paid the insurance company in premiums?
    (*b*) How much did his beneficiaries receive?
    (*c*) How much more did his beneficiaries receive than he paid the insurance company?

15. Determine the non-forfeiture benefits from the table on page 334 for each whole life policy issued at age 20.

	POLICY CANCELLED AFTER	FACE	BENEFIT DESIRED
(*a*)	3 years	$15 000	Cash value
(*b*)	10 years	25 000	Paid-up insurance
(*c*)	20 years	15 000	Cash value
(*d*)	4 years	10 000	Extended term insurance

16. Penelope P. Prudence (age 35) is considering taking out either a limited-payment life or an endowment to age 65 policy for $20 000. Determine the difference in premiums on the 2 policies over the 30-year interval, if she paid premiums annually in each case.

11(a) $192.02 (e) $26.19
(c) $3.39
12. $80
13. $145.50
14(a) $1623 (c) $18 377
15(a) $15
(b) $5400
(c) $2475
(d) 1 year, 4 months
16. $7566

17. Compare the total premiums paid over the life of the policy, on a quarterly basis, for a $10 000 policy issued at age 20, for each of the following types of policies.
    (*a*) 15-year term, renewed at age 35
    (*b*) limited-payment life (to age 65)
    (*c*) endowment to age 60

That is, coverage is for 30 years.

18. Assuming that premiums are paid over the life of the $20 000 policy, which would be more expensive
    (*a*) a limited-payment policy purchased at age 55, with premiums payable semi-annually, or
    (*b*) an endowment to age 60 policy purchased at age 45, with premiums payable quarterly?

17(a) $1599 (c) $7005.44
(b) $6444

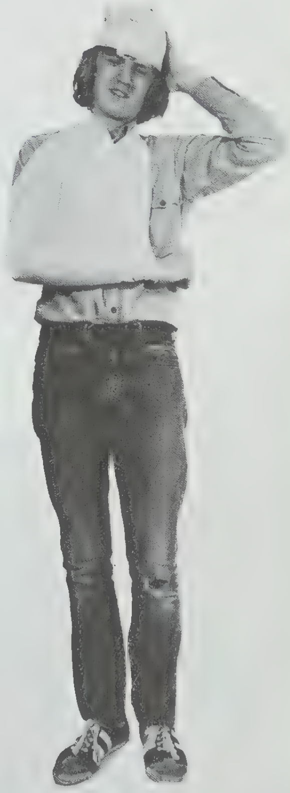
18. The limited-payment policy is more expensive by $1215.20.

Investigation

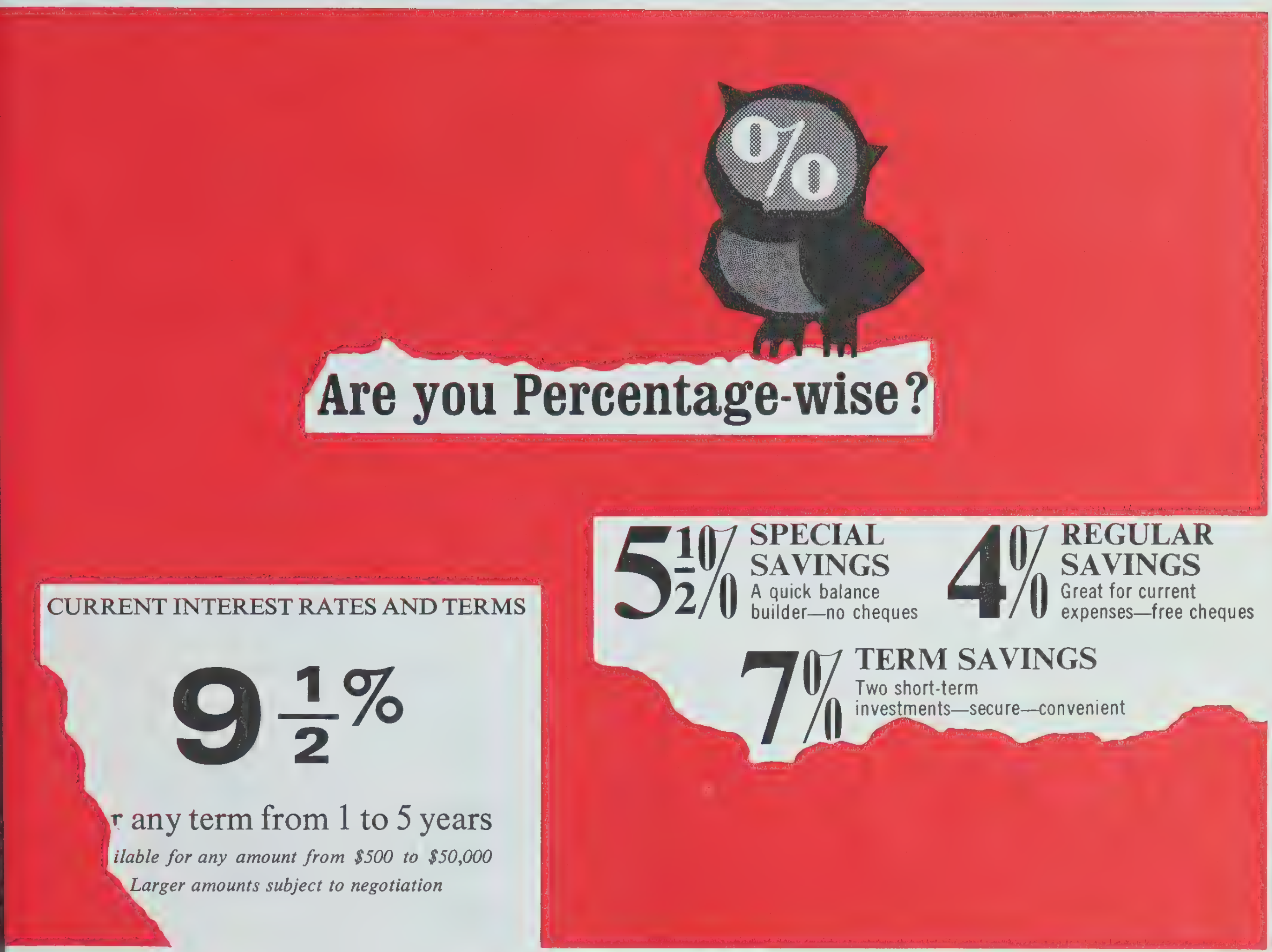
There are types of protection available other than those studied in this unit. Some are listed below.

workmen's compensation
sick leave
maternity leave
reinsurance
unemployment insurance
pensions
welfare
disability insurance
car and home insurance that include life insurance
mortgage insurance
insurance for snowmobiles, boats, cottages
Lloyd's of London, and unusual insurances
floater policy

Research a few of the above and write a short note on each.



24
4



Are you Percentage-wise?

CURRENT INTEREST RATES AND TERMS

$9\frac{1}{2}\%$

r any term from 1 to 5 years

ilable for any amount from $500 to $50,000

Larger amounts subject to negotiation

$5\frac{1}{2}\%$ SPECIAL SAVINGS
A quick balance builder—no cheques

4% REGULAR SAVINGS
Great for current expenses—free cheques

7% TERM SAVINGS
Two short-term investments—secure—convenient

Unit 24
Simple Interest

As consumers, we are all involved from time to time in the purchasing of goods or services provided by others. Occasionally we find it necessary to borrow money. You will recall that *interest* is money paid for the use of another person's money. Interest is an income to the lender, and an expense to the borrower.

%

Get-Set Exercise — Percent

Percent indicates the ratio of some number to 100. Hence, 8% may be expressed as $\frac{8}{100}$.

$$\therefore\ 8\% = \frac{8}{100} = 0.08$$

That is, a percent can be expressed either as a fraction or as a decimal.

1. State the fractional form (in lowest terms) of each of the following.
   (*a*) 3% (*c*) 0.06 (*e*) 10%
   (*b*) 5% (*d*) 0.13 (*f*) 0.25

Remember!

$\frac{1}{4} = \frac{25}{100} = 25\%$

2. State as percents.
   (*a*) $\frac{1}{4}$ (*c*) $\frac{1}{3}$ (*e*) 0.7
   (*b*) $\frac{3}{4}$ (*d*) 0.4 (*f*) 0.18

3. State the missing terms in each part of the table.

Percent	*Decimal*	*Fraction*
40%		
	0.07	
		$\frac{3}{5}$
15%		
	0.24	
		$\frac{7}{20}$

24.1 Calculating Interest

A Consider a situation in which a person wishes to buy a car. To do so, he or she borrows a sum of $1000 for a year at an interest rate of 12%; that is, the cost of the loan is $12/year for every $100 borrowed.

The amount borrowed ($1000) is the *principal*.
The period of the loan (1 year) is the *time*.
The percent (12%) is the *rate of interest* or *rate*.
The cost of the loan ($120) is the *interest*.
The sum of principal and interest ($1120) is the *amount*.

Note that rate of interest, unless otherwise stated, is always an annual rate (interest *per annum*).

Recall

$\because I = Prt$
$\therefore I = 1000 \times \frac{12}{100} \times 1$
$= 120$

∴ The interest is $120.

$\because A = P + I$
$\therefore A = 1000 + 120$
$= 1120$

∴ The amount is $1120.

B Problems involving percent occur often in our daily lives, as illustrated by the following examples.

Example 1

Millicent M. Matchmaker, owner of a Lonely Hearts club, pays her telephone bill monthly. If it is paid before the 10th of the month, she earns a cash discount of 3%. How much money will she save by early payment of a bill of $80.90?

Solution

The cash discount will be 3% of $80.90

$$= \frac{3}{100} \times \$80.90$$

$$= \frac{\$242.70}{100}$$

$$= \$2.43$$

Why to 2 decimals?

∴ Millicent will save $2.43 by early payment.

Example 2

The Kleer-Vu Cable Television Company offers one free month's viewing if you pay annually in advance. What percent savings does this represent for one year?

Solution

∴ Percent savings is $\frac{1}{12} \times 100\%$

$$= 8\tfrac{1}{3}\%$$

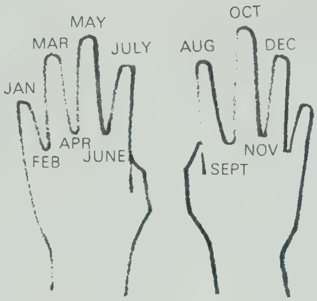
Why times 100%?

C The formula relating interest, principal, rate and time is called the simple interest formula.

$$\boxed{I = Prt}$$

I is interest. ($)
P is principal ($)
r is rate. (%)
t is time. (years)

In using this formula, you will encounter situations such as those illustrated in the following examples.



"Fingertips" are 31 days; "hollows" are 30 days (except Feb.)!

*
Why isn't $t=3$?

Look at (f)! Who'd ever pay that rate?

Example 3

Determine the number of days from March 12 to June 26.

Solution

Number of days remaining in March	19
Number of days in April	30
Number of days in May	31
Required days in June	26
Total	106

March has 31 day
∴ (31 − 12) or
19 days remain.

There are 106 days from March 12 to June 26.

Example 4

Harry Hardluck borrows $150 from Sylvester Softouch for a period of 3 months at 15% per annum. How much will the loan cost Harry?

Solution

$$\begin{aligned} I &= Prt \\ &= 150 \times \frac{15}{100} \times \frac{1}{4} \\ &= 5.63 \end{aligned}$$

$P = \$150$
$r = 15\%$
$t = \frac{1}{4}$ year*

∴ The loan will cost Harry $5.63.

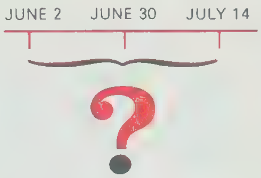
Exercise 24.1

Discussion

1. State the interest for 1 year.
   (*a*) $100 at 10% (*c*) $1000 at 5% (*e*) $2600 at 3%
   (*b*) $350 at 10% (*d*) $200 at 7% (*f*) $50 at 24%

2. State the interest on $6000 at 8% for each of the following periods.
   (*a*) 1 year (*c*) 6 months (*e*) 8 months
   (*b*) 3 years (*d*) 1 month (*f*) $\frac{3}{4}$ year

3. State the number of days in each of the following loan periods.
   (*a*) Sept. 11 to Sept. 25
   (*b*) June 2 to July 14
   (*c*) Jan. 19 to Feb. 1
   (*d*) April 27 to May 31



Written Solutions

4. Stella Stereo earns 8% commission on her total sales each week. How much will she earn on commission if her sales in 1 week total $3000?

5. Peter Paperpacker delivers papers and is paid 20¢ per customer per week. What are his earnings in 1 week if he has 110 customers? What percent does he earn of total sales if the paper sells for 80¢/week?

6. Sue Stashaway deposits $80 in her bank account and 1 year later finds that her account contains $86. What interest rate is the bank paying for the use of Sue's money?

What use does the bank make of Sue's money?

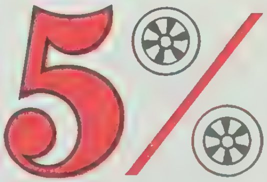
7. William Wellredd forgets to return a library book. The library charges 5¢/day for overdue books or 90% of the list (new) price for lost books. If the list price of William's book is $8, should he pay the fine after 150 days or purchase the borrowed book?

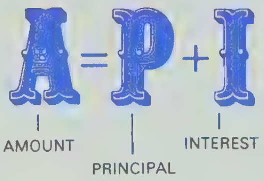
8. Determine the cost of a $500 loan at 12% for 5 months.

9. Beatrice Bargainhunter buys a dress on sale for $19.95, less 20% clearance discount. Passing another store on the way home, she sees an identical outfit at the regular price of $15.99. Did Beatrice buy too soon?

10. Sigmund Spinout wants to buy a set of charioteer wheel discs, regularly selling for $80, that are on a "1 day special" at a discount of 5%. To do so, however, he will have to borrow the money and repay it after 3 months at 10% interest. Will Sigmund save money by borrowing and buying now, or should he wait 12 days until payday to make his purchase?

4. **$240**
5. **$22, 25%**
6. **$7\frac{1}{2}$%**
7. **Purchase and save 30¢**
8. **$25**
9. **No, she saved 3¢**
10. **Borrow now and save $2.10**



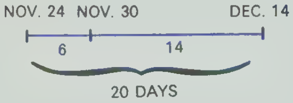


24.2 Finding the Amount

A When a loan is repaid, both the principal and interest must be paid to the lender. This sum of money is the *amount*.

Example 1

What sum of money is needed to repay a loan of \$500 at 15% from November 24 to December 14?



Solution

$$\begin{aligned} I &= Prt \\ &= 500 \times \frac{15}{100} \times \frac{20}{365} \\ &= 4.11 \end{aligned}$$

$\therefore$ The interest is \$4.11.

$\therefore$ The amount is \$500 + \$4.11 = \$504.11.

$P = \$500$
$r = 15\%$
$t = \frac{20}{365}$ years

Note that time must be in years!

1. Determine the amount after 1 year on a loan of \$1000 at the following rates.
   (*a*) 4% (*b*) 7.5% (*c*) $9\frac{3}{4}\%$ (*d*) 16%

B Money is often allowed to accumulate interest over several interest periods. The interest earned each period is added to the principal, and this amount becomes the new principal for the next interest period. Hence, each time an interest calculation is made, it will be made on a larger sum of money than the previous calculation. Such interest is said to be *compound interest*.

Consider the situation in which \$1000 is loaned at 10% per annum, and the interest is allowed to accumulate for 3 years. The table below shows the accumulated amount of the loan at the end of each year.

Interest Period	1	2	3
Principal	\$1000	\$1100	\$1210
Rate	10%	10%	10%
Time (years)	1	1	1
Interest	\$100	\$110	\$121
Amount	\$1100	\$1210	\$1331

∴ We say that \$1000 bearing interest at 10%, *compounded annually*, amounts to \$1331 in 3 years.

Interest may also be compounded semi-annually (twice a year), or quarterly (4 times a year), or even more frequently.

Example 2

Lender B. Ware makes a loan of \$200 to Wally Withit at 10%, compounded semi-annually. What amount will repay the loan after 2 years?

Solution

Interest Period	*1*	*2*	*3*	*4*
Principal	\$200	\$210	\$220.50	\$231.53
Rate	10%	10%	10%	10%
Time (years)	0.5	0.5	0.5	0.5
Interest	\$10	\$10.50	\$11.03	\$11.58
Amount	\$210	\$220.50	\$231.53	\$243.11

∴ \$243.11 will repay the loan after 2 years.

On a simple interest basis, however,

$I = Prt$

$= \overset{2}{\cancel{200}} \times \frac{10}{\cancel{100}} \times 2$

$= 40$

∴ The amount would be only \$200 + \$40 or \$240.

Exercise 24.2

Discussion

$$A = P + I$$

1. State the amount of a loan of \$1000 at 12% for the times shown.
   (*a*) 1 month
   (*b*) 3 years
   (*c*) 4 years
   (*d*) 18 months

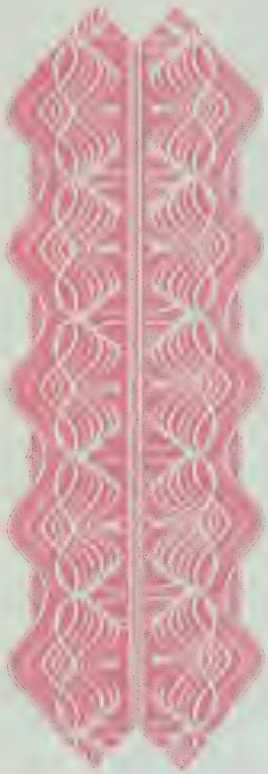
2. If you had a choice of borrowing money with interest compounded annually, semi-annually, or quarterly, which would you choose? Why?

3. If you had a choice of calculating the interest on \$500 at 9% for 3 years by (*a*) the simple interest method, or (*b*) compounding the interest annually, which method would you choose? Which method is easier?

Simple interest

$I = Prt$

where $t = 3$ years



Written Solutions

4. Calculate the amount of $800 at 10% after 1 year by
   (*a*) the method of simple interest, and
   (*b*) compounding the interest semi-annually.

5. Truman Trustworthy borrows $350 for 2 years at 9%, compounded annually. What amount will settle the debt after 2 years?

6. Trudy Tardy is late paying her hydro bill for $19.70 and must pay a late penalty of 3.5%. What amount must she pay to clear her account?

7. A bank offers a special Christmas Savings Plan and in order to encourage early participation, offers interest at 8% compounded quarterly. If Edna Earlybird deposits $100 in the plan at the start of the year, how much will she have saved to spend on gifts the following Christmas?

8. A $500 Canada Savings Bond bearing interest at $9\frac{3}{4}\%$, compounded semi-annually, is cashed after 3 interest periods with the coupons still attached. What is the amount received?

9. In a product promotion campaign, the Silent Sneaker Slipper Company offers a fabulous "Sweepstakes" cash prize of $1200. The advertising gives a table showing how this money, compounded annually at 6%, will amount to nearly $17 000 by the year 2025. Show the portion of the table for the first 4 years.

4(a) $880 (b) $882
5. $415.84
6. $20.39
7. $106.12
8. $573.14
9.

Year	*Amount*
1	$1272.00
2	1348.32
3	1429.22
4	1514.97

24.3 Manipulating Interest Formulas

In using the simple interest formula we are usually concerned with the calculation of interest (I), given the principal, rate, and time. In some cases, however, we may wish to calculate time or rate or principal, rather than interest. The examples following show how the interest formula may be rearranged to determine these other quantities.

If $P = \$100$
$r = 5\%$
$t = 1$ year
$\therefore I = Prt$
$= \cancel{100} \times \frac{5}{\cancel{100}} \times 1$
$= 5$
∴ Interest is $5.

Example 1

How many months will it take for \$1000 to earn \$30 interest at a rate of 12%?

That is, 12% per annum!

Solution

$$\begin{aligned} t &= \frac{I}{Pr} \\ &= \frac{30}{1000 \times \frac{12}{100}} \\ &= \frac{30}{120} \\ &= \frac{1}{4} \end{aligned}$$

$$\boxed{\begin{aligned} I &= \$30 \\ P &= \$1000 \\ r &= 12\% \end{aligned}}$$

If $I = Prt$

$\therefore \frac{I}{Pr} = \frac{\cancel{Pr}t}{\cancel{Pr}}$

$\therefore t = \frac{I}{Pr}$

∴ The time required is $\frac{1}{4}$ year or 3 months.

Example 2

A principal of \$800 amounts to \$822 after 3 months. Determine the interest rate charged.

If $A = P + I$, then $I = A - P$

Solution

$$\begin{aligned} I &= A - P \\ &= 822 - 800 \\ &= 22 \end{aligned}$$

∴ Interest is \$22.

$$\begin{aligned} r &= \frac{I}{Pt} \\ &= \frac{22}{800 \times \frac{1}{4}} \\ &= \frac{22}{200} \\ &= \frac{11}{100} \\ &= 11\% \end{aligned}$$

$$\boxed{\begin{aligned} I &= \$22 \\ P &= \$800 \\ t &= \tfrac{1}{4} \text{ year} \end{aligned}}$$

If $I = Prt$

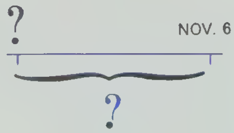
$\therefore \frac{I}{Pt} = \frac{\cancel{P}r\cancel{t}}{\cancel{Pt}}$

$\therefore r = \frac{I}{Pt}$

∴ The interest rate charged was 11%.

Example 3

On its date of issue, Frugal Freddy buys a \$500 Canada Savings Bond bearing interest at 9.5%, compounded annually, but is in need of cash a short time later and cashes the bond during the first year. If Freddy receives \$509.50 for the bond on November 6, find the date of issue of the bond.



$t = \frac{I}{Pr}$

where $I = ?$

$P = \$500$

$r = 9.5\%$

But $I = A - P$!

Solution

It is evident that the *time* must be calculated before the date may be determined.

$$\text{Since } A = P + I$$
$$\therefore I = A - P$$
$$= 509.50 - 500$$
$$= 9.50$$

$\therefore$ The interest is \$9.50.

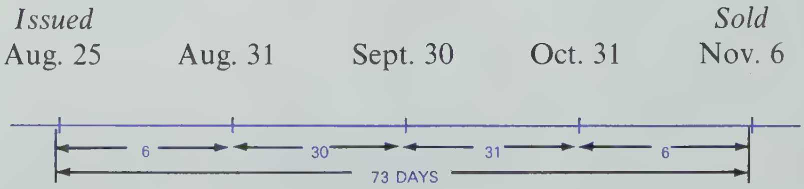
$$t = \frac{I}{Pr}$$
$$= \frac{9.50}{500 \times \frac{9.5}{100}}$$
$$= \frac{\cancel{9.50}}{\cancel{500}_{5}} \times \frac{\cancel{100}}{\cancel{9.5}}$$
$$= \frac{1}{5}$$

$\therefore$ The time is $\frac{1}{5}$ year or 73 days.

$\frac{1}{5}$ year

$= \frac{1}{5} \times 365$ days

$= 73$ days



$73 - (6 + 31 + 30)$

$= 73 - 67$

$= 6$

$\therefore$ 6 days before Aug. 31 (that is, Aug. 25)

Working back 73 days from November 6, we find the date of issue was August 25.

Exercise 24.3

Discussion

1. State the correct *units* when referring to each of the following in the simple interest formulas.
   (*a*) interest
   (*b*) principal
   (*c*) amount
   (*d*) rate
   (*e*) time

2. With reference to the formula A = P + I, state expressions for P and for I.

3. With reference to the formula I = P*rt*, state expressions for P, *r*, and *t*.

2. $A = P + I$
$P = ?$
$I = ?$

3. $I = Prt$
$P = ?$
$r = ?$
$t = ?$

Written Solutions

4. Find the principal that will earn $40 interest in 4 months at 8%.

5. Draw and complete the following table.

Problem	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)	(*h*)	(*i*)	(*j*)
Interest			$3.00	$3.50	$2.50	$70	$1.20	$4.50		$8
Principal	$50	$300	$150	$75		$1000		$150	$4000	$400
Rate	12%	10%	8%	14%	12%		6%		7%	
Time (years)	1	0.5			$\frac{1}{6}$	1	$\frac{1}{4}$	$\frac{1}{3}$	0.5	$\frac{1}{6}$

Remember
$t = \frac{I}{Pr}$ years
$= \frac{I}{Pr} \times 365$ days

6. Determine the amount of a $600 loan at 11% made on May 10 and paid on October 3.

7. A $2500 Savings Bond is issued on July 9 and bears interest at 7%, compounded semi-annually. Find the date on which the bond will yield $2523.97.

8. Using your experience in computer programming, write a program for determining interest from the formula I = P*rt*. Can you also produce a program for determining P, *r*, or *t*?

4. $1500
5(a) $6
(c) $\frac{1}{4}$ year
(e) $125
(g) $80

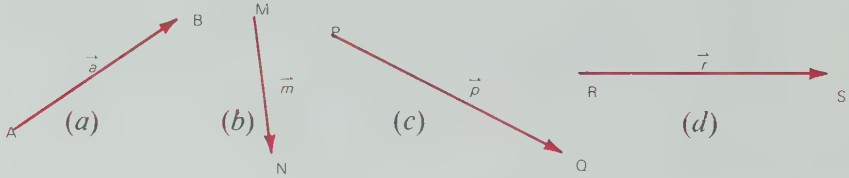
6. $626.40

7. August 28

Maintaining Basic Skills

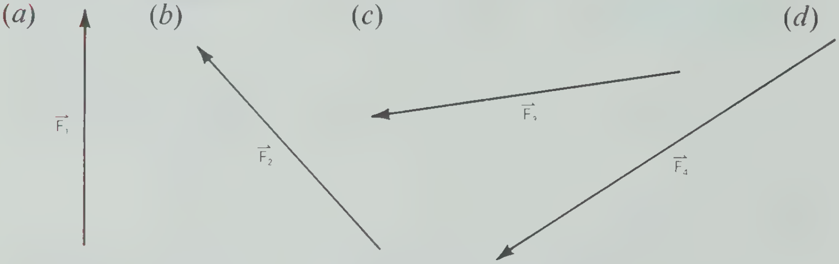
Exercise 5

1. Name each vector in 2 or more ways.



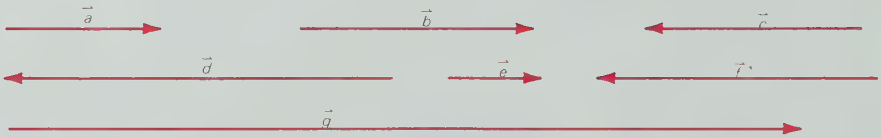
B
M
P
$\vec{a}$
$\vec{m}$
$\vec{p}$
$\vec{r}$
R
S
A
(a)
(b)
(c)
(d)
N
Q

2. Using a scale of 1 cm = 10 N, determine the size of the force represented by each vector.



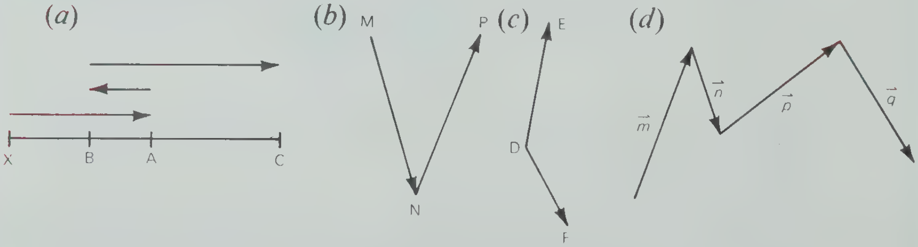
(a)
(b)
(c)
(d)
$\vec{F}_1$
$\vec{F}_2$
$\vec{F}_3$
$\vec{F}_4$

3. Using a 1 cm scale, express each given vector as a scalar multiple of $\vec{a}$.



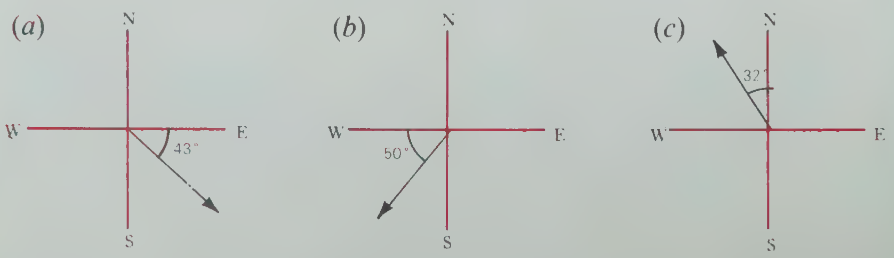
$\vec{a}$
$\vec{b}$
$\vec{c}$
$\vec{d}$
$\vec{e}$
$\vec{f}$
$\vec{g}$

4. Show the resultant of each system of vectors.



(a)
(b)
M
P
(c)
E
(d)
X
B
A
C
N
D
F
$\vec{m}$
$\vec{n}$
$\vec{p}$
$\vec{q}$

5. Determine the bearing and its opposite.



(a)
N
W
E
43°
S
(b)
N
W
E
50°
S
(c)
N
32°
W
E
S

1(a) $\vec{a}$, $\overrightarrow{AB}$, $-\overrightarrow{BA}$
2(a) 30 N
3. $\vec{b} = 1.5\,\vec{a}$
$\vec{d} = -2.5\,\vec{a}$
$\vec{f} = -1.8\,\vec{a}$
5(a) 133°, 313°

6. Calculate the amount of the premium returned* by the company in each case. Use the tables on pages 448 and 449.

	Face	*Construction*	*Type of Policy*	*Policy Cancelled After*
(*a*)	$25 000	Frame	standard	8 months
(*b*)	41 000	Brick	deluxe	130 days
(*c*)	53 000	Brick	standard	257 days

7. Flame Fizzleout insured her apartment and its contents under a standard $18 000 Homeowner's Policy, even though its value was $28 000. If the apartment has a "brick" rating and is damaged by fire to the extent of $14 000, how much of this loss would Miss Fizzleout have to pay? Assume an 80% co-insurance clause is contained in the policy.

8. Find the annual premium and the total premium paid over the life of each of the following life insurance policies.

	Face	*Age at Issue*	*Type of Policy*	*Premium Payment*
(*a*)	$10 000	19	whole life	annually
(*b*)	25 000	32	limited-payment	quarterly
(*c*)	30 000	40	endowment to age 60	monthly

9. Copy and complete the table.

	Percent	*Decimal*	*Fraction*
(*a*)			$\frac{4}{5}$
(*b*)		0.875	
(*c*)			$\frac{19}{20}$
(*d*)		1.35	
(*e*)	$16\frac{2}{3}\%$		

10. Copy and complete the table.

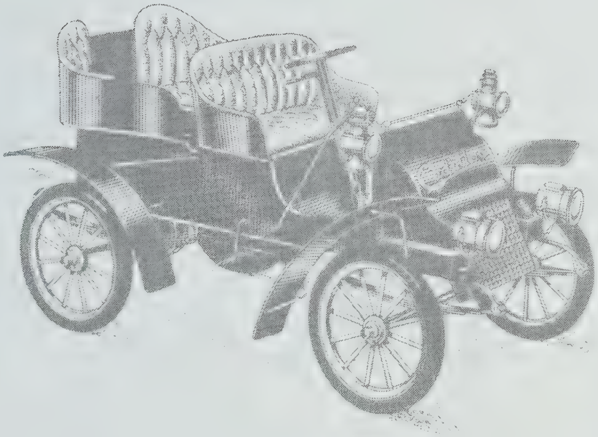
Problem	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
Interest		$2.50	$6.40		$42
Principal	$180	$200	$480	$600	
Rate	6%	5%		7.5%	8%
Time	4 months		2 months	46 days	5 months

11. Determine the amount of a $1500 loan at 12% made on July 16 and paid on September 27.

*Careful here!

6(a) $18.36 (c) $32.40
7. $2750
8(a) $125, $8250
(c) $1207.44, $24 148.80
9(a) 80%, 0.8 (e) $0.1\dot{6}$, $\frac{1}{6}$
(c) 95%, 0.95
10(a) $3.60 (e) $1260
(c) 8%
11. $1536

25



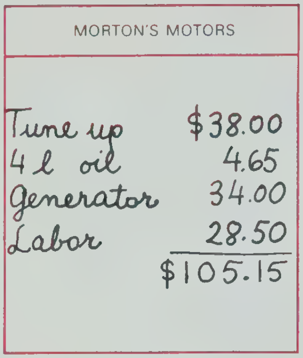
Unit 25
Owning an Automobile

The mobility of our modern society depends to a very great extent upon the automobile. During the past 70 years, automobiles have caused more changes in daily living than any other machine.

Since automobile ownership imposes a heavy financial burden on the owner, careful consideration should be given to the various factors involved in purchasing and operating an automobile.

France claims the first automobile, driven by steam and having 3 wooden wheels. It was invented about 200 years ago.

25.1 Operating Expenses


MORTON'S MOTORS

Tune up	$38.00
4 l oil	4.65
Generator	34.00
Labor	28.50
	$105.15

Operating an automobile involves expenses such as insurance, licence, repairs, gas, oil, lubrication, and depreciation.

Day to day expenses such as gas, oil, lubrication, and repairs are direct costs necessary to keep the automobile in working order.

Licence and insurance fees are annual costs, and will vary from province to province, depending upon government legislation, size of the automobile, use of the automobile, and other factors.

Depreciation refers to the loss in value due to use and the passage of time. An automobile generally decreases in value as it grows older; wear on moving parts of the engine and deterioration of body parts due to corrosion are contributing factors here.

Usually the more expensive the car, the greater the rate of depreciation. An average automobile will depreciate so that its value at the end of 5 years is only about 25% or 30% of its original cost price.

The table below gives approximate annual depreciation rates for a medium-priced automobile.

End of Year	*Value of Automobile (Percent of Original Cost)*
1	70%
2	56%
3	42%
4	33%
5	26%

Example 1

Mr. Willet Rustout purchased a new car for $4200 cash. Find its depreciated value at the end of 3 years, using the table above.

Solution

The value of Mr. Rustout's car at the end of 3 years would be 42% of $4200 or $1764.

$$\frac{42}{100} \times 4200 = 1764$$

Example 2

Rhonda Roadrunner drives a truck carrying fresh produce from the Okanagan Valley to the northern parts of British Columbia and drives about 100 000 km every year. Rhonda estimates yearly expenses on her truck as follows: gas, oil, and lubrication, $5000; tires, $2400; insurance, $465; licence, $60; repairs, $400; depreciation, $2400. Determine

(*a*) Rhonda's average monthly cost of operating her truck, and
(*b*) the cost/100 km to operate the truck.

Solution

(*a*) gas, oil, lubrication	$ 5 000
tires	2 400
insurance	465
licence	60
repairs	400
depreciation	2 400
	$10 725

The average monthly cost of operation is $10 725 ÷ 12 or $892.92.

(*b*) The cost/100 km is $10 725 ÷ 1000 or $10.73/100 km.

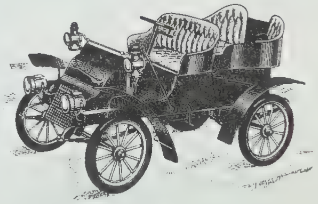
Exercise 25.1

Discussion

1. (*a*) Why would an automobile depreciate at the greatest rate during its first year?
   (*b*) Do all makes of cars depreciate at the same rate?
   (*c*) Give an example of a car that *appreciates* with age.

2. Determine the approximate cost associated with each of the following items.
   (*a*) 1 ℓ of gasoline
   (*b*) a tankful of gasoline
   (*c*) an oil change
   (*d*) a brake job
   (*e*) repair of a flat tire
   (*f*) a new set of tires
   (*g*) wheel alignment
   (*h*) a new set of spark plugs
   (*i*) replacement of a fender
   (*j*) replacement of a window

3. What factors might influence the licence fee or the insurance cost for a car?



Original cost?
Price today?

4. $3850, $3080, $2310, $1815, $1430

5. 3-year-old car

6(a) $122.92, 6.4¢
(b) $79.42, 6.8¢
(c) $215.33, 8.6¢
(d) $70.42, 5.6¢

7(a) $65 (b) 13.7¢

8. $8600

Written Solutions

4. Using the table on page 354, calculate the depreciated value of a car at the end of each of the first 5 years of its life, if its original cost was $5500.

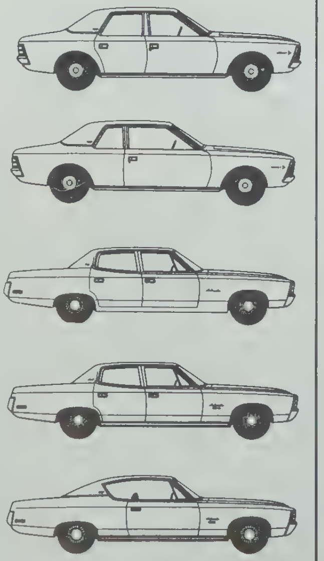
5. Which would have the greater value: a car 3 years old whose original cost was $4100, or a car 5 years old whose original cost was $5000?

6. For each situation indicated in the table, calculate the average monthly cost of operation and the operating cost/100 km. All figures shown represent annual costs.

Car	*Gas, Oil, Lubrication*	*Insurance*	*Licence*	*Repairs*	*Depreciation*	*Annual Distance Driven (km)*
A	$420	$225	$30	—	$ 800	23 000
B	316	153	27	39	418	14 000
C	732	362	45	285	1160	30 000
D	270	104	25	21	425	15 000

7. Millicent Memorymaster, a retired schoolteacher, drives 5700 km in 1 year. She always purchases her gasoline from the same service station for $0.16/ℓ, and averages 15ℓ/100 km. She is very careful to have her oil changed once every 3 months and uses 4 ℓ of top grade 5-W-30 oil costing $1.39/ℓ. She spends $92 for insurance, $124 for repairs, and $45 for licence fees. If these figures all apply to the fourth year in which she owns the car (which cost $4000 new), calculate
(*a*) the average monthly cost of operating the car, and
(*b*) the cost/100 km to operate the car.

8. The Farenwide Trucking Company consists of a fleet of 8 trucks, all 3 years old. If the depreciated value of the fleet is $28 896, determine the original cost price of each truck. Use the depreciation table on page 354.



25.2 Insuring an Automobile or Motorcycle

A *Automobile Insurance*

Even the best drivers may be involved in automobile accidents, and most drivers buy insurance to protect themselves financially from the costs of an accident. Automobile insurance may be divided into 2 broad categories as indicated in the following table.

Protection for Others		*Protection for the Owner*
Public liability insurance	Third-party liability insurance	Medical
Property damage insurance		Collision or Upset
		Comprehensive

Public Liability insurance protects the owner against claims made on behalf of persons injured or killed in an accident involving the insured car.

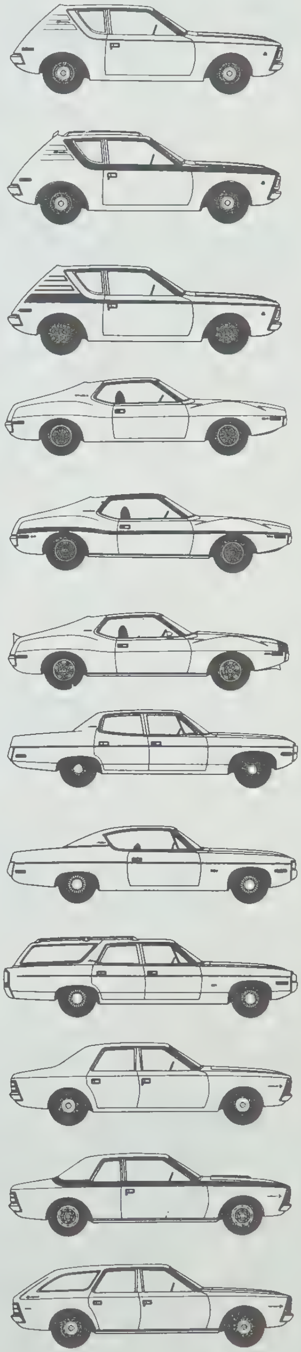
Property Damage insurance protects the owner against claims arising from an accident in which the insured car damages property of others.

Medical insurance pays for medical or funeral expenses of a person riding in the insured car at the time of an accident.

Collision insurance pays for damages to the owner's own automobile.

Comprehensive insurance covers loss or damage to the insured vehicle by a means other than collision or upset. This coverage includes loss caused by fire, theft, wind, and glass breakage.

With collision and comprehensive insurance, there is usually a *deductible clause* in the insurance policy that states that the owner must pay the first portion (usually \$25, \$50, or \$100) of the repair costs, and the insurance company will pay the remainder.



The tables below give typical annual rates or *premiums* for various types of insurance.

Why is such a large sum needed? →

Table 1 Typical annual premiums for a $200 000 inclusive Public Liability and Property Damage (PL and PD) Insurance policy.

Notice that the age of the car has no effect on the premium!

	URBAN AREA			RURAL AREA		
AGE OF DRIVER(S)	AUTOMOBILE USED FOR			AUTOMOBILE USED FOR		
	BUSINESS	COMMUTING	PLEASURE	BUSINESS	COMMUTING	PLEASURE
Over 25	$129	$104	$ 81	$108	$ 82	$ 67
One over 25; one under 25	$250	$200	$159	$210	$161	$132
Under 25	$310	$283	$245	$262	$214	$195

Table 2 Typical annual premiums for a $50-deductible Collision Insurance policy.

"Old" is 5 years!

	URBAN AREA						RURAL AREA					
AGE OF DRIVER(S)	AUTOMOBILE USED FOR						AUTOMOBILE USED FOR					
	BUSINESS		COMMUTING		PLEASURE		BUSINESS		COMMUTING		PLEASURE	
	NEW	OLD	NEW	OLD	NEW	OLD	NEW	OLD	NEW	OLD	NEW	OLD
Over 25	$101	$ 51	$ 75	$ 38	$ 56	$ 24	$ 90	$ 45	$ 69	$ 35	$ 55	$ 23
Under 25	$220	$ 88	$220	$ 88	$220	$ 88	$212	$106	$212	$106	$212	$106

Table 3 Typical annual premiums for a Comprehensive Insurance policy.

Why does it cost more if you're under 25?

AGE OF DRIVER	URBAN AREA		RURAL AREA	
	NEW	OLD	NEW	OLD
Over 25	$17	$ 9	$19	$10
Under 25	$23	$12	$26	$13

The *medical insurance* premium is a flat rate of $3 for $5000 coverage.

It should also be noted that some insurance companies offer up to 20% discount on premiums for students who have successfully completed a driver-training course.

That's nice to know!

Example 1

Theodore Throttlepusher, age 22, having passed his driver-training course, buys a new car and obtains insurance coverage for it as indicated.

PL and PD $200 000 inclusive
$50-deductible collision
Comprehensive insurance
$5000 medical insurance

Public Liability and Property Damage are often shortened to PL and PD

If Theodore lives in an urban area and drives his car mainly to and from work, calculate his total annual insurance premium. (The insurance company allows a 15% discount due to the driver-training course certificate.)

Solution

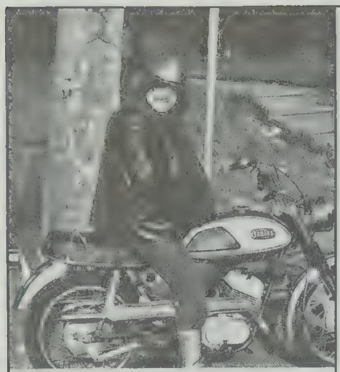
Coverage	*Premium*
PL and PD	$283
Collision	220
Comprehensive	23
Medical	3
	$529
Less 15% discount	79.35
Total premium payable	$449.65

from Table 1
from Table 2
from Table 3

Due to driver training certificate!

∴ Theodore's annual insurance premium is $449.65.

* * *



B *Motorcycle Insurance*

Although automobiles remain a favorite form of transportation for most young people, many teen-agers find the motorcycle (or motorbike) an exciting alternative.

Motorcycles may not be as comfortable as cars and certainly do not have the same passenger-carrying ability; nevertheless, they are fun to ride, relatively inexpensive to purchase and operate, and have the added feature of being able to travel trails and pathways inaccessible to the automobile.

Insurance rates for motorcycles are based on corresponding rates for automobiles, as indicated below.

Table 4 PL and PD Insurance ($200 000 inclusive) These rates are based on "pleasure" rates for automobiles and then adjusted as indicated. Premium = Basic rate × Adjustment factor

(*a*) Basic rate

AGE	URBAN	RURAL
Over 25	$ 81	$ 67
Under 25	$245	$195

(*b*) Adjustment factor

BIKE SIZE *	FACTOR
0 - 100	40%
101 - 200	60%
201 - 500	100%
above 500	125%

* Bikes are rated by piston displacement. "500" means 500 cm^3.

Table 5 Collision Insurance ($50-Deductible)
Premium = Basic rate × Adjustment factor

(*a*) Basic rate

AGE	RATE
Over 25	$ 56
Under 25	$118

(*b*) Adjustment factor

LIST PRICE	FACTOR
$0 - $ 250	40%
251 - 500	60%
501 - 1000	80%
1001 - 2000	100%
above 2000	125%

Table 6 Comprehensive Insurance

These figures apply to a new bike only!

LIST PRICE	PREMIUM
$0 - $ 250	$12
251 - 500	$18
501 - 1000	$24
1001 - 2000	$30
above 2000	$36

Medical Insurance There is a flat rate of $3 for $5000 coverage.

Example 2

Ima Cyclebuff pays $1200 for a "300" motorbike on her 19th birthday, since she requires transportation to and from work in downtown Vancouver. If Ima purchases $200 000 of PL and PD insurance, $50-deductible collision, comprehensive and medical insurance, determine her annual insurance premium.

Solution

From the tables on page 360, Ima's annual premium will be calculated as follows.

PL and PD ($200 000)	$245 × 100% =	$245	← from Table 4
Collision ($50-deductible)	$118 × 100% =	118	← from Table 5
Comprehensive		30	← from Table 6
Medical		3	
Total		$396	

∴ Ima's annual premium is $396.*

* WOW! (Compare this to the value of Ima's bike!)

Exercise 25.2

Discussion

1. Explain the influence of each of the following factors on the insurance premium for an automobile.
   (*a*) area or location
   (*b*) type of automobile (sports car or family sedan)
   (*c*) age of the driver
   (*d*) use of the automobile (business or pleasure)
   (*e*) the driver's accident record
   (*f*) an increasing accident rate for the area
   (*g*) driver-training certificate held by driver

2. At what stage in the life of an automobile or a motorcycle would it be unwise to continue to purchase collision insurance?

3. For collision insurance, the higher the amount that is deductible, the lower the premium. Explain.

4. Why should PL and PD insurance premiums for motorbikes increase with the size of the engine?

5. Why should collision insurance premiums for motorbikes increase with the list price of the bike?

Use the tables!

Written Solutions

6. Calculate the total annual insurance premium for each situation shown below. (Assume a new car in each case, driven in the city.)

Car	Used for	Age of Driver	Insurance Coverage Required			
			PL and PD	Collision	Comprehensive	Medical
(a)	Pleasure	45	$200 000	$50-Deductible	Yes	No
(b)	Commuting	18	200 000	No	No	Yes
(c)	Business	36	200 000	50-Deductible	Yes	No
(d)	Commuting	24	200 000	50-Deductible	Yes	Yes

7. Determine the total annual premium in each case. (Assume a new bike.)

Engine size is given in cubic centimetres

Bike	List Price	Engine Size	Use	Age of Driver	Insurance Coverage Required			
					PL and PD	Collision	Comprehensive	Medical
(a)	$ 550	80	Urban	33	Yes	Yes	Yes	Yes
(b)	1185	450	Rural	22	Yes	No	No	No
(c)	1850	750	Rural	19	Yes	Yes	No	Yes
(d)	1300	500	Rural	28	Yes	Yes	Yes	No

8. Careful Carl has a 5-year old sedan that he drives to work daily within the city. Once a month he drives 50 km to visit his son at college. Each year Carl purchases a $200 000 PL and PD policy, which includes $50-deductible collision and comprehensive coverage, and $5000 medical coverage. Calculate Carl's annual insurance premium.

9. Ted and Teresa Travelmore are 20-year-old twins living in an urban center. For transportation to and from work, Teresa purchases a new compact car for $2500, and Ted buys a new "350" motorbike for $1200. Who pays more in annual premiums for $200 000 PL and PD, $50-deductible collision, comprehensive, and medical insurance?

10. Thelma Thinker, age 18, moves from the country to the city to attend university. Thelma had just purchased $200 000 PL and PD, $50-deductible collision, and comprehensive insurance on her new "500" motorbike worth $1050. Calculate the increase in her annual insurance premium.

6(a) $154 (d) $529
(b) $286
7(a) $104.20 (d) $153
8. $154
9. Teresa pays $133 more.
10. $50

11. E. Z. Ryder purchases insurance for his new "750" motorcycle (worth $1500), given to him on his 21st birthday, as indicated below.

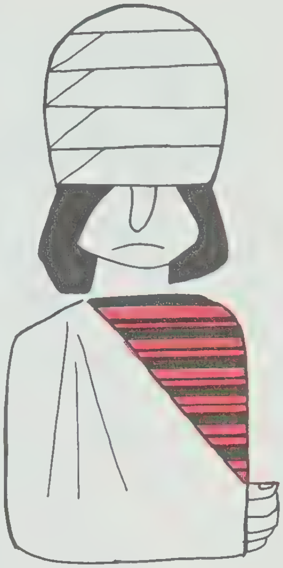
    $200 000 PL and PD
    $50-deductible collision and comprehensive
    $5000 medical

    (*a*) If E.Z. drives his motorcycle only to and from work within the city, calculate his annual premium.

    (*b*) Mr. Ryder is involved in an accident that causes $280 damage to his bike and $575 damage to the other person's car, and incurs $1285 medical expenses to his passenger, who suffered facial cuts, a broken leg, and 3 fractured ribs. If Mr. Ryder is found to be totally at fault, determine

    (*i*) the total amount paid out in claims by E.Z.'s insurance company, and

    (*ii*) the cost of the accident to E.Z. himself.



12. I. M. Lucky pays $55 each year for 4 years for a $100-deductible collision insurance policy. At the end of the fourth year, Ms. Lucky has an accident that results in damages of $843. How much did she save by carrying the insurance for 4 years?

13. Nineteen-year-old Graham Graduate completes a driver-training course successfully and is allowed an 18% discount on the total insurance premium for his new car. Graham plans to use his car for pleasure driving only, within the city. Calculate his total annual insurance premium for a $200 000 PL and PD, $50-deductible collision, $5000 medical policy.

Remember?
ha means
"hectares"

14. Farmer Fulton and his 4 children manage a 100 ha farm. Mr. Fulton's eldest son has just become eligible to drive the family car in helping with the business of the farm. Since the car is 5 years old, Mr. Fulton does not purchase collision insurance, but obtains $200 000 PL and PD and $5000 medical coverage. What was Mr. Fulton's increase in premium due to the fact that his son was able to drive?

11(a) $457.25
(b) (i) $2090
(ii) $50
12. $523
13. $383.76
14. $102

*The information in this section would also apply to the purchase of a motorcycle!

25.3 Purchasing an Automobile*

The decision to purchase an automobile is one that should be made with great care. In your lifetime, you may spend many thousands of dollars providing convenient transportation in the form of a car.

Some costs involved in owning an automobile are listed below.

Remember insurance, depreciation, and operating costs, too!

1. *Initial cost.* This is the cash price of the automobile.
2. *Interest charges.* These may be considered in terms of
   (*a*) forgone interest; that is, lost interest that might have been earned had the money been used for investment, and
   (*b*) interest paid for borrowed money when the car is bought on instalment.
3. *Replacement cost.* Usually an automobile has some trade-in value when it is replaced with a new car. Hence, the replacement cost will not be the full cash price of the new automobile.

Example 1

Kenneth Kompact purchases a new car worth $4000 by paying $1000 down, and paying off the balance in 24 monthly instalments of $135 each. If Kenneth had been able to invest the $1000 down payment at 7%/year, determine the total interest cost.

Solution

The interest lost on the $1000 down payment would be

$$\begin{aligned} I &= Prt \\ &= \$1000 \times \frac{7}{100} \times 2 \\ &= \$140 \end{aligned}$$

$P = \$1000$
$r = 7\%$
$t = 2$ years

$t = 2$ years because 24 months $= 2$ years

The cost of buying the car by instalments (the carrying charge) would be

$$\begin{aligned} &\text{"Time" price} - \text{Amount owing} \\ &= 24\ (\$135) - \$3000 \\ &= \$3240 - \$3000 \\ &= \$240 \end{aligned}$$

$$\underset{\substack{\uparrow \\ \text{total} \\ \text{cost}}}{\$4000} - \underset{\substack{\uparrow \\ \text{down} \\ \text{payment}}}{\$1000} = \underset{\substack{\uparrow \\ \text{owing}}}{\$3000}$$

∴ The total interest cost to Kenneth is $140 + $240 or $380.

Example 2

Due to the nature of her job, Connie Commuter requires a car only from Monday to Friday. Connie estimates that, if she purchases a car, annual expenses will be as follows: $1000 average "cash price" cost; $60 in lost interest; $216 for insurance; $750 for operating expenses and repairs; $35 for licence fees. On the other hand, she can rent a car for $102/month, including insurance, with gasoline expenses estimated at $465 for the year. Should Connie lease a car or buy one? What amount would she save annually by choosing the less expensive plan?

In leasing arrangements, all operating costs, except the cost of gasoline, are paid by the leasing company.

Solution

The total costs in purchasing a car would be calculated as follows.

"Cash price"	$1000
Lost interest	60
Insurance	216
Operating expenses	750
Licence fees	35
	$2061

The total rental cost would be

Annual rental cost + Gasoline expenses

= 12 ($102) + $465

= $1224 + $465

= $1689

∴ Connie should lease a car. She will save $372 annually.

Exercise 25.3

Discussion

1. What factors should be considered in deciding whether to buy a new car or a used car?
2. What are some of the advantages of owing your own car? Of leasing a car?

Condition? Repairs? Initial cost? Prestige? Anticipated lifetime? Insurance?

Don't forget to calculate average "cash price" costs.

Remember depreciation? See page 354.

3. $880
4. $135
5. $8\frac{1}{2}\%$
6. $15.53/100 km

Written Solutions

3. Find the total interest cost to Dawn Brakes who purchases a new car worth $3800 by paying $1500 down and paying the balance in 30 monthly instalments of $96 each, allowing for lost interest at the rate of 8%.

4. Percival Parameter purchases a car for $2700 and estimates its annual replacement cost at $500. Over the first 3 years he has the car, Percival's annual expenses are as follows: lost interest, $250; insurance, $190; operating expenses, $630; and licence fees $30. If Percival had been able to lease a car for $165/month, with annual gasoline expenses of $475, how much would he have saved over the 3-year period?

5. Mabel Moneywise purchases a new car worth $3600 by paying $800 down, and paying off the balance in 18 monthly instalments of $170 each. If Mabel calculates the total interest cost (lost interest plus carrying charges) to be $362, what interest rate did she use to determine the interest lost on the $800 down payment?

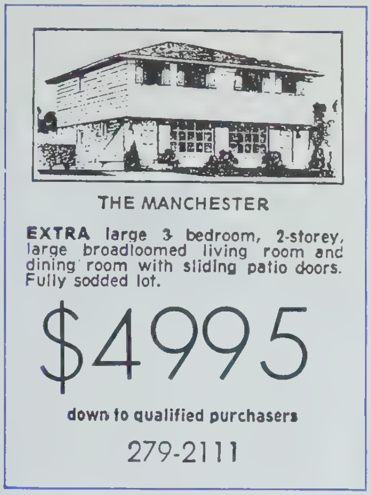
6. N. E. Wunn, a 24-year-old executive living in an urban center, purchases a new car worth $4200 by paying $1200 down and paying off the balance in 24 monthly instalments of $150 each. Mr. Wunn estimates that it costs him an average of $700 annually for operating expenses (gasoline, oil, lubrication, licence). Since he will be using the car for business, he purchases insurance coverage as follows: $200 000 PL and PD, $50-deductible collision, $5000 medical. If Mr. Wunn drives 29 000 km each year, determine the average cost/100 km for his transportation over the first 2 years he owns the car. (Include down payment, lost interest at 8%, insurance, depreciation, and operating costs.)

7. Contact a local automobile or motorcycle dealer and price the car or bike of your choice. Having obtained specifications and costs concerning the machine, approach a local insurance company to determine the type and cost of appropriate insurance. Compare your results with those of your classmates.

More Nutty Math: An Apitude Test

1. Do they have a 1st of July in England?
2. Why can't someone living in Toronto be buried west of Winnipeg?
3. How many birthdays does the average person have?
4. If you had only one match and entered a room in which there was a kerosene lamp, an oil burner and a wood burning stove, which would you light first?
5. Some months have 30 days and some have 31. How many have 28 days?
6. If a doctor gave you 3 pills and told you to take 1 every half hour, how long would they last?
7. A person builds a rectangular house. Each side has a southern exposure. A big bear comes wandering by. What color is the bear?
8. How far can a dog run into the woods?
9. What is the minimum number of active baseball players on the field during any part of an inning? How many outs in each inning?
10. I have in my hand 2 Canadian coins that total 55¢ in value. One is not a nickel. What are the coins?
11. A farmer had 17 sheep. All but 9 died. How many were left?
12. How many animals of each species did Moses take aboard the Ark with him?
13. Two people playing checkers played 5 games, and each won the same number of games. How can this be?
14. Take 2 apples from 3 apples and what do you have?
15. An archaeologist claimed to have found some gold coins dated 55 B.C. Do you believe this? Why?
16. A woman gives a beggar 50¢. The woman is the beggar's sister, but the beggar is not the woman's brother. How can this be?
17. What word is misspelled in this test?

26

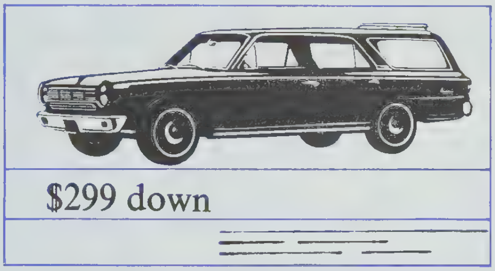

THE MANCHESTER

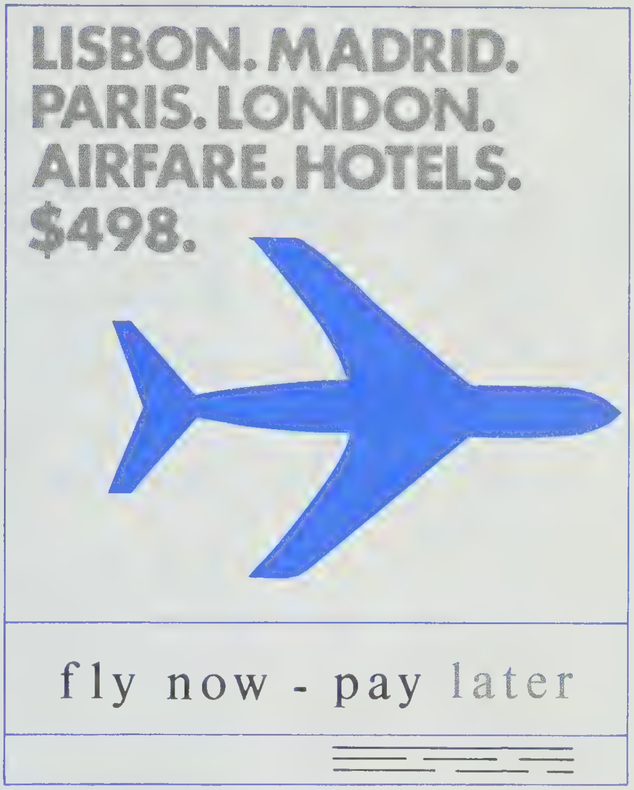
EXTRA large 3 bedroom, 2-storey, large broadloomed living room and dining room with sliding patio doors. Fully sodded lot.

$4995

down to qualified purchasers

279-2111


$299 down


LISBON. MADRID. PARIS. LONDON. AIRFARE. HOTELS. $498.

fly now - pay later

Unit 26
Direct Financing

Most of us attempt to control our expenses and keep them within our income. From time to time, however, we may be justified in purchasing one item or another "on time." That is, we may wish to enjoy the use of an article bought sooner than we could purchase it for cash, or it may be advantageous for us to purchase an article now because of an anticipated increase in its price in the future.

Such purchases fall into the category of *credit buying*, in which *financing* of one form or another may be involved.

In this unit we will deal with instances in which credit is given directly by the company supplying the goods or services. This is called *direct financing*.

26.1 Discounts on Cash Sales

A In spite of the trend to easy credit, there are individuals or companies that still offer cash discounts for prompt payment of bills or invoices. Such companies usually depend upon cash for the day-to-day operation of their businesses, and offer cash discounts to anyone settling an account by a given date. Many utilities such as gas, water, and electric companies offer cash discounts.

A sample invoice is shown at the bottom of the page. The *terms* of the invoice are usually clearly stated. Here are some common terms, and their meanings.

The net amount is the amount billed.

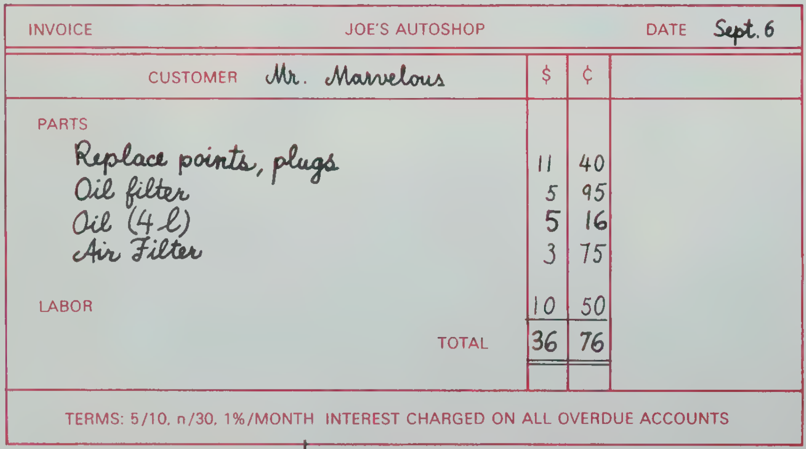
Terms 2/10, n/30. Here 2/10 means that the customer will receive a cash discount of 2% if the goods or services are paid for within 10 days of the billing date. The n/30 indicates that the *net amount* of the invoice must be paid within 30 days.

"e.o.m." means "end of month".

Terms 3/15 e.o.m., n/30 e.o.m. In this case, a 3% discount is offered if the bill is paid within 15 days of the end of the month, and the net amount is due within 30 days of the end of the month.

In addition to the terms of the invoice, some firms specify a late payment penalty (usually 1% per month) that will be charged on overdue accounts.

1. State the meaning of each of the following invoice terms.
   - (*a*) 5/10, n/30
   - (*b*) n/90
   - (*c*) 5/10 e.o.m., n/60
   - (*d*) net cash



INVOICE	JOE'S AUTOSHOP	DATE Sept. 6	
CUSTOMER Mr. Marvelous		$	¢
PARTS			
Replace points, plugs		11	40
Oil filter		5	95
Oil (4 l)		5	16
Air Filter		3	75
LABOR		10	50
	TOTAL	36	76

TERMS: 5/10, n/30, 1%/MONTH INTEREST CHARGED ON ALL OVERDUE ACCOUNTS

Example 1

The invoice is for automobile repair work at Joe's Autoshop. Determine the amount required to pay the invoice on

(*a*) Sept. 10,
(*b*) Oct. 1,
(*c*) Nov. 6.

Solution

(*a*) Since the invoice is paid within 10 days, the 5% cash discount applies. The cash discount is 5% of \$36.76 or \$1.84.

∴ The amount needed to pay the invoice on September 10 is \$36.76—\$1.84 or \$34.92.

10% of \$36.76 is \$3.68
∴5% of \$36.76 is \$1.84

(*b*) Oct. 1 does not fall within the cash discount period but is within the 30-day period.

∴ The full invoice price of \$36.76 must be paid.

(*c*) Nov. 6 is 1 month past the net due date, and the late penalty of 1% per month must be charged.

∴ Interest is \$36.76 × $\frac{1}{100}$ or \$0.37.

∴ The amount to settle the invoice is \$36.76 + \$0.37 or \$37.13.

Note that the penalty applies on the first day late!

B *Retail discounts* are those offered by merchants selling to the general public, and are offered to encourage the customer to make a purchase now, rather than at a later date. The regular or *list price* is subjected to a discount, and the new (lower) price is called the *net price*.

Example 2

The U.B. Proud Furniture Company offers an E-Z Rest chair in its annual Fall Sale at 30% off the regular list price of \$99.99. What price would a customer pay for the chair?

Solution

30% of \$99.99 is \$30.
∴ The customer pays \$99.99 – \$30.00 or \$69.99.

30% of \$99.99 = \$29.997

C *Trade discounts* are offered to retail merchants by manufacturers or wholesalers as an incentive to buy in quantity from the wholesaler, or to adjust prices in a catalogue. Often a series of discounts on the list price will be offered.

Example 3

10% discount with a purchase of 20 or more and a further 20% discount with a purchase of 50 or more.

The Super-Service Wholesale Appliance Company offers washing machines to Arlene's Appliances under the terms 20/10%, 50/20%. If Arlene buys 60 washers at a list price of $300 each, calculate the net price on one machine.

Solution

Since Arlene purchased more than 50 machines, both discounts will apply.

	List price	$300
10% of $300	Less 1st discount	30
		270
20% of $270	Less 2nd discount	54
	Net price	$216

∴ Arlene's net price is $216 per machine.

Example 4

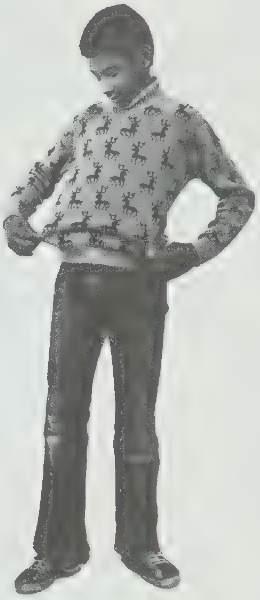
Determine the single discount equivalent to a series of discounts of 10%, 10%, 5%.

Solution

Consider the list price as $100.

	List price	$100
10% of $100	1st discount	10
		90
10% of $ 90	2nd discount	9
		81
5% of $ 81	3rd discount	4.05
	Net price	$ 76.95
	Hence, list price	$100
	Less net price	76.95
	Discount on $100	$ 23.05

∴ The single discount equivalent to a discount series of 10%, 10%, 5% is 23.05%.



Exercise 26.1

Discussion

1. Why might a company offer 2/15 e.o.m., n/30 e.o.m. to one customer, but demand c.o.d. (cash on delivery) from another customer?
2. State some reasons why many retail clothing stores have annual "Spring (or Fall) Clearance" Sales.
3. How can a wholesaler afford to offer trade discounts for quantity purchases?
4. Collect blank invoices from several local shops or service stations and compare the terms.
5. State the amount of the discount and the net price.

	List price	*Rate of discount*
(*a*)	$ 150	20%
(*b*)	$ 300	5%
(*c*)	$1200	2%
(*d*)	$ 200	7.5%

6. State the rate of discount.

	List price	*Discount*
(*a*)	$400	$ 40
(*b*)	$200	$ 10
(*c*)	$350	$ 35
(*d*)	$600	$150

$$\frac{40}{400} = \frac{10}{100} = 10\%$$

Written Solutions

7. An invoice of goods for $360 is dated January 15 and has terms 2/10, n/30. Find the sum that will settle the invoice on
   (*a*) January 22, (*b*) February 9.
8. At a sale, Bertie Bargainbeater purchases a shirt at $10.95, less 25%, and a pair of slacks at $17.50, less 15%. What did Bertie pay for his new outfit?

7(a) $352.80
(b) $360
8. $23.09

9. $20.83

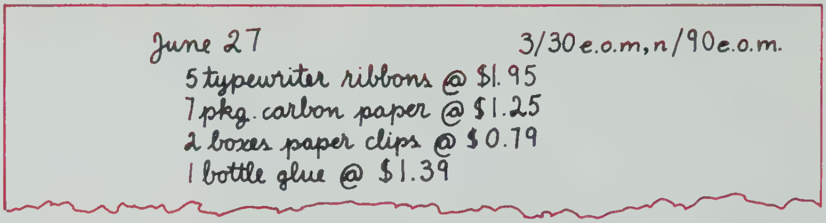
10(a) $67.50 (e) $102.38
(c) $1373.04

11(a) 14.5%
(c) 43.48%

12(a) $80
(b) $16\frac{2}{3}$%

13. $15

9. Determine the sum required to pay the invoice shown below on July 20.



June 27 3/30 e.o.m., n/90 e.o.m.
5 typewriter ribbons @ $1.95
7 pkg. carbon paper @ $1.25
2 boxes paper clips @ $0.79
1 bottle glue @ $1.39

10. Find the net price in each case.

	List price	*Trade discounts*
(*a*)	$75	10%
(*b*)	$217	15%
(*c*)	$1490	5%, 3%
(*d*)	$2500	20%, 10%
(*e*)	$195	30%, 25%
(*f*)	$400	20%, 10%, 10%

11. Determine the single discount equivalent to the given series of discounts.
(*a*) 10%, 5% (*b*) 40%, 25%, 10% (*c*) 30%, 15%, 5%

12. The Panoramic Television Company sells a set listed at $480 for $400. Calculate (*a*) the discount, and
(*b*) the rate of discount.

Notice that the rate of discount is calculated on the list price.

13. The Hi-Flyer home trampoline set is listed at $450 less 33$\frac{1}{3}$% and 20%. If the second discount is changed to 15%, find the increase in the net price.

26.2 Instalment Buying

A People frequently wish to enjoy the use of an item *now* and pay for it over a given period of time. Whether the present need justifies the added expense of buying on time must be decided by the individual.

When you buy an article on the instalment plan, you are really borrowing money from the company selling you the article. It is most important that you are aware of the *true interest rate** when you buy under these conditions.

* This will enable you to compare the cost of such borrowing with what you might pay elsewhere for the use of this money.

Example 1

A radio costing \$100 is bought by paying the purchase price, plus interest, in 12 equal monthly instalments. The dealer says the interest charge is only 6%, so that each monthly payment will be $\frac{\$100 + \$6}{12}$ or \$8.83. Determine the actual rate of interest charged.

This is really \$8.83 !

Solution

Since the amount owing is reduced by \$8.83 each month, it is necessary to find the average loan.

$$
\begin{aligned}
\text{Average loan} &= \frac{\text{Sum of monthly payments}}{\text{Number of months paid}} \\
&= \frac{\$106 + \$97.17 + \$88.34 + \$79.51 + \cdots + \$8.87}{12} \\
&= \frac{\$689.22}{12} \\
&\doteq \$57.44
\end{aligned}
$$

Can you see why this payment is \$ 8.87 instead of \$ 8.83?

$$
\begin{aligned}
\therefore \text{The actual rate of interest} &= \frac{\text{Monthly interest}}{\text{Average monthly loan}} \times 100\% \\
&\doteq \frac{\$6}{\$57.44} \times 100\%, \text{ or about } 10.5\%
\end{aligned}
$$

Hence, the old saying "Buyer beware!" is especially true when purchasing on the instalment plan!

For example, if an article is bought by making equal monthly payments for 1 year, then a quoted rate of

4%/year is actually about 7%/year
5%/year is actually about 9%/year
6%/year is actually about 11%/year
7%/year is actually about 13%/year
8%/year is actually about 15%/year
9%/year is actually about 17%/year
10%/year is actually about 18%/year
1%/month is actually about 22%/year
1.5%/month is actually about 33%/year

B As you may recall, the formula below gives an approximation of the true interest rate on an instalment purchase.

$$r = \frac{2NC}{P(n+1)} \quad \text{where} \quad r = \text{true interest rate per annum}$$

N = number of payment intervals per annum
C = carrying charge (total instalment price − cash price)
P = principal (cash price − down payment)
n = number of payments made

Example 2

A TV set has a cash price of \$250, and may also be purchased for \$25 down and \$24/month for 10 months. Calculate the true rate of interest being charged.

"\$25 down" means that \$25 is paid toward the purchase price when the set is picked up.

Solution

$$\begin{aligned} r &= \frac{2NC}{P(n+1)} \\ \therefore r &= \frac{2 \times 12 \times 15}{225 \times 11} \\ &\doteq 0.1454 \\ &= 14.54\% \end{aligned}$$

∴ The true interest rate is about 14.5%.

* N = 12 payments per year
C = \$265 − \$250 = \$15
P = \$250 − \$25 = \$225
n = 10 payments made

*Instalment price is \$25 + \$24 x 10 = \$265

1. Calculate
   (*i*) the instalment price, and
   (*ii*) the carrying charge
   in each case.

	Item	*Cash Price*	*Instalment Terms*
(*a*)	TV set	\$320	\$50 down, \$26.50/month for 1 year
(*b*)	camera	250	\$25 down, \$24/month for 10 months
(*c*)	kitchen set	250	\$25 down, \$13.75/month for 18 months
(*d*)	skis	75	\$15 down, \$3/week for 23 weeks
(*e*)	guitar	125	\$15 down, \$8/month for 15 months

Exercise 26.2

Discussion

1. Which would be the more expensive method of instalment buying:
   (*a*) a small down payment with payments spread over a long interval of time, or
   (*b*) a large down payment with the period of repayment as short as possible?
   Explain.

2. Why might a company encourage you to purchase an article on "easy credit terms" rather than paying cash?

3. State as many advantages and disadvantages of instalment buying as you can.

sickness?
bankruptcy?
business expansion?
repossession?
convenience?

4. State the instalment price in each case.
   (*a*) \$50 down, \$10/week for 8 weeks
   (*b*) \$120 down, \$40/month for 12 months
   (*c*) \$75 down, \$15/month for 6 months

Written Solutions

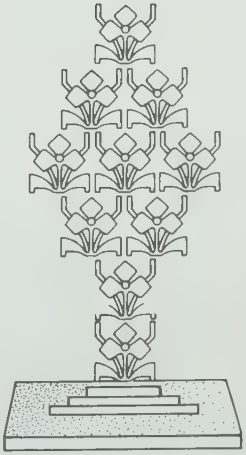
5. Find the missing item in each case.

	Item	*Instalment Price*	*Instalment Terms*
(*a*)	lamp	\$ 40	\$10 down, □/month for 6 months
(*b*)	bicycle	175	\$25 down, \$3/week for □ weeks
(*c*)	radio	50	\$5 down, □/week for 15 weeks
(*d*)	boat	300	\$30 down, \$15/month for □ months

6. Brenda Breezalong can purchase a 750 ml used motorcycle for \$400 cash, or pay \$64 down and make 24 monthly payments of \$17. Calculate the true rate of interest Brenda must pay, if she buys the bike on instalment.

7. The A. Ladinz Lamp Company produces a special brass lamp that may be purchased for \$25 in cash, or may be bought on the instalment plan for \$5 down and \$2.25/month for 10 months. Determine the true interest rate charged for the instalment plan.

8. The Illustrated Encyclopedia is offered for \$125 cash, or \$5 down and \$9.50/month for 15 months. What true rate of interest is paid by the instalment purchaser?



5(a) \$5 (b) 50 weeks
6. 20.6%
7. 27.3%
8. 28.1%

9. Randy Rithumaker, drummer for the newly formed Rockin' Rollers band, purchased a set of drums for \$450 and made a down payment of \$90. The carrying charge was 8% of the unpaid balance. The total of the unpaid balance and the carrying charge was paid by Randy in 12 equal monthly instalments. How much did he pay each month?

10. Find, in your local newspaper, 2 advertisements that give details concerning instalment buying. Calculate the true interest rate in each case.

26.3 Retail Buying and Credit

Many companies, such as department stores, furniture and appliance stores, oil companies, and automobile dealers offer *charge accounts* to individuals with good credit ratings.

Two main reasons for the use of charge accounts are the convenience to the buyer and the fact that items can be purchased before the buyer actually has the cash to pay for them.

The customer must pay for all purchases within 1 month of the company's billing date if there is to be no charge for the use of credit. However, if payment is delayed for more than one month after the billing date, a *service charge* will be added to the next month's bill.

You must resist the temptation to purchase more goods than you can comfortably pay for in the next month!

Credit cards are also used to buy credit. There are 2 main types of credit cards used.

(*a*) Credit cards issued by an oil company. These cards provide a convenient means of purchasing gasoline, tires, batteries, and other auto accessories, especially during trips when it is not desirable to carry a large amount of cash. When a purchase is made, the station attendant or dealer records the credit card number and details of the sale on the bill.

These bills are sent monthly to the company's head office and the purchaser is invoiced the following month. As with charge accounts, credit card statements must be paid within a specified period of time to avoid interest charges.

(*b*) Credit cards issued by a national credit card company. These cards are used for credit purchases of such varied items as meals, gasoline, transportation, and accommodation. This type of card is usually issued only to individuals who have very good credit ratings who are earning a specified minimum salary. Purchases and payments are treated in a fashion similar to charge accounts.

Revolving Credit Account (also called budget charge account) is another common form of credit, offered mainly by department, furniture and clothing stores. A *maximum debt limit* for each individual is set by the store, and is determined by the individual's estimated payment ability. The amount owed at the end of each month determines the payment that must be made during the following month. Payments are determined from a monthly payment chart. Part of such a chart is shown below.

Monthly Payment Chart

Amount of Unpaid Balance	*Monthly Payment Required*
up to $25.00	$3.00
25.01 to 35.00	4.00
35.01 to 55.00	5.00
55.01 to 75.00	6.00
75.01 to 100.00	7.00
100.01 to 125.00	8.00
125.01 to 155.00	9.00
155.01 to 200.00	10.00

Unlike regular charge accounts or credit cards, a revolving credit account charges interest on the unpaid balance for the previous month.

Chargex

Diner's Club

Carte Blanche

AMERICAN EXPRESS

master charge

Example

Mrs. I.C. Straight had a balance in her account of $23 on October 1. During October and November she purchased items as follows.

Date	*Purchase*
Oct. 3	$14.90
Oct. 12	11.05
Oct. 28	23.95
Nov. 19	7.00
Nov. 23	33.08
Nov. 25	9.99

She also returned goods worth $19.11 on Oct. 6, and made payments of $3 and $5 on Oct. 31 and Nov 30, respectively. If the store charges 1.5% on the unpaid balance of the previous month, calculate Mrs. Straight's balance on Dec. 1.

Solution

Date	*Item*	*Charge*	* *Payment*	*Returns*	*Balance Owing*
Oct. 1					$23.00
3	Purchase	$14.90			37.90
6				$19.11	18.79
12	Purchase	11.05			29.84
28	Purchase	23.95			53.79
31	Interest	0.35			54.14
31	Cheque		$3.00		51.14
Nov. 19	Purchase	7.00			58.14
23	Purchase	33.08			91.22
25	Purchase	9.99			101.21
30	Interest	0.77			101.98
30	Cheque		5.00		96.98

$$\begin{aligned} I &= Prt \\ &= 23 \times \frac{1.5}{100} \times 1 \\ &= \$0.35 \end{aligned}$$

$$\begin{aligned} I &= Prt \\ &= 51.14 \times \frac{1.5}{100} \times 1 \\ &= \$0.77 \end{aligned}$$

∴ Mrs. Straight's balance on Dec. 1 is $96.98.

* Minimum payments determined from the table on page 379.

Exercise 26.3

Discussion

1. What factors might affect a person's credit rating?

salary? occupation? overspending?

2. State as many advantages and disadvantages as you can for these ways of buying on credit.
   (*a*) charge account
   (*b*) credit card
   (*c*) revolving credit account

Written Solutions

3. Determine the annual rate of interest equivalent to
   (*a*) 1% per month, and
   (*b*) 1.5% per month.

4. Ted Turnaround has a revolving credit account that charges 1% on the unpaid balance of the previous month. His balance on May 1 is $40.10, and he makes purchases during May and June as indicated.

Date	*Purchase*
May 4	$10.19
May 19	24.00
May 23	9.50
May 24	6.32
June 10	35.95
June 23	4.37

He also returned goods on May 2 and June 17 worth $25.60 and $13.55 respectively.

Calculate Ted's balance on July 2.

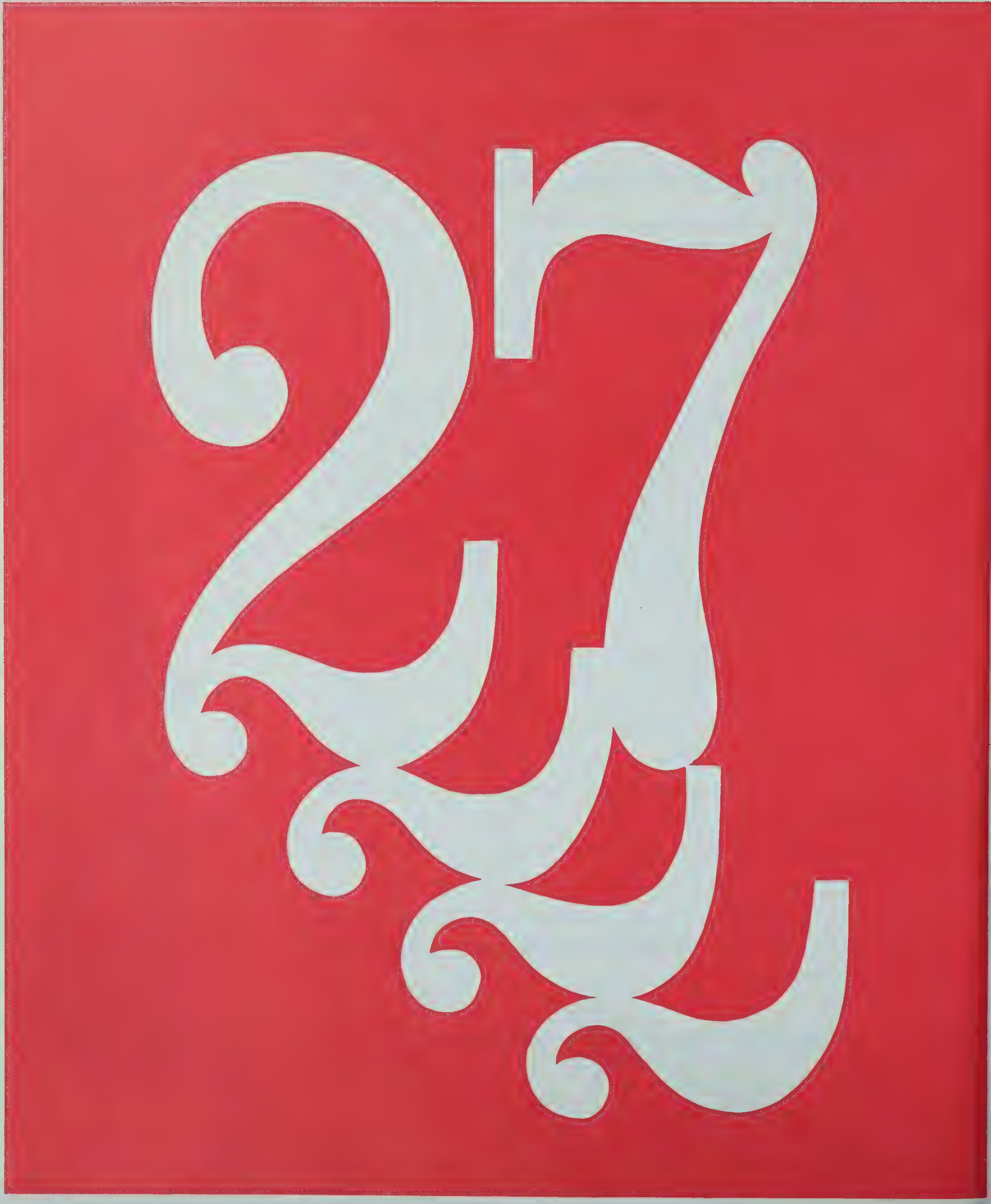
Remember to use the Monthly Payment Chart on page 379!

5. How does a credit card company make money?

6. Investigate the requirements for obtaining credit cards, such as Chargex or American Express, and compare their terms of payment.

* * *

3(a) 12% (b) 18%
4. $81.28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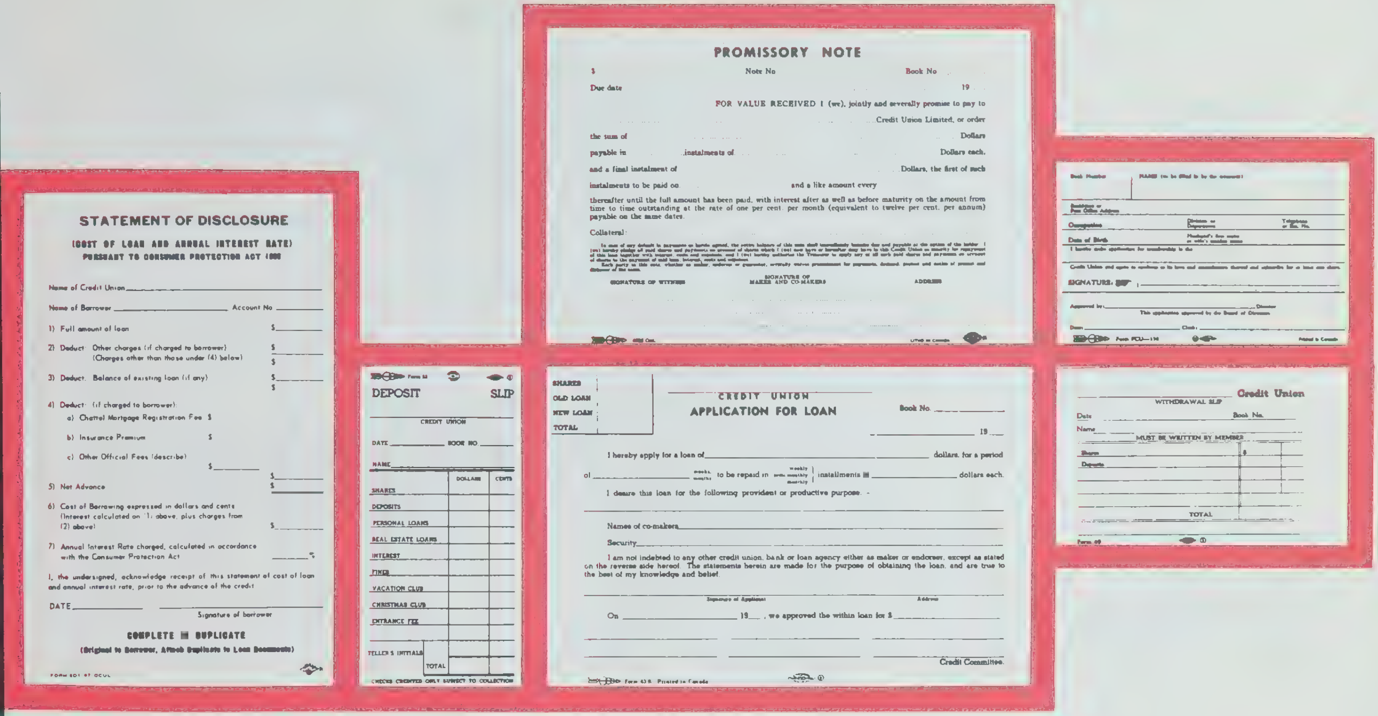


STATEMENT OF DISCLOSURE

(COST OF LOAN AND ANNUAL INTEREST RATE)

Name of Credit Union __________

Name of Borrower __________ Account No ______

1) Full amount of loan $______

2) Deduct: Other charges (if charged to borrower) (Charges other than those under (4) below) $______ $

3) Deduct. Balance of existing loan (if any) $______ $

4) Deduct: (if charged to borrower):

a) Chattel Mortgage Registration Fee $

b) Insurance Premium $

c) Other Official Fees (describe) $______ $______

5) Net Advance $______

6) Cost of Borrowing expressed in dollars and cents (Interest calculated on (1) above, plus charges from (2) above) $______

7) Annual Interest Rate charged, calculated in accordance with the Consumer Protection Act ______%

I, the undersigned, acknowledge receipt of this statement of cost of loan and annual interest rate, prior to the advance of the credit

DATE ______ __________

Signature of borrower

COMPLETE IN DUPLICATE

PROMISSORY NOTE

$ Note No Book No

Due date 19

FOR VALUE RECEIVED I (we), jointly and severally promise to pay to

Credit Union Limited, or order

the sum of Dollars

payable in instalments of Dollars each,

and a final instalment of Dollars, the first of such

instalments to be paid on and a like amount every

thereafter until the full amount has been paid, with interest after as well as before maturity on the amount from time to time outstanding at the rate of one per cent. per month (equivalent to twelve per cent. per annum) payable on the same dates.

Collateral:

SIGNATURE OF WITNESS | SIGNATURE OF MAKERS AND CO-MAKERS | ADDRESS

DEPOSIT SLIP

____

CREDIT UNION

DATE ______ BOOK NO. ______

NAME __________

	DOLLARS	CENTS
SHARES		
DEPOSITS		
PERSONAL LOANS		
REAL ESTATE LOANS		
INTEREST		
FINES		
VACATION CLUB		
CHRISTMAS CLUB		
ENTRANCE FEE		
TELLER'S INITIALS		
TOTAL		

CHECKS CREDITED ONLY SUBJECT TO COLLECTION

SHARES
OLD LOAN
NEW LOAN
TOTAL

__________ CREDIT UNION

APPLICATION FOR LOAN

Book No. ______

__________ 19__

I hereby apply for a loan of __________ dollars, for a period of ______ weeks / months to be repaid in weekly / semi-monthly / monthly installments of ______ dollars each.

I desire this loan for the following provident or productive purpose. -

____

Names of co-makers __________

Security __________

I am not indebted to any other credit union, bank or loan agency either as maker or endorser, except as stated on the reverse side hereof. The statements herein are made for the purpose of obtaining the loan, and are true to the best of my knowledge and belief.

____

Signature of Applicant Address

On ______ 19__, we approved the within loan for $______

______ ______ ______

______ ______ ______

Credit Committee.

Form 43 B. Printed in Canada

Occupation

Date of Birth

SIGNATURE:

Approved by: ______ Director

__________ **Credit Union**

WITHDRAWAL SLIP

Date ______ Book No. ______

Name __________

MUST BE WRITTEN BY MEMBER

	$	
Shares		
Deposits		
TOTAL		

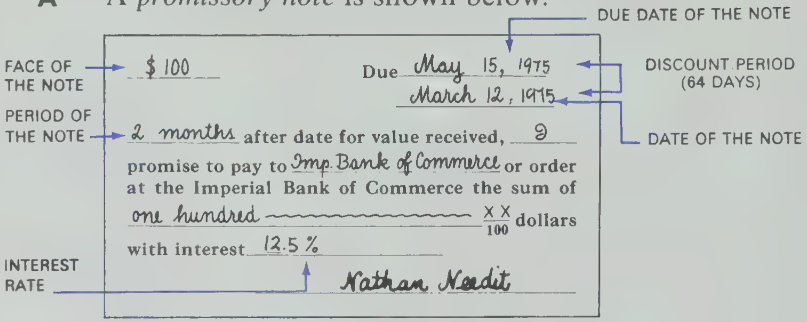
Unit 27
Third-Party Financing

Most people at one time or another borrow money in order to finance the purchase of a car or major household appliances such as a stove or washing machine, or to pay for medical aid, education, or a vacation. In such cases, credit is extended by a third party, that is, by someone other than the person or firm supplying the goods or services.

"Third Party"
1. Bank
2. Credit union
3. Loan company

27.1 Borrowing from a Bank

A A *promissory note* is shown below.


DUE DATE OF THE NOTE
FACE OF THE NOTE
DISCOUNT PERIOD (64 DAYS)
PERIOD OF THE NOTE
DATE OF THE NOTE
INTEREST RATE

$ 100 Due May 15, 1975
March 12, 1975
2 months after date for value received, I
promise to pay to Imp. Bank of Commerce or order
at the Imperial Bank of Commerce the sum of
one hundred ——— xx/100 dollars
with interest 12.5 %
Nathan Needit

The bank usually collects its interest on the day the note is made and deducts this amount from the face of the note. Such interest is called the *discount* and the interest rate is called the *discount rate*. The sum of money the borrower receives is called the *proceeds* of the note.

According to Canadian law, *3 days of grace** must be allowed on promissory notes.

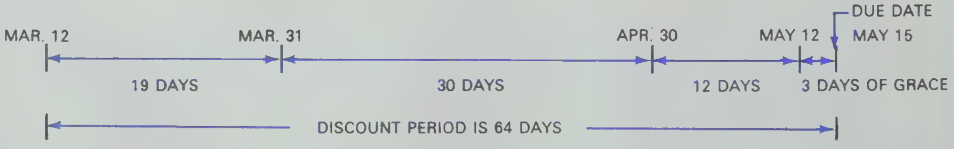
* That is, 3 days are added to the period of the note.

Example 1

Determine the proceeds of the above note.

Solution

A sketch like the one below helps to determine the *discount period* of the note.


MAR. 12
MAR. 31
APR. 30
MAY 12
DUE DATE
MAY 15
19 DAYS
30 DAYS
12 DAYS
3 DAYS OF GRACE
DISCOUNT PERIOD IS 64 DAYS

Recall I = Prt

$$\begin{aligned}\text{Proceeds} &= \text{Face} - \text{Discount}\\ &= \$100 - \left(\$100 \times \frac{12.5}{100} \times \frac{64}{365}\right)\\ &= \$100 - \$2.19\\ &= \$97.81\end{aligned}$$

Example 2

Melinda Musiklover saves \$300 towards the purchase of a new \$495 stereo set, and finances the remainder through a \$200 90-day promissory note bearing interest at 13%. Determine

(*a*) the proceeds of the note, and
(*b*) the actual rate of interest.

Solution

(*a*) Proceeds = Face − Discount

$$= \$200 - \left(\$200 \times \frac{13}{100} \times \frac{93}{365}\right)$$

Why 93?

$$= \$200 - \$6.62$$

$$= \$193.38$$

(*b*) $\because \quad I = Prt$

$$\therefore \quad r = \frac{I}{Pt}$$

$$= \frac{6.62}{193.38 \times \frac{93}{365}}$$

$$= \frac{6.62 \times 365}{193.38 \times 93}$$

$$= 0.134$$

$$\boxed{\begin{aligned} I &= \$6.62 \\ P &= \$193.38 \\ t &= \frac{93}{365}\text{ year} \end{aligned}}$$

The actual amount of money available to Melinda.

∴ The actual rate of interest is approximately 13.4%.

B *A Personal Loan Plan* is available to an individual wishing to borrow money. The individual signs a *personal instalment loan plan note*, similar to a promissory note. As the name implies, however, such a note is repaid by instalments, rather than by a lump sum at the end of the period. There may also be *service charges* on such a note, due to the extra time required for calculations performed by the bank's staff.

New car, home repairs, and so on.

The true interest rate here is calculated in the same way as it is for instalment purchases.

$$r = \frac{2NC}{P(n+1)}$$

Example 3

On August 10, Orville Owemor borrowed $2000 from his bank to buy a new car. He signed a personal loan plan note for $2292, including interest and service charges, that called for repayment in 24 monthly instalments of $95.50 each. Calculate the true rate of interest.

Solution

$$
\begin{aligned}
r &= \frac{2NC}{P(n+1)} \\
&= \frac{2 \times 12 \times 292}{2000\,(24+1)} \\
&= \frac{24 \times 292}{2000 \times 25} \\
&\doteq 0.1402
\end{aligned}
$$

N = 12 payments per year
C = $292 *
P = $2000
n = 24 payments in total

* The interest paid is actually the "carrying charge" here.

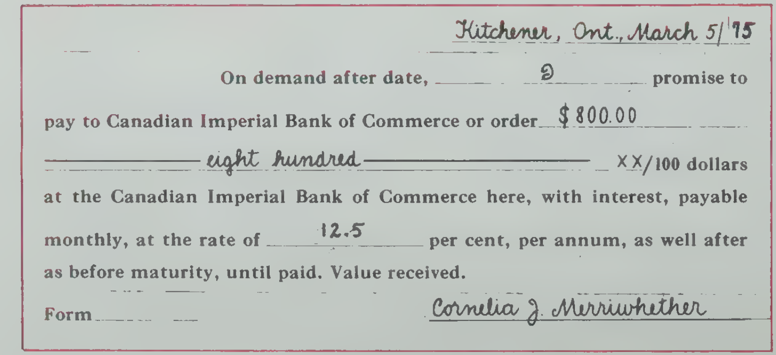
$\therefore$ The true rate of interest is approximately 14%.

C A *demand note*, as the name suggests, is payable on demand. Although the bank can legally demand payment at any time, it rarely does so. As there is no definite time of payment, a demand note has no days of grace. Payments are usually of a periodic nature (generally monthly).

A demand note is not discounted, and thus the borrower receives the full amount of the note. Interest is paid each month on the balance owing.

Example 4

Cornelia J. Merriwhether borrows $800 from the bank on a demand note as shown below.



Kitchener, Ont., March 5/'75

On demand after date, I promise to pay to Canadian Imperial Bank of Commerce or order $800.00 eight hundred xx/100 dollars at the Canadian Imperial Bank of Commerce here, with interest, payable monthly, at the rate of 12.5 per cent, per annum, as well after as before maturity, until paid. Value received.

Form

Cornelia J. Merriwhether

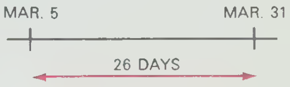
Ms. Merriwhether makes payments on the loan according to the following schedule:
March 31, \$100; April 30, \$100; May 31, \$100; June 30, \$100; July 31, \$200; August 31, \$200.

Calculate the total interest paid for the loan.

Solution

The table below indicates the monthly interest payments.

Date	*Payment*	*Balance Owing*	*Period*	*Interest Calculation*	*Interest Paid*
Mar. 5	—	\$800	Mar. 5 to Mar. 31	$I = 800 \times \frac{12.5}{100} \times \frac{26}{365}$	\$ 7.12
Mar. 31	\$100	\$700	Mar. 31 to Apr. 30	$I = 700 \times \frac{12.5}{100} \times \frac{1}{12}$	7.29
Apr. 30	\$100	\$600	Apr. 30 to May 31	$I = 600 \times \frac{12.5}{100} \times \frac{1}{12}$	6.25
May 31	\$100	\$500	May 31 to June 30	$I = 500 \times \frac{12.5}{100} \times \frac{1}{12}$	5.21
June 30	\$100	\$400	June 30 to July 31	$I = 400 \times \frac{12.5}{100} \times \frac{1}{12}$	4.17
July 31	\$200	\$200	July 31 to Aug. 31	$I = 200 \times \frac{12.5}{100} \times \frac{1}{12}$	2.08
Aug. 31	\$200			Total:	\$32.12



$= \frac{26}{365}$ year

1 month $= \frac{1}{12}$ year

∴ The total interest paid for the loan is \$32.12.

Exercise 27.1

Discussion

1. Distinguish between a 90-day promissory note and a 3-month promissory note.

90 days from June 2 is Aug. 31
3 months from June 2 is Sept. 2

2. In your own words, tell what is meant by each of the following terms.
   - (*a*) face
   - (*b*) period
   - (*c*) date
   - (*d*) due date
   - (*e*) discount
   - (*f*) proceeds
   - (*g*) days of grace
   - (*h*) discount period

3. Compare promissory notes and demand notes by considering
   (*a*) days of grace,
   (*b*) loan repayment, and
   (*c*) interest charges.

4. Discuss promissory notes and personal instalment loan plans under the headings
   (*a*) service charges,
   (*b*) loan repayment, and
   (*c*) calculation of interest.

5. In certain situations, a bank may require a borrower to provide some guarantee that the loan will be repaid. The guarantee usually takes the form of stocks, bonds, or life insurance policies, and is called *collateral* for the loan. Under what circumstances might a bank demand collateral as security for a loan?

6. An individual's *credit rating* often determines the need for collateral. How might a person establish a good credit rating?

Written Solutions

7. Determine the number of days in the discount period of each of the following promissory notes.

	Face	*Date of Note*	*Period*
(*a*)	$200	Nov. 9	1 month
(*b*)	$150	Apr. 19	60 days
(*c*)	$1000	June 28	3 months
(*d*)	$800	May 13	90 days
(*e*)	$480	Aug. 11	5 months

Remember
3 days of grace!

8. Calculate the proceeds of each note in Question 7, if each is discounted at the rate of 12.5%/year.

9. Trudy Truenote discounts her $550 note at the bank in order to purchase a trumpet so she can start her own rock group. Her 90-day promissory note bears interest at 13%. Determine
   (*a*) the proceeds of the note, and
   (*b*) the actual rate of interest on the loan.

7(a) 33 (e) 156
(c) 95

8(a) $197.74 (e) $454.36
(c) $967.47

9(a) $531.78 (b) 13.4%

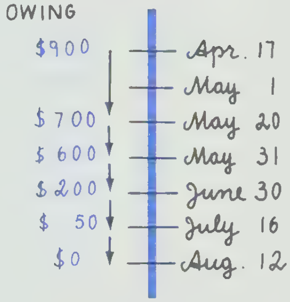
10. Fenmore Finaltouch obtains $3000 from his bank to finish the recreation room in his house by signing a personal loan plan note for $3636, including interest and service charges. If he repays the loan in 36 monthly instalments of $101 each, calculate the rate of interest charged.

11. In order to buy a motorbike for transportation to and from work, Gary Goodguy borrows $500 from the bank on August 20 by signing a demand note bearing interest at 12.5%/year. Gary repaid the loan as follows: Aug. 31, $100; September 30, $200; October 31, $100; November 30, $100. Calculate the total interest paid on the loan.

12. Glenda Goodgal signs a demand note for $900 on April 17 and makes payments as follows: May 20, $200; May 31, $100; June 30, $400; July 16, $150; August 12, the remainder. If the note bears interest at $13\frac{1}{4}\%$, calculate the total interest Glenda paid.

10. 13.75%
11. $9.14
12. $21.76

12.



27.2 Borrowing from a Credit Union or Loan Company

A A *credit union* is an association of people with a common interest: they may have the same occupation, for example, or be employed by the same company. Upon payment of a small entry fee (25¢), members may invest money in the union by purchasing shares, (usually valued at $5) or borrow money from it.

A member may borrow money from the union by applying for a loan through the *loan committee.* If the value of the shares held by the member is not sufficient as collateral for the loan, other collateral may be required.

Collateral
Property given to the lender as security that the loan will be repaid.

Loans are repaid in monthly or weekly instalments with interest charged at the end of each month on the unpaid balance.

There is a maximum interest rate allowed by law. At the end of each year in which profits are made, there is usually a rebate of interest to borrowers and a dividend declared to the shareholders. In this way, members who have borrowed money from the credit union pay a lower rate of interest, and those who have a savings account usually receive a higher interest rate on their savings than is paid by a bank.

Example 1

1%/month = 12%/year

Timothy Terrific borrows \$500 from his credit union on March 1 to help purchase a new car. He agrees to repay the loan in 10 monthly instalments of \$50 each as well as interest at the rate of 1%/month on the unpaid balance. Calculate Timothy's total interest payments.

Solution

The table below shows the calculation of interest on the loan.

Date	*Calculation of Interest*	*Payments*		*Balance Owing*
		Interest	*Principal*	
March 1				\$500
March 31	500 × 0.01	\$5.00	\$50	450
April 30	450 × 0.01	4.50	50	400
May 31	400 × 0.01	4.00	50	350
June 30	350 × 0.01	3.50	50	300
July 31	300 × 0.01	3.00	50	250
August 31	250 × 0.01	2.50	50	200
September 30	200 × 0.01	2.00	50	150
October 31	150 × 0.01	1.50	50	100
November 30	100 × 0.01	1.00	50	50
December 31	50 × 0.01	0.50	50	0

$\frac{1}{100} = 0.01$

∴ The total interest paid is \$27.50.

Example 2

If a rebate of 15% is given on December 31 in Example 1 above, calculate the annual rate of interest Timothy paid after taking the rebate into consideration.

Solution

$\frac{15}{100} \times 27.5 = 4{\cdot}13$

The amount of the rebate would be 15% of \$27.50 or \$4.13. Hence, Timothy's loan cost him only \$23.37.

The actual interest rate is calculated as follows.

$$\begin{aligned} r &= \frac{2NC}{P(n+1)} \\ &= \frac{2 \times 12 \times 23.37}{500 \times 11} \\ &\doteq 0.102 \end{aligned}$$

N = 12 payments per year
C = \$23.37
P = \$500
n = 10 payments in all

∴ The annual rate of interest is approximately 10.2%.

B A *loan or finance company* is a business corporation that provides personal cash loans in the form of *instalment loans*. That is, in return for a cash loan, the borrower agrees to make monthly payments to repay the loan over a specified period of time.

Also called a "small loans company."

The interest rate charged is usually higher than that charged by banks or credit unions. Because the higher rate covers the added risk, loan companies do not require as much collateral as other lending institutions. Individuals who would otherwise be unable to obtain funds may do so from a loan company.

It is most important to be able to determine the actual interest rate per annum from the information applying to a loan from a finance company.

Example 3

The Ernest N. Dever Finance Company offers $300 in cash in return for 12 equal monthly instalments, based on principal plus interest. If the total interest and service charges are quoted at $39, determine

(*a*) the amount of each monthly instalment, and

(*b*) the true rate of interest charged.

True interest rate

$$r = \frac{2NC}{P(n+1)}$$

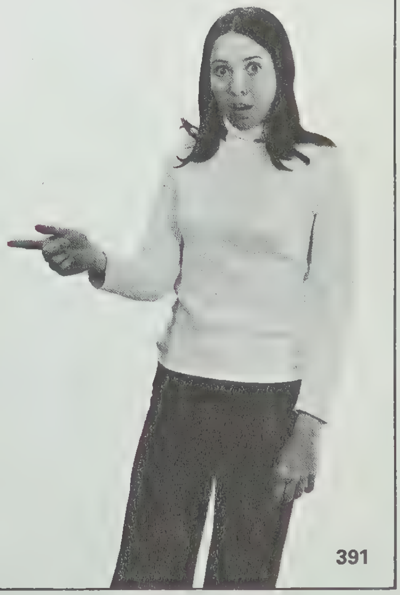
Solution

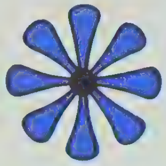
(*a*) The total amount to be repaid is $339. Thus, each monthly instalment will be $\frac{\$339}{12}$ or $28.25.

(*b*) $$\begin{aligned} r &= \frac{2NC}{P(n+1)} \\ &= \frac{2 \times 12 \times 39}{300(12+1)} \\ &= \frac{24 \times 39}{300 \times 13} \\ &= 0.24 \end{aligned}$$

$$\boxed{\begin{aligned} N &= 12 \\ C &= \$39 \\ P &= \$300 \\ n &= 12 \end{aligned}}$$

∴ The true rate of interest charged is 24% per annum.





Exercise 27.2

Discussion

1. Discuss the advantages and disadvantages of borrowing money from a bank, a credit union, and a finance company.
2. Identify each of the following terms.
   - (*a*) loan committee
   - (*b*) rebate
   - (*c*) instalment loan
   - (*d*) discount
   - (*e*) days of grace
   - (*f*) demand note
   - (*g*) face of a note
   - (*h*) collateral
3. Why are interest rates charged by loan companies generally higher than those charged by banks or credit unions?

Written Solutions

4. Make a table similar to the one in Example 1 on page 390 to show payments of interest and principal and balance owing on the following loans from a credit union.

(*a*)

Date borrowed	April 1
Amount of loan	$1500
Monthly payment	$250(on principal)
Number of payments	6
Interest rate	$\frac{2}{3}\%$/month

(*b*)

Date borrowed	June 15
Amount of loan	$400
Monthly payment	$40
Number of payments	10
Interest rate	9%/year

$$\frac{2}{3}\% = \frac{2}{3} \times \frac{1}{100}$$
$$= \frac{2}{300}$$

5. Penny Farthing borrows $600 from the credit union where she works, and agrees to repay it in 12 equal monthly instalments. If interest is charged at the rate of 1%/month on the unpaid balance, calculate
   - (*a*) the total interest paid, and
   - (*b*) the actual interest rate charged if a rebate of 20% is given.

5(a) $39
(b) 9.6%

6. Determine the monthly payment on each of the following finance company loans.

	Cash You Get	*Interest Charged*	*Number of Payments*
(*a*)	$146	$16	9
(*b*)	$1000	$80	12
(*c*)	$600	$69	12
(*d*)	$631	$107	18
(*e*)	$1300	$338	36

7. Calculate the actual annual rate of interest for each loan of Question 6.

8. Henry Homeowner received $899.53 in cash from a finance company after agreeing to repay the loan in 24 equal monthly instalments of $45 each. What is the true rate of interest on Henry's loan?

9. Contact a credit union and a loan company in your community and compare their loan requirements and rates charged.

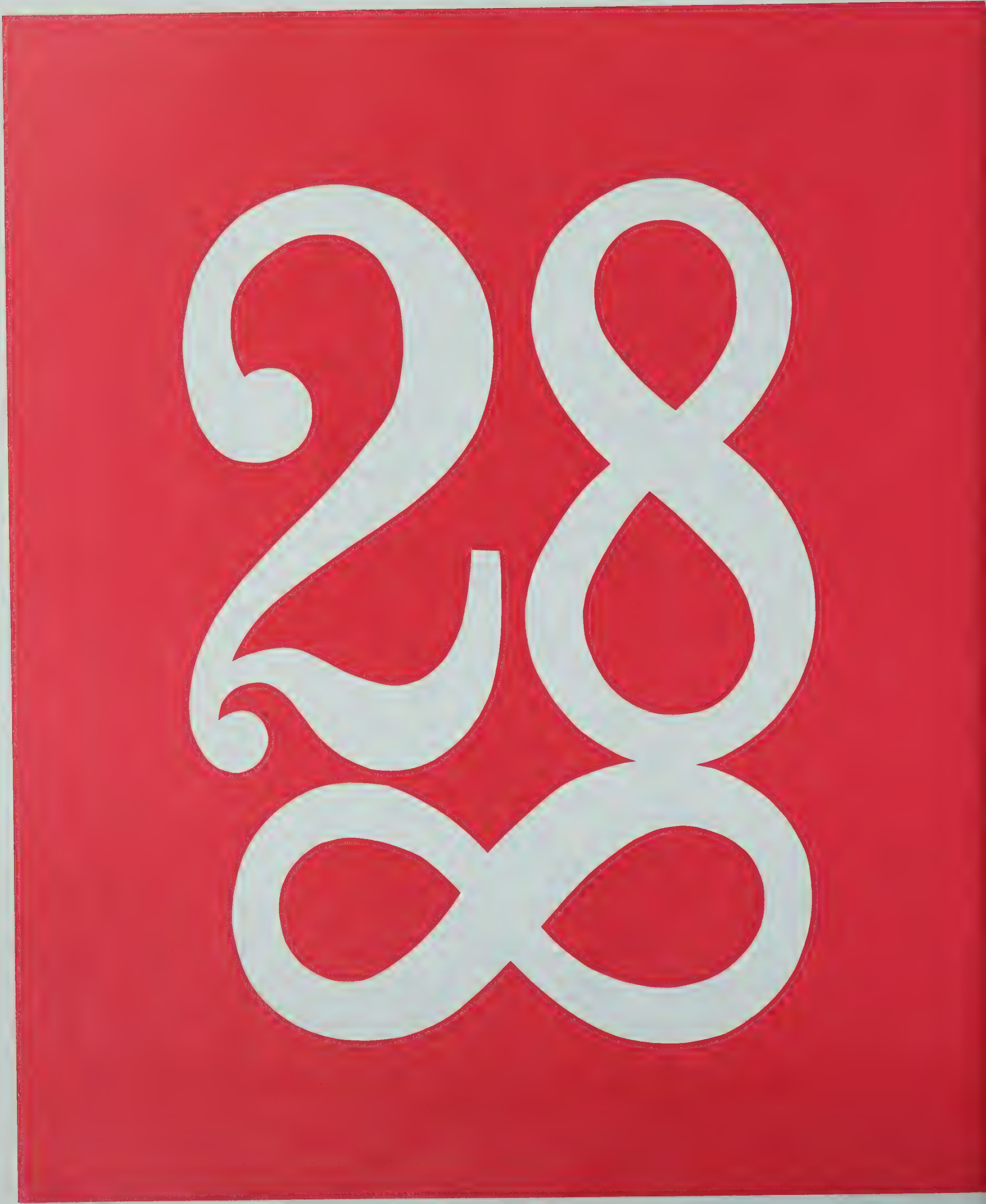
6(a) $18 (e) $45.50
(c) $55.75

7(a) 26.3% (d) 21.4%
(b) 14.8% (e) 16.9%
(c) 21.2%

8. 19.3%

Recall

$$r = \frac{2NC}{P(n+1)}$$


28

Unit 28
Saving and Investing Money

“Cash or charge?” This question is asked of all of us when we are buying merchandise. Many people choose to make their purchases using a charge account and learn to live with several monthly payments (including debt charges!). Others avoid extra charges, either by paying cash or by paying the account before interest charges can be applied. The relatively large sums required for furniture, appliances, or a car can be accumulated by saving the necessary money before the purchase is made. In this unit you will study a wide variety of methods for saving money.

28.1 Deposit Accounts

A A bank or trust company provides you with a ready method for saving money. You can open a *savings account* that is strictly for savings, and you are paid interest at a rate that can be as high as 10%. This rate changes from time to time and depends on the amount of money the company earns when it uses your money.

The usual procedure is to calculate the interest for each month on the *minimum* monthly balance and credit this to your account every 6 months.

Example 1

The minimum balances in Joe Smart's savings account are as follows: November $70, December $40, January $50, February $50, March $100, April $120. Find the interest credited to Joe's account on April 30, if the rate is 9%/year.

Solution

Month	*Min. Balance*	*Calculation*	*Interest*
November	$70	I = $70 \times 0.09 \times \frac{1}{12}$	$0.53
December	40	$40 \times 0.09 \times \frac{1}{12}$	0.30
January	50	$50 \times 0.09 \times \frac{1}{12}$	0.38
February	80	$80 \times 0.09 \times \frac{1}{12}$	0.60
March	100	$100 \times 0.09 \times \frac{1}{12}$	0.75
April	120	$120 \times 0.09 \times \frac{1}{12}$	0.90
			$3.46

Recall

$I = Prt$

∴ The interest credited to Joe's account on April 30 is $3.46.

You can see that it is important for you to know the method used by your bank in calculating interest. By delaying the withdrawal of cash for 2 or 3 days, it is possible to avoid losing the interest on the amount of the withdrawal for the whole month. This may be very little on a $25 withdrawal, but can be considerable if several hundred dollars is involved.

B Other bank or trust accounts are provided for people who have frequent deposits and withdrawals. These are called *personal chequing accounts* (p.c.a.), and their purpose is to assist you in keeping a close check on your income and expenditures.

No interest is paid on money deposited in this type of account. You can write cheques, for which there is a fixed service charge (currently 14¢ each). A monthly statement is sent to you that outlines all transactions that took place during the month. Attached to the statement are the cheques cashed against the account during the month. This is the kind of chequing account that you will be most likely to use yourself.

C Other types of accounts combine the interest features of savings accounts and the chequing features of personal chequing accounts. Interest is earned at a lower rate than on a savings account and the cost of writing each cheque is higher than that charged for cheques on a p.c.a.

bonus savings
premium savings
blue chip savings
true savings
programmed savings plan
hockey college account
chequing savings
chequable savings
true chequing
current account

Example 2

April Showers has a bank account that pays 4% interest on the minimum monthly balance and allows one cheque for each \$100 in the minimum balance. It costs 20¢ to write each additional cheque. The following transactions take place during the month of April: deposits of \$25 on April 1 and on April 15; cheques cashed for \$12.50, \$5.40, and \$18.00 on April 7, 16, and 25, respectively. If the balance on March 31 was \$270, prepare the entries in April's bank book, and calculate the interest and service charge for the month.

Solution

Date	*Deposits*	*Withdrawals*	*Balance*
Mar. 31			\$270.00
April 1	\$25.00		295.00
April 7		\$12.50	282.50
April 15	25.00		307.50
April 16		5.40	302.10
April 25		18.90	284.10

The minimum balance is \$282.50. Therefore, two cheques are free and the third cheque costs \$0.20.

$\therefore$ Service charge is \$0.20 for April.

$$\text{Interest} = \$282.50 \times 0.04 \times \tfrac{1}{12}$$
$$= \$0.94$$

If April's account were a personal chequing account, the 3 cheques would have resulted in a service charge of \$0.42.

1. What is the net result of the service charge and the interest on April's bank balance?

2. How much is she saving by having this special chequing account instead of a p.c.a.?

4%

D *Credit unions* provide another means of saving money. The members of the credit union operate it on a non-profit basis and return any excess of revenue over expenditures to the members at the end of the year.

Credit unions accept deposits on which interest is paid, grant loans to members, and may offer chequing privileges. The calculation of interest is simplified in a savings account of a credit union by basing the calculation on the minimum monthly balance which is a multiple of $5 or $10. Thus, if a balance of $261.50 were in the account, the interest would be calculated on $260.

Deposits in a credit union are really purchases of $5 or $10 *shares* and the interest paid can be thought of as a *dividend.*

A dividend is the profit earned on money you invest.

Example 3

Harry Hardnose is the football coach at Hotsprings High. As a teacher he is eligible to belong to the credit union operated by the teachers. If a dividend of 7% is credited to each account on Nov. 30 and May 30 each year and Harry deposits $75 on the first day of each month from September to May, find the amount in his account on May 30.

Solution

Month	*Deposit*	*Balance*	*Dividend*
Sept.	$75	$ 75	$75 \times 0.07 \times \frac{1}{12} = 0.44$
Oct.	75	150	$150 \times 0.07 \times \frac{1}{12} = 0.88$
Nov.	75	225	$225 \times 0.07 \times \frac{1}{12} = 1.31$
Nov. 30	2.63	227.63	
Dec.	75	302.63	$300 \times 0.07 \times \frac{1}{12} = 1.75$
Jan.	75	377.63	$375 \times 0.07 \times \frac{1}{12} = 2.19$
Feb.	75	452.63	$450 \times 0.07 \times \frac{1}{12} = 2.63$
Mar.	75	527.63	$525 \times 0.07 \times \frac{1}{12} = 3.06$
Apr.	75	602.63	$600 \times 0.07 \times \frac{1}{12} = 3.50$
May	75	677.63	$675 \times 0.07 \times \frac{1}{12} = 3.94$
May 30	17.07	694.70	

0.44
0.88
1.31
2.63

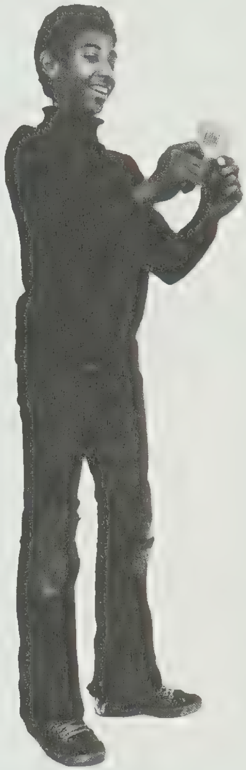
∴ The balance as of May 30 is $694.70.

What is this $17.07 deposit?

Exercise 28.1

Discussion

1. Convert to decimals.
   (*a*) 6% (*b*) 4.5% (*c*) 7.75% (*d*) $5\frac{1}{4}$%
2. Convert to fractions.
   (*a*) 5% (*b*) 4% (*c*) 6.5% (*d*) $8\frac{3}{4}$%
3. Why does a savings account pay more interest than a chequing-savings account?
4. One "advantage" of a savings account with no chequing privileges is that you have to make a special trip to the bank to withdraw any cash. Discuss.
5. Since it costs more money to buy something on "time" and you can actually make money by saving ahead and paying cash, it seems obvious that everyone would pay cash. What advantages exist that make people go into debt? Explain why occasional instalment purchases might be justified.



Written Solutions

6. Find the interest on the following minimum balances for one month using the rate given.
   (*a*) $250 at 8%
   (*b*) $540 at 5.5%
   (*c*) $1200 at $6\frac{1}{4}$%
   (*d*) $352.80 at 7.5%
   (*e*) $173.95 at $5\frac{3}{4}$%
   (*f*) $642.50 at 9.5%
7. In a credit union dividends are based on the number of $5 shares credited to each member. Find the amount of the dividend for the period stated using the rate given.
   (*a*) $175 for 4 months at 10.5%
   (*b*) $300 for 3 months at 8.5%
   (*c*) $832 for 6 months at 6%
   (*d*) $578.50 for 2 months at 9%
   (*e*) $1438.00 for 5 months at 8.5%
   (*f*) $372.50 for 4 months at $7\frac{3}{4}$%

6(a) $1.67 (e) 83c
(c) $6.25

7(a) $6.13 (e) $50.82
(c) $24.90

What types of accounts are offered by your bank?

8. The following amounts are kept in a savings account under the conditions outlined. Find the balance in the account at the end of the period. (Assume interest is added at that time.)
   (*a*) \$100 for 3 months at 9%, calculated monthly
   (*b*) \$350 for 4 months at 8%, calculated monthly
   (*c*) \$500 for 6 months at 8.5%, calculated monthly
   (*d*) \$1200 for 5 months at 8.5%, calculated monthly

9. (*a*) Prepare the entries in Sally Sayvall's bank book for the month of May if the following transactions take place: deposits of \$10 each week on the 7th, 14th, 21st, and 28th; and cheques cashed for \$21.50 on the 4th, \$5.80 on the 15th, and \$3.25 on the 25th. The balance on April 30th is \$375.
   (*b*) In this account interest is paid at the rate of 4.5%/year, based on the minimum balance for the month, and the charge for each cheque is \$0.15. Find the amount of interest and the service charge for the month.

10. The policy of the High Trust Company on its savings accounts is to pay an interest rate of 9%/year on the minimum monthly balance in each account. However, this interest is not added to the balance until the last day of April and the last day of October each year. Prepare a chart showing
    (*a*) the minimum monthly balance
    (*b*) the monthly interest
    (*c*) the accumulated value on May 1,
    if the following deposits are made.
    Nov. 1, \$250; Dec. 15, \$100; Jan. 31, \$200; March 15, \$150. No withdrawals are made and the account was opened on November 1.

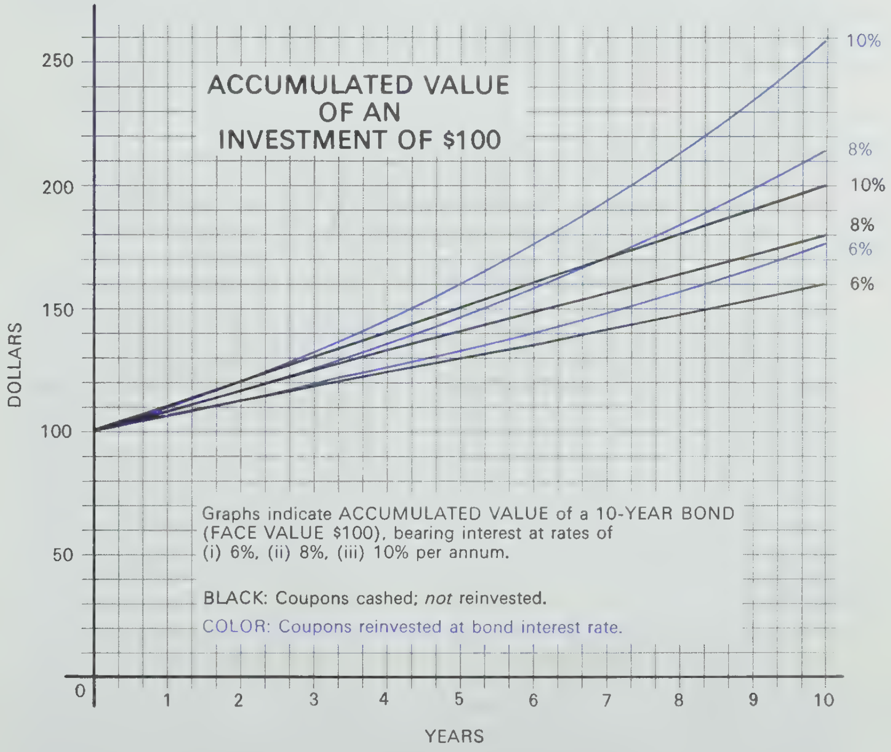
11. The Steelworkers' Credit Union of Iron City encourages members to make regular monthly deposits on which it pays 7%/year, calculated on the minimum number of paid-up \$10 shares per month and credited annually to their accounts. Iris Inspector has paid for her present car and plans to keep it for another 2 years. Since her monthly car payments have been \$95, Iris decides to save \$100/month for the next 2 years. Calculate the balance in her account on December 31, 1976 if her deposits begin January 1, 1975 and continue until December 1, 1976.

8(a) \$102.25 (c) \$521.25
9(b) \$1.33, \$0.45
10(c) \$719.88
11. \$2577.80

28.2 Investing in Short-Term Notes, Bonds, and Stocks

You have probably read advertisements about Canada Savings Bonds and other forms of investment that pay greater interest than is paid on bank and trust accounts. This means that once your savings build to a level that permits you to handle regular expenses as well as the occasional large purchase, then you should speak to your bank manager about other forms of investing that pay a higher return.

There are many factors to be considered in making a decision concerning investment. For how long a period are you willing to leave the money invested? Is it important for you to be able to turn your investment back into cash at short notice? How much risk are you prepared to take? Usually the greater the risk, the greater the return.



A Special types of short-term savings certificates are made available by banks and trust companies. These may be for as short a period as 30, 60, or 90 days or for as long as 5 years.

Typical Short-Term Rates
($5000 minimum investment)

NUMBER OF DAYS	RATE
30–59	10.75%
60–89	10.5%
90–179	10.25%
180–269	10%
270–364	9.75%

Example 1

Wyn Wise receives an inheritance of $5000 for which she has no immediate use. She decides to invest it in a 90-day certificate paying interest at $10\frac{1}{4}\%$/year. Find the accumulated value of the certificate after 90 days.

Solution

$$\boxed{\begin{aligned} P &= \$5000 \\ r &= 10\tfrac{1}{4}\% \\ t &= \tfrac{90}{365} \text{ year} \end{aligned}}$$

$$\begin{aligned} I &= Prt \\ &= 5000 \times \frac{10.25}{100} \times \frac{90}{365} \\ &= 126.37 \end{aligned}$$

$\therefore$ The note is worth $5126.37 after 90 days.

Example 2

Anne and Mike, a newly married couple, plan to invest their combined savings of $1500 in a Trust Certificate for a period of 2 years. If the rate of interest is 9% compounded annually, find the value of their certificate 2 years later.

Solution

For the first year, I = $1500 × 0.09 = $135
$\therefore$ Certificate value is $1500 + $135 = $1635
For the second year, I = $1635 × 0.09 × 1 = $147.15
$\therefore$ Certificate value is $1635 + $147.15 = $1782.15
$\therefore$ Their certificate is worth $1782.15 after 2 years.

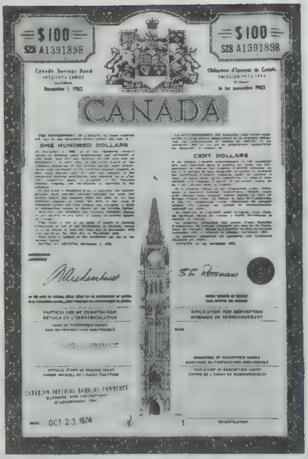
Why $1635 for the second year?

B *Bonds*. What is a bond? When a government or company needs cash, it sometimes seeks to borrow from the public by issuing bonds. A bond is really a promise by the organization issuing it to repay the buyer the face value at a set date in the future. It also agrees to pay interest at regularly stated intervals. The most common bond held by Canadians is the Canada Savings Bond (CSB) issued each November by the federal government in Ottawa. You can buy a CSB for as little as $50 and as much as $25 000.

The 1974/75 series issued on November 1, 1974 matures 9 years later in 1983. On a $100 bond, 9 interest coupons are attached, each valued at $9.75. One coupon is payable each year on November 1, from 1975 to 1983. On November 1, 1983, the total value of such a $100 bond would be $187.75.

Typical Long-Term Rates

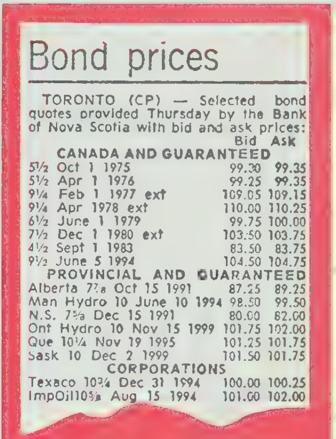
NUMBER OF YEARS	RATE	MINIMUM INVESTMENT
1	9.5%	$1000
2	9%	
3		$ 500
4		
5	10.5%	


$100
S28 A1391898
Canada Savings Bond
CANADA
ONE HUNDRED DOLLARS
CENT DOLLARS
OCT 23 1974

The simple interest formula, I = P*rt* gives interest of \$87.75 on a \$100 principal, when deposited for 9 years at $9\frac{3}{4}\%$. This is the *rate of interest* for the 1974/75 CSB series.

One of the special features of a CSB is that you can cash it in *at any time* for its face value plus interest coupons payable up to that time. This is not true of other bonds.

Study the Bond Quotations in the margin. "Bid" and "Ask" values are quoted on a \$100 bond. Since bonds issued a few years ago paid interest rates of 4% to 7%, their value today is much less than their face value. Why is this?



Bond prices

TORONTO (CP) — Selected bond quotes provided Thursday by the Bank of Nova Scotia with bid and ask prices:

	Bid	Ask
CANADA AND GUARANTEED		
5½ Oct 1 1975	99.30	99.35
5½ Apr 1 1976	99.25	99.35
9¼ Feb 1 1977 ext	109.05	109.15
9¼ Apr 1978 ext	110.00	110.25
6½ June 1 1979	99.75	100.00
7½ Dec 1 1980 ext	103.50	103.75
4½ Sept 1 1983	83.50	83.75
9½ June 5 1994	104.50	104.75
PROVINCIAL AND GUARANTEED		
Alberta 7⅞ Oct 15 1991	87.25	89.25
Man Hydro 10 June 10 1994	98.50	99.50
N.S. 7⅝ Dec 15 1991	80.00	82.00
Ont Hydro 10 Nov 15 1999	101.75	102.00
Que 10¼ Nov 19 1995	101.25	101.75
Sask 10 Dec 2 1999	101.50	101.75
CORPORATIONS		
Texaco 10¾ Dec 31 1994	100.00	100.25
ImpOil10⅝ Aug 15 1994	101.00	102.00

Example 3

Betty Banker buys a \$1000 Canada Savings Bond whose interest coupons pay $8\frac{1}{4}\%$ for the first 3 years. After holding the bond for 2 full years, Betty cashes it in so she can buy some furniture. What amount does she receive?

Solution

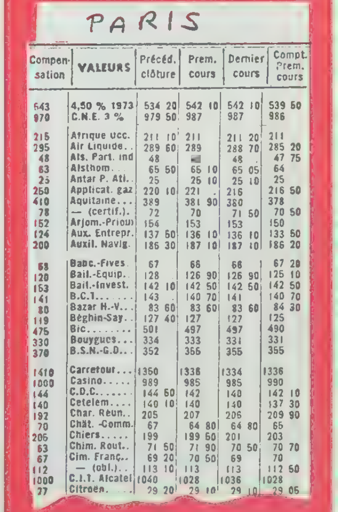
$$I = Prt$$
$$I = \$1000 \times 0.0825 \times 2$$
$$= \$165$$

$\therefore$ Betty receives \$1000 + \$165 = \$1165.

C *Stocks.* A company issues common or preferred stock to raise capital that may be necessary if more cash is required than is available through profits. After the original issue, buying and selling of these stocks takes place every business day as the ownership of the stocks changes from person to person. The market place for this trading is called a Stock Exchange; in Canada these are found in Montreal, Toronto, Calgary, and Vancouver. Only the stocks listed for trading at a particular exchange can be bought and sold there.

You can arrange to buy or sell stock through a stock broker. It is also possible for you to invest in a *mutual fund*, which in turn owns the stocks of many companies.

As the holder of the stock of a company, you are technically a part owner. This entitles you to receive regular reports on the progress of the business and any dividend that may be declared on the stock you hold. In addition you can vote on certain issues, but in practice the vote is decided by those who hold large blocks of shares.



PARIS

Compensation	VALEURS	Précéd. clôture	Prem. cours	Dernier cours	Compt. Prem. cours
543	4,50 % 1973	534 20	542 10	542 10	539 50
970	C.N.E. 3 %	979 50	987	987	986
215	Afrique occ.	211 10	211	211 20	211
295	Air Liquide..	289 60	289	288 70	285 20
48	Als. Part. ind	48		48	47 75
63	Alsthom...	65 50	65 10	65 05	64
25	Antar P. Atl..	25	25 10	25 10	25
250	Applicat. gaz	220 10	221	216	216 50
410	Aquitaine...	389	381 90	380	378
78	— (certif.).	72	70	71 50	70 50
152	Arjom.-Priou)	154	153	153	150
124	Aux. Entrepr.	137 50	136 10	136 10	133 50
200	Auxil. Navig.	186 30	187 10	187 10	186 20
68	Babc.-Fives.	67	68	68	67 20
120	Bail.-Equip..	128	126 90	126 90	125 10
153	Bail.-Invest.	142 10	142 50	142 50	142 50
141	B.C.T.......	143	140 70	141	140 70
80	Bazar H.-V...	83 60	83 60	83 60	84 30
119	Béghin-Say..	127 40	127	127	125
475	Bic........	501	497	497	490
330	Bouygues...	334	333	331	331
370	B.S.N.-G.D...	352	356	355	355
1410	Carrefour...	1350	1336	1334	1336
1000	Casino.....	989	985	985	990
144	C.D.C......	144 60	142	140	142 10
140	Cetelem....	140 10	140	140	137 30
192	Char. Réun..	205	207	206	209 90
70	Chât. -Comm.	67	64 80	64 80	65
206	Chiers.....	199	199 50	201	203
63	Chim. Rout..	71 50	71 90	70 50	70 70
67	Cim. Franç..	69 20	70 50	69	70
112	— (obl.)..	113 10	113	113	112 50
1000	C.I.T. Alcatel	1040	1028	1036	1028
27	Citroën....	29 20	29 10	29 10	29 05

Example 4

Ivan Investor bought the following amounts of shares in Stelco at the prices quoted.

25 shares at \$20.50
15 shares at \$22
10 shares at \$23.50

Find (*a*) his average cost per share,
(*b*) his yearly dividend if the dividend is \$1.40 per share annually, and represent this as a rate of interest.

50 shares

Solution

(*a*)
$$\text{Average cost} = \frac{(25 \times 20.50) + (15 \times 22) + (10 \times 23.50)}{50}$$
$$= \frac{512.50 + 330 + 235}{50}$$
$$= \frac{1077.50}{50}$$
$$= 21.55$$

Recall weighted average.

$\therefore$ The average cost per share is \$21.55.

(*b*) Total yearly dividend = 50 × \$1.40 = \$70.
Thus Ivan is earning \$70 per annum on an investment of \$1077.50. Using the interest formula we can find the rate of interest.

$$r = \frac{I}{Pt}$$
$$= \frac{70}{1077.50}$$
$$= 0.0649$$
$$\doteq 6.5\%$$

Why would Ivan be satisfied to earn a 6.5% return on his investment?

Exercise 28.2

Discussion

1. "If you hold cash at home instead of investing it, it loses value." Discuss this statement.
2. Why is a Canada Savings Bond a safer investment than a bond issued by an industrial company? What effect does this have on the interest rate paid on the bond?

3. Frequently, shares of common stock pay no dividend at all. Why is an investor willing to invest in such a stock? Where do the profits of such a company go if no dividend is paid? What effect does this usually have on the market value of the stock?

4. What is a mutual fund? What are some of the advantages of investing in such a fund?

5. What are the advantages of buying a Canada Savings Bond through a payroll deduction plan?

STOCK ?
MUTUAL FUND
TRUST CERTIFICATE
CSB ?

Written Solutions

6. The following amounts are invested for the stated period in a trust certificate at the given interest rate. Calculate the amount of the cash certificate at the end of the period if the interest is compounded annually.

	Amount Invested	*Rate*	*Time* (*years*)
(*a*)	\$500	7%	1
(*b*)	800	8%	1
(*c*)	1000	$7\frac{1}{2}$%	2
(*d*)	2500	8%	4
(*e*)	750	$8\frac{1}{4}$%	5

7. Lorna Ledger is treasurer of her Students' Council. On a dance, the Council realizes a profit of \$500. Knowing that this money will not be required for 1 year, Lorna decides to invest it. A savings account will pay 9%/year, compounded semi-annually, and a trust certificate pays $9\frac{3}{4}$%/year, compounded annually. Which is the better investment and by what amount?

8. A \$500 Canada Savings Bond is held for 3 full years. The interest coupons are based on a rate of 6.5%/year. If the bond is cashed in at the end of the 3 years, what amount is realized?

9. ABC Mines pays a quarterly dividend of \$0.08 per share on its common stock. If Wilbur Windfall owns 500 shares, what is his quarterly dividend? Find the annual dividend per share and express it as a per cent of the current \$5 price of 1 share.

6(a) \$535 (d) \$3401.22
(b) \$864 (e) \$1114.81
(c) \$1155.63

7. The trust certificate by \$2.74

8. \$597.50

9. Quarterly dividend of \$40, 6.4%

29

Unit 29
Earning Money

You have all earned money on a casual basis by babysitting, shovelling snow, picking fruit, and so on. Some of you may now be working steadily on a part-time job, especially if you are 16 years old. Every dollar counts, and all of us need to be sure that we are receiving exactly the sum that is due for the work we have done.

In this unit you will study several different methods for calculating wages and some of the deductions normally subtracted before you receive your take-home pay. This topic is a good example of applied mathematics. The arithmetic required is not difficult, but no error can be tolerated.

29.1 Calculating Gross Earnings

A The simplest method of payment for work done is by *salary*. Office staff, municipal employees such as fire fighters and police officers, members of the clergy, and teachers are examples of people who receive a salary. Usually the conditions of employment allow them a specified number of days off due to sickness without loss of pay. Although they are not paid by the hour, it is not uncommon for them to be required to punch or sign *in* and *out*.

Example 1

A stenographer is offered one job at \$125/week, and another at \$6400/year. Which is the better offer?

Solution

$$\begin{aligned} \$125/\text{week} &= (\$125 \times 52)/\text{year} \\ &= \$6500/\text{year} \end{aligned}$$

∴ The better offer is \$125/week.

1. Convert to an annual rate.
   (*a*) \$450/month (*b*) \$115/week
2. Change to a weekly rate.
   (*a*) \$13 000/year (*b*) \$590/month*

*
Hint: Convert
to an annual
rate, and then
to a weekly rate.

B Another common method of earning is on an hourly basis. In most occupations where this method is used, the worker has a time card and punches *in* when arriving for work and punches *out* when leaving. Workers in construction trades and factory employees are usually paid an hourly *wage*.

Three factors that can affect earnings are overtime, shift premiums, and penalties for lateness.

Overtime is normally paid at the rate of $1\frac{1}{2}$ times the hourly rate.

Shift premium may be paid for work performed on the afternoon or night shift. The hourly rate is increased by an agreed-upon amount—perhaps 20¢ for the afternoon shift and 30¢ for the night shift.

Day shift
08:00 to 16:00

Afternoon shift
16:00 to midnight

Night shift
midnight to 08:00

Usually a *lateness penalty* is applied if the time card is punched 3 or more minutes late. This amounts to a quarter or half hour's wages.

Example 2

Jacquie Hammer is a carpenter for OK Construction, working from 08:00 to 16:00. Overtime starts after 8 h work in any one day, and the lateness penalty is a half hour's wages. Use the information on the time card at the right to compute Jacquie's wages for the week if her hourly rate is $4.80.

WEEK ENDING	May 4		
NAME	Jacqueline Hammer		
NUMBER	3602		
DAY	IN	OUT	HOURS
M	07:50	16:05	
T	07:52	18:08	
W	07:48	17:02	
T	08:05	16:05	
F	07:50	16:00	
S			
S			
REGULAR TIME	HOURS	RATE	EARNINGS
OVERTIME			
TOTAL			

Solution

WEEK ENDING	May 4		
NAME	Jacqueline Hammer		
NUMBER	3602		
DAY	IN	OUT	HOURS
M	07:50	16:05	8
T	07:52	18:08	10
W	07:48	17:02	9
T	08:05	16:05	7½
F	07:50	16:00	8
S			
S			
REGULAR TIME	HOURS	RATE	EARNINGS
	39½	4.80	189.60
OVERTIME	3	7.20	21.60
TOTAL			$211.20

Why 7½ h?

Example 3

Harry Hoist is a crane operator for the Western Lumber Co. Harry works 8 h shifts; his latest pay included 3 afternoon shifts and 5 night shifts, as well as 2 day shifts. Western pays an extra 15¢/h for "afternoons" and an extra 25¢/h for "nights". How much is Harry's pay increased by these shift premiums?

Solution

Each afternoon-shift premium totals $8 \times \$0.15$ or \$1.20.
Each night-shift premium totals $8 \times \$0.25$ or \$2.00.

$$\therefore \text{Total shift premium for Harry} = (3 \times \$1.20) + (5 \times \$2.00) = \$3.60 + \$10.00 = \$13.60$$

C Many people are paid according to the amount of work they do instead of the time spent. Doctors and many sales representatives are in this category and so are thousands of production workers in automobile assembly plants, primary manufacturing companies, and so on. Frequently there is a basic rate that is increased by an amount determined by the *incentive rate*.

Example 4

Nora Nimblefingers is a line operator working in a textile plant. Nora's hourly rate is \$5.20 for her 40 h week. In addition, Nora receives \$0.105 for each 10 m of cloth she processes. During a recent week, Nora's production was 5000 m. Find her earnings for the week.

Solution

Basic earnings = 40 × \$5.20 = \$208.00
Incentive earnings = 500 × \$0.105 = 52.50
∴ Total earnings \$260.50

50%
10%
2%

Example 5

The Lively Life Insurance Co. pays its sales representatives a commission based on a percentage of premiums paid on policies sold. During April, Melvin Mutual sold new policies on which the premiums totalled \$2000. On these policies his commission is 50% of the premium. In addition, he received 10% commission on premiums for policies which had been in force for 1 year to 3 years. These premiums totalled \$4000. On policies still in force for more than 3 years, he received 2% of the premiums totalling \$12 000. Find Mel's earnings during April.

Solution

Earnings on new policies = 50% of \$ 2 000 = \$1000
Earnings on 1 year to 3 year policies = 10% of \$ 4 000 = 400
Earnings on remaining policies = 2% of \$12 000 = 240
∴ Total earnings in April \$1640

Exercise 29.1

Discussion

1. How does a salary of \$140/week compare to one of \$600/month? What is the best method for making this comparison?

2. Rich Mann is paid on the 15th and 30th of each month at an annual rate of \$11 400. How would you find his gross earnings for each pay period?

3. If overtime is paid at $1\frac{1}{2}$ times the regular rate, state the overtime rate for each of the following hourly earnings.
   (*a*) \$2.50 (*b*) \$4.00 (*c*) \$3.50 (*d*) \$5.40

4. A production line worker is paid \$0.15 for each article Q and \$0.20 for each article B produced. During an 8 h shift the line produces 200 of article Q, and 100 of article B. Find each worker's
   (*a*) earnings on article Q and article B, and
   (*b*) total earnings for the 8 h shift.

5. How might a company decide on the incentive rates it will use?

6. A sales rep receives 4% of her gross sales of merchandise. Find her earnings on sales of
   (*a*) \$1000, (*b*) \$10 000, (*c*) \$100 000.

Written Solutions

7. Convert the following salary rates to those stated.
   (*a*) \$135/week to an annual wage
   (*b*) \$720/month to an annual wage
   (*c*) \$9600/year to a monthly rate
   (*d*) \$14 500/year to a weekly rate
   (*e*) \$650/month to a weekly rate
   (*f*) \$192/week to a monthly rate

8. Rod Sparer earns money babysitting. At present, he charges 50¢/h before midnight and 75¢/h after midnight. He is considering a change to a flat 60¢/h. Compare his earnings if Rod regularly "sits" from 20:30 to 01:30 for a neighbor.

7(a) \$7020/year
(c) \$800/month
(e) \$150/week

8. 60¢/h pays 12¢ more.

9. Pete Packham works Thursday and Friday evenings and all day Saturday in a supermarket. Calculate his earnings if he is paid $2.50/h for completed half hours.

	In	*Out*
Thursday	16:30	22:00
Friday	17:00	21:35
Saturday	07:20	12:00
	12:57	18:00

10. Copy the following time cards and calculate the total earnings in each case. Overtime is paid at $1\frac{1}{2}$ times regular rate for time in excess of 8 h/day from 07:00 to 15:00, and a quarter-hour penalty is deducted for lateness.

(*a*)

WEEK ENDING	June 1
NAME	David Jones
NUMBER	50281

DAY	IN	OUT	HOURS
M	06:50	15:02	
T	06:48	15:05	
W	07:10	15:04	
T			
F			
S	06:55	16:30	
S	06:52	16:00	

	HOURS	RATE	EARNINGS
REGULAR TIME		4.10	
OVERTIME			
TOTAL			

(*b*)

WEEK ENDING	May 15
NAME	Robert Rowat
NUMBER	63412

DAY	IN	OUT	HOURS
M			
T	07:00	15:30	
W	07:20	16:00	
T	06:59	16:30	
F	07:01	16:02	
S	07:00	14:40	
S			

	HOURS	RATE	EARNINGS
REGULAR TIME		3.20	
OVERTIME			
TOTAL			

11. Sally Forth is an agent for the Square-Sell Real Estate Company. The terms of her agreement with the firm give her 4% of gross sales up to $100 000, and 5% of gross sales beyond that level. During April, Sally sold houses at prices of $60 000, $65 000, and $70 000. Find her earnings for the month.

9. $48.75
10(a) $178.35 (b) $143.20
11. $8750

12. The following table shows the premium income on life insurance policies sold by Susan Salesworthy for the month of May. Calculate Susan's total earnings.

Age of Policy	*Premium Income*	*Commission*
New	\$ 1 500	60%
1 to 2 years	2 800	15%
3 to 5 years	4 000	5%
6+ years	\$12 000	2%

13. For the week of April 25 to 29, the sales representatives of Jack Spratt Meats sold the amounts indicated below. Using the commission rates shown, calculate their gross earnings.

Sales Rep	*Sales*	*Commission*
Timothy E. Bacon	\$2700	$5\frac{1}{4}\%$
Carol J. Chop	2500	$4\frac{3}{4}\%$
Stephen D. Ham	2850	5.5%
James R. Hogg	4280	4 %
Sonya W. Ribb	3500	5 %

14. The operating crew on a finishing line in the Strong Steel Company consists of an operator, coiler, flipper, dipper, and helper. Using the table below and production of 2380 t, calculate their gross earnings for a 40 h week.

Job Classification	*Basic Hourly Rate*	*Incentive Rate per Tonne*
Operator	\$2.75	\$0.085
Coiler	2.25	0.070
Flipper	2.10	0.065
Dipper	2.10	0.065
Helper	2.00	0.050

15. Marjorie Morton has been offered 2 jobs. One pays a salary of \$675/month with a \$75/month raise after 6 months. The other job pays \$4.25/h basic wage for a 40 h week, with incentive earnings of \$0.030 per unit for a weekly average production of 500 units. From a financial standpoint, which is the better offer and by how much is it better over a period of 1 year?

12. \$1760

13. \$141.75
118.75
156.75
171.20
175.00

14. \$312.30
256.60
238.70
238.70
199.00

15. Hourly rate job pays \$1070 more in 1 year.

t means "tonnes"

29.2 Calculating "Take-Home" Pay

As soon as you begin to work regularly, whether on a part-time or full-time basis, you will receive less pay than your gross earnings. Federal laws dictate that several deductions must be made by your employer. These are for Income Tax, Unemployment Insurance, and the Canada Pension Plan.

Also, deductions made by the provinces– for example, Medicare.

Other deductions made are voluntary. These may include a payment on a Canada Savings Bond, a contribution to charity, or a special group life insurance plan.

A *Income Tax.* As soon as you start working full time you will begin paying Income Tax. This revenue enables the Federal and Provincial Governments to provide many services for us. Health services is one example. What are some others?

Some students earn enough part time to pay tax.

Each employee completes a TD1 Employee's Tax Deduction Return when hired. This form is used to certify the amount of income that is to be exempt from tax. For example, in 1975 a single person had a basic exemption* of $1878, an employee supporting a spouse had a tax exemption of $3522, and an employee supporting 2 children under 16 had an exemption of $2562.

* Exemption from tax means "tax-free" income.

The government provides each employer with a set of tables showing the amount of the required tax deduction. The table on page 452 shows the weekly tax deductions, depending on total earnings and total personal exemptions, for 1975.

1. State the deduction made each week if the following earnings and exemptions exist.

	Earnings	*Exemptions*
(*a*)	$102.00	$1878
(*b*)	114.50	3522
(*c*)	93.99	2914

It is the employer's responsibility to make regular payments to the Federal Government of the amounts of tax deducted. At the end of the year, the employer provides you with a T4 Statement of Remuneration Paid, showing your earnings as well as the tax deducted from your income during the year. You use this form when you complete your Income Tax Return for that year.

Table of Net Claim Codes

Net Claim for 1975		*Net Claim Code*
From	*To*	
$1 878	$1,981	1
1 982	2,201	2
2 202	2,421	3
2 422	2,641	4
2 642	2,862	5
2 863	3,081	6
3 082	3,302	7
3 303	3,519	8
3 520	3,742	9
3 743	3,962	10
3 963	4,182	11
4 183	4,499	12
4 500	4 799	13
4,800 and up		X
Exemption from Tax Deduction as claimed below		0

B *Unemployment Insurance.* Unemployment is an unpleasant possibility for thousands of wage earners today. You will be quite fortunate if you are employed at all times in the future. To assist people who are unable to obtain employment, the Unemployment Insurance Act was passed by our Federal Government and went into force in 1941. Since that time over $10 000 000 000 in benefits has been paid to unemployed Canadians.

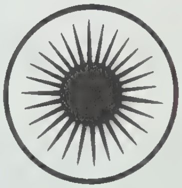
WOW!

10 billion dollars!

Unemployment insurance operates in the same way as all other types of insurance. A large number of contributors pay a relatively small premium so that the few who become unemployed can receive some benefits. It should be understood that this payment is not government support for the unemployed as welfare payments are. Strict regulations apply to those receiving benefits from this fund so that abuses are minimized. Using computers, the government is able to check on each person receiving benefits to ensure that the person is entitled to them.

The revenues necessary to pay these benefits are drawn from a fund to which every worker who falls within the definition stated in the Act contributes—that is about 96% of Canadian employees. The remaining 4% of the labor force consists of employees over 70 years of age, those who earn less than $35/week in Prince Edward Island and $37/week in the other provinces, casual labor, employees who are married to or dependents of the employer, and other minor categories.

The employee's share of the insurance premium is 1.4% of money earned up to $185/week. Employees earning more than $185/week pay a set amount. (See the table, page 451). The employer's premium is 1.4 times the employee's premium.



> *Note*: The deduction for UIC is made *before* income tax is calculated.

1. State the UIC payroll deduction for each weekly salary.
   (*a*) $162.50 (*b*) $115 (*c*) $153.75

UIC

A worker who is laid off receives an unemployment insurance book upon leaving. To obtain benefits, the worker must register at the local office of the Unemployment Insurance Commission (UIC). Contributions made during the time of employment are the basis of the calculation of weekly benefits. The worker must have worked for at least 8 weeks in order to qualify. For example, a single person receives $66\frac{2}{3}\%$ of his or her average weekly earnings or \$20, whichever is greater. These weekly payments do not continue indefinitely, as it is up to the person to find another job as soon as possible. The duration of the benefit is from 8 to 51 weeks.

* Canada Pension Plan

C *Canada Pension Plan*. Pensions are regular income payments received when people have reached a specified age. (This age is often the time of retirement.) To pay for this regular future income, you will start contributing to the CPP* the day you start working. By law, a deduction will be made from your earnings to finance this plan. Starting in 1966, everyone in Canada earning more than a minimum yearly salary was required to contribute to this plan set up by our Federal Government. In 1975, the contributions were calculated by subtracting \$700 from your earnings and taking 1.8% of the remainder. The maximum payment into the plan changes each year with the cost of living; in 1975, the maximum payment was \$120.60. This was the total deduction for anyone earning \$7400 or more during 1975. The employer contributes the same amount as the employee.

Note: The deduction for CPP is made *before* income tax is calculated.

1. State the deduction for CPP made for the following earnings. Use the table on page 451.
   (*a*) \$132.50/week (*b*) \$130.50/week

CPP

Example 1

If the weekly earnings of a person are \$152.50, the CPP deduction is \$2.50. For how many weeks in 1975 would this deduction be made from the person's pay?

Solution

The maximum deduction is \$120.60. At \$2.50/week, the number of deductions required

$$= \frac{120.60}{2.50} = 48.24$$

Therefore 49 weekly deductions would be made. However, the last deduction would be somewhat less than \$2.50.

Last deduction
$= \$120.60 - (48 \times \$2.50)$
$= \$120.60 - \$120.00$
$= \$0.60$

Example 2

Richard Reddicash decides to leave school at the end of Grade 10. His first job pays him \$2.25/h. Calculate Richard's gross earnings and take-home pay for a 40 h week with 5 h overtime if he has a \$10 deduction for a CSB, \$1.40 for health insurance, and \$1.25 for union dues.

Solution

Regular rate = \$2.25
∴ Earnings at \$2.25 = 40 × \$2.25 = \$ 90.00
Overtime rate = $1\frac{1}{2} \times \$2.25 = \$3.375$
∴ Overtime pay = 5 × \$3.375 = \$ 16.88
∴ Gross earnings are \$106.88

CPP deduction	\$ 1.68
UIC deduction	1.50
Total	\$ 3.18

Taxable pay = \$106.88 − \$3.18 = \$103.70

Other deductions

Income tax	\$ 10.10
Canada Savings Bond	10.00
Health Insurance	1.40
Union dues	1.25
Total	\$ 22.75

∴ Richard takes home \$103.70 − \$22.75 = \$80.95.

Based on personal exemption of \$1878.

Note also that the \$1.68 for CPP and the \$1.50 for UIC are deducted from \$106.88 before income tax is calculated!

Exercise 29.2

Discussion

1. Describe some of the services provided in your province by
   (*a*) the Federal Government, and
   (*b*) the Provincial Government.

2. Besides Income Tax, what other sources of revenue has the Federal Government? The Provincial Government?

3. What facts must you know to find a person's weekly deduction for Income Tax in the table?

4. Which deductions required by the Federal Government must be subtracted from every pay during the year? Which deduction has a maximum amount for any one year and might not be deducted late in the year?

5. Why do you think our Government introduced
   (*a*) unemployment insurance, and
   (*b*) the Canada Pension Plan?

Written Solutions

6. State the amount deducted for Income Tax in each case.

	Earnings	*Personal Exemptions*
(*a*)	$100/week	$1878
(*b*)	$3/h for 40 h	1878
(*c*)	$3.35/h for 38 h	3874
(*d*)	$115/week	3522

7. Find the total deduction for Income Tax, Unemployment Insurance, and Canada Pension Plan.
   (*a*) Tom supports a wife and 2 children and earns $145/week.
   (*b*) Janet is single and earns $125/week.
   (*c*) Jim is married, works a 40 h week at $3.20/h, and each week puts in 5 h overtime.

8. What difficulties do you foresee in writing a computer program to determine CPP and Income Tax deductions for a company's various employees?

EXEMPTIONS
(1975)

self	$ 1878
spouse	$1644
each child	$ 352

6(a) $8.55 (c) $5.45
7(a) $12.30 (c) $17.47
(b) $18.71

9(a) $116.88, $99.63
(b) $147.00, $120.54

10. Dinah takes home $96.60 which is $14.94 more than Paisley.

9. For each of the following employees calculate
   (*i*) gross earnings, and
   (*ii*) net earnings if each person is single and pays $1.00 weekly union dues.

   George Wiley works the 08:00 to 16:00 shift, and Harry Stone works 15:00 to 23:00.

   (*a*)

WEEK ENDING	May 18
NAME	George Wiley
NUMBER	3058

DAY	IN	OUT	HOURS
M	07:50	16:30	
T	07:55	17:30	
W	08:00	18:00	
T	07:57	16:30	
F	08:00	16:00	
S			
S			

	HOURS	RATE	EARNINGS
REGULAR TIME		2.50	
OVERTIME			
TOTAL			

   (*b*)

WEEK ENDING	May 18
NAME	Harry Stone
NUMBER	4217

DAY	IN	OUT	HOURS
M	14:50	23:02	
T	14:53	00:00	
W	14:52	01:30	
T	14:59	00:30	
F	15:00	00:00	
S			
S			

	HOURS	RATE	EARNINGS
REGULAR TIME		3.00	
OVERTIME			
TOTAL			

10. Dinah Myte works 07:00 to 15:00 on a steady 40 h week at $3.20/h. She is single, has union dues of $1.25/week, and her miscellaneous deductions total $10.25/week. Paisley Print is also single and works in the office on a salary of $450/month, which is paid weekly. Her miscellaneous deductions total $9.50/week. Compare Dinah's and Paisley's take-home pays.

Project

1. Find the names of 2 unions and write a short account of each union describing the following points.
   - type of membership
   - type of executive and how selected
   - union affiliations
   - union dues
   - union benefits
2. Debate the issue "Unions do more good than harm."

“““
30
”””

Unit 30
Payroll

One figure that each of us can calculate without a mistake is the amount of our pay! This puts a real pressure on those in a business whose responsibility it is to produce the payroll.

The payroll system used, whether in a small company where one person does the complete job or in a large company with a computer-assisted payroll department, must include checks and cross-checks to avoid mechanical errors.

In addition, a company has a legal responsibility to the government and other organizations who receive the deductions subtracted from the gross pay. Unit 29 looked at earnings from the viewpoint of an employee. Now we turn to some of the reports and procedures used by an employer.

Payroll forms used by payroll departments of various companies are quite different from one another. The payroll forms used for the examples in this unit, however, are general in nature and will serve to demonstrate basic payroll techniques.

30.1 The Cumulative Earnings Record

A Each employer must keep careful records of the earnings and deductions of its work force. Large companies, with thousands of employees, applied the computer very quickly to this task. However, in small businesses, a "hand operation" is still used. Wherever you work, it is important that you retain your pay stubs so that you have your own personal check on the accuracy of the figures.

Income tax, unemployment insurance, vacation pay, in fact all your deductions should be checked both at the time of each pay and when the cumulative record is made available to you. Unemployment insurance benefits and vacation pay are affected by the total you earn over a long period. The limit on the Canada Pension Plan should be watched closely as the year goes by.

The following form is a typical earnings record used in a manual payroll operation.

Cumulative Earnings Record

Name	Joanne Jones		
Address	1101 Straight St.	*Exemptions*	$1878
Department	Finishing	*Rate*	$4.25/h

Date	*Total Hours*	*Reg. Pay*	*OT Pay*	*Gross Pay*	*UIC**	*CPP**	*Taxable Pay*	*Income* Tax*	*Union* Dues*	*Total Ded'ns*	*Net Pay*
Apr. 5	40										
Apr. 12	45										
Apr. 19	42										
Apr. 26	40										
Totals											

**Deductions*

1. Draw the earnings record shown on page 422 in your notebook. Anything over 40 h/week is overtime and union dues are $1.75/week.

2. Using the tables on pages 451 and 452, complete the record for the month of April.

3. This record is kept throughout the year. How would the employer make use of the totals for
   (*a*) UIC,
   (*b*) CPP,
   (*c*) income tax,
   (*d*) union dues?

B *Vacation Pay.* An employer is obligated by law to provide vacation pay for every worker. Normally, workers receive their regular weekly pay for the holiday period. However, those employed for just part of a year are entitled to 4% of gross earnings as vacation pay. The minimum holiday allowance is 2 weeks after 1 year of employment.

Example

During 1974, Clem Intyne earned $2350 at the Acme Farm Machinery Co. before he was laid off in April. Find the value of his vacation pay.

Solution

$$\begin{aligned}\text{Vacation pay} &= 4\% \text{ of } \$2350\\ &= 0.04 \times \$2350\\ &= \$94\end{aligned}$$

A common method used to handle vacation pay in cases like the example is for the employer to give a Vacation Pay book containing vacation pay stamps to the employee when he or she leaves the company. This book can be cashed on or after June 30 of each year at any chartered bank.

Exercise 30.1

Written Solutions

1. Copy and complete the following cumulative payroll records using the tables on pages 451 and 452.

(*a*)

Cumulative Earnings Record

Name	Isobel Ringing	*Employee No.*	1483
Address	2394 Gilded Cres.	*Exemptions*	$4578
Department	Shipping	*Rate*	$3.75/h

Date	*Total Hours*	*Reg. Pay*	*OT Pay*	*Gross Pay*	*UIC**	*CPP**	*Taxable Pay*	*Income* Tax*	*Union* Dues*	*Total Ded'ns*	*Net Pay*
Mar. 7	40										
Mar. 14	40										
Mar. 21	40										
Mar. 28	40										
Apr. 4	40										
Apr. 11	40										
Apr. 18	40										
Apr. 25	40										
Totals											

**Deductions*

(*b*)

Cumulative Earnings Record

Name	John Delaware	*Employee No.*	32015
Address	1640 Bronze Ct.	*Exemptions*	$3874
Department	Receiving	*Rate*	$4.80/h

Date	*Total Hours*	*Reg. Pay*	*OT Pay*	*Gross Pay*	*UIC**	*CPP**	*Taxable Pay*	*Income* Tax*	*Union* Dues*	*Total Ded'ns*	*Net Pay*
Feb. 2	45										
Feb. 9	42										
Feb. 16	48										
Feb. 23	46										
Totals											

**Deductions*

2. Calculate the vacation pay.
    (*a*) earnings of $3864.25
    (*b*) earnings of $4575
    (*c*) earnings of $1586.50

3. Olive Pitt is a pickle-taster for Paula's Premium Pickles; she receives $3.20/h. Olive has a personal tax exemption of $1878, the usual deductions for UIC, CPP, and income tax, union dues of $1/week, a Canada Savings Bond deduction of $20/week, and a health plan that requires a weekly deduction of $1.75. Prepare a Cumulative Earnings Record for Olive for the month of January. During pay periods ending January 6, 13, 20, and 27, she worked 52 h, 48 h, 42 h, and 45 h.

4. Mike Moulder is a tire builder for Smoothride Ltd. He receives $125/week plus $0.052 for each tire produced. During March the production was 250 tires, 350 tires, 320 tires, 400 tires, and 280 tires for weeks ending March 2, 9, 16, 23, and 30, respectively. Prepare a Cumulative Earnings Record for Mike who has a $4226 personal exemption, the usual federal deductions, union dues of $1.25/week, and a medical deduction of $3.50/week.

1(a) Total net pay $1103.92
(b) Total net pay $758.83
2(a) $154.57
3. Total net pay $433.56
4. Total net pay $628.69

Remember! Vacation pay is 4% of gross earnings.

30.2 The Payroll Summary

A Employers today are constantly taking steps to automate their operations. In many cases, automation is the result of the high cost of labor.

Hence, from the company's point of view, it is important that the labor cost of each article, and a breakdown of this cost for each step in production, be known. Payroll summaries or registers can be used for this purpose. They can also be used to determine the total deductions for income tax, UIC, and CPP, which must be sent regularly to the Federal Government.

1. Copy and complete the following payroll summary. Overtime is paid for time in excess of 8 h/day, and the personal exemption is $1878, in each case.

Emp.	*Hours*						*Total Hours*		*Wage*		*Gross*	*Deductions*			*Total*	*Net*
No.	*M*	*T*	*W*	*T*	*F*	*Rate*	*Reg.*	*O.T.*	*Reg.*	*O.T.*	*Earnings*	*CPP*	*UIC*	*Inc. Tax*	*Ded'ns*	*Earnings*
38	8	8	8	8	8	$2.50										
39	8	9	$7\frac{1}{2}$	8	8	2.95										
40	8	10	11	9	8	2.40										
41	10	8	—	8	10	3.50										
42	9	8	8	$7\frac{1}{2}$	$8\frac{1}{2}$	2.25										
Total																

B Business owners must operate their businesses at a profit and they need to know which areas of their operation are more profitable than others. This means that the labor cost must be broken down by department.

1. The following is an analysis of weekly labor costs for the A-Z Supermarket. Copy the table and complete it.

Department	*Job Description*	*No. of Employees*	*Weekly Rate*	*Total Cost*	*Dept. Totals*
Administration	Manager	1	$400		
	Asst. Manager	1	325		
	Head Cashier	1	175		
	Cashiers	8	125		
	Packers (part-time)	5	30		
Grocery	Department Head	1	250		
	Clerks	5	180		
	Pricers (part-time)	3	30		
Produce	Department Head	1	250		
	Trimmers	2	190		
	Clerks	2	180		
Meat	Department Head	1	250		
	Butchers	2	200		
	Packagers	3	190		

Exercise 30.2

Discussion

1. The employer's contribution to CPP equals the employee's share and to UIC is 1.4 times the employee's share. What amount should be forwarded to the Federal Government, if the total deduction on a company payroll for CPP is $1200, and for UIC is $1000?

2. If you are working at certain jobs, your pay will be in cash instead of a cheque. Two such jobs are casual farm labor and construction work. Using the largest possible denomination of bills (up to a $20 bill) or coins, how would you make up each of the following amounts?

   (*a*) $75.00 (*b*) $82.50 (*c*) $118.70

3. Overtime is usually paid at "time and a half." How many regular hours would you work to earn the same amount as for each of the following overtime periods?

   (*a*) 4 h (*b*) 8 h (*c*) 5 h

Written Solutions

4. (*a*) Complete the following payroll summary. Use a personal exemption of $1878 for each employee.

Employee Number	*Hours*		*Rate*	*Gross Pay*	*UIC**	*CPP**	*Taxable Pay*	*Income* Tax*	*Union* Dues*	*Total Ded'ns*	*Net Pay*
	Reg.	*O.T.*									
30243	40	5	$2.60								
30247	40	6	3.00								
30248	40	3	2.70								
30249	40	0	3.50								
30252	40	0	3.10								
30255	40	4	2.60								
30256	40	6	2.70								
Totals											

**Deductions*

(*b*) If these 7 employees represent a crew responsible for the production of 50 hand-finished, solid walnut tables during the week, find the cost of labor per table.

(*c*) State the amounts that would be forwarded to the Federal Government.

4(b) $18.78/table.
(c) $29.24 for C.P.P.
$30.46 for U.I.C.
$114.00 for income tax

Careful here! — Allow for employer's share! (See Question 1.)

5(b) 4.8%
6. Total net earnings $949.15

5. (*a*) The sales representatives for Custom Distributors are on a straight commission basis for their earnings. Complete the payroll summary for these sales reps, using the data shown for the week of March 7 to 14.

Sales Rep	*Sales*	*Percent Commission*	*Gross Earnings*	*Personal Exemption*	*Deductions*			*Total Ded'ns*	*Net Earnings*
					CPP	*UIC*	*Inc. Tax*		
J. R. Sel	$2500	5%		$3874					
R. E. Good	2000	5.5%		3874					
O. J. Day	2350	6%		4578					
R. L. Budicky	3500	4%		3522					
L. Wong	3200	4.5%		4226					
A. James	2650	4%		3522					
S. St. Jean	2160	5%		1878					
R. Proulx	3240	4.5%		1878					

(*b*) Calculate the average commission rate.

6. Champion Construction Ltd. uses cash to pay 7 employees working on a remote project. Complete the following Currency Analysis sheet, which shows how many of each denomination bill and coin are required for each employee's pay.

L.B. Blue
8 × $20 = $160.00
1 × 5 = 5.00
1 × 2 = 2.00
1 × 1 = 1.00
2 × 0.25 = 0.50
1 × 0.10 = 0.10
$168.60

Name	*Net Earnings*	*Number of each Denomination*							
		$20	$10	$5	$2	$1	0.25	0.10	0.05
L. B. Blue	$168.60	8		1	1	1	2	1	
H. Dumpty	135.20								
R. R. Hood	152.80								
Ms. Muffet	115.25								
B. Peep	128.10								
R. Stiltskin	140.45								
T. Tinker	108.75								
Total									
		× 20	× 10	× 5	× 2	× 1	× 0.25	× 0.10	× 0.05
Check									

Calculate the value of each denomination and record across the bottom row. The total net earnings should agree with the total value of the bottom row.

7. Pat and Mike's Motors Ltd. sells and services new and used cars. Using the weekly wages shown for each employee, prepare an analysis of labor costs for this dealership.

Office Staff	1 office manager	$220
	1 accountant	180
	1 cashier	130
Sales Representatives	2 reps	300
	3 reps	250
	2 reps	200
Service Dept.	2 clerks	120
	3 apprentice mechanics	130
	5 mechanics	225
	1 service manager	350

8. Arrange for a visit to local companies that calculate their payrolls (*a*) by hand, and (*b*) by computer. Discuss the advantages and disadvantages of each system.

9. Obtain the necessary T1 income tax forms from your local District Taxation Office. Assume vital information concerning a fictitious person of your choice (J. Dokes: salary $10 500, 3 dependents, union dues, and so on), and then determine the total federal and provincial income tax payable.

Maintaining Basic Skills

Exercise 6

1. For each situation, calculate the average monthly cost of operation and the operating cost/100 km. All figures represent annual costs.

Car	*Gas, Oil*	*Insurance*	*Licence*	*Repairs*	*Depreciation*	*Distance Driven* (km)
(*a*)	$400	$220	$30	$12	$ 780	21 000
(*b*)	335	182	27	92	450	15 000
(*c*)	784	419	45	158	1230	30 000
(*d*)	193	116	25	43	326	10 000

2. Determine the amount required to settle each invoice on the given date. (Late payment penalty is 1% per month.)

Invoice	*Terms*	*Date of Invoice*	*Amount*	*Settled on*
(*a*)	5/10, n/30	*August 19*	$ 83.90	August 23
(*b*)	n/90	*January 4*	$125.65	April 17
(*c*)	5/10 e.o.m., n/60	*March 12*	$356.80	April 8

3. Find the net price in each case.

	List Price	*Trade Discount*
(*a*)	$420	10%, 5%
(*b*)	$640	25%, 20%, 10%
(*c*)	$180	10%, 20%

4. Determine the single discount equivalent to the given series of discounts.

(*a*) 5%, 15%
(*b*) 20%, 25%, 40%
(*c*) 25%, 15%, 5%
(*d*) 5%, 15%, 25%

5. Find the missing information in each case.

Item	*Instalment Price*	*Instalment Terms*
mower	$80	$20 down; ☐/week for 12 weeks
sofa	$310	☐ down; $30/month for 9 months
car	☐	$100 down; $70/month for 24 months
radio	$50	$10 down; $2.50/week for ☐ weeks

1(a) $6.87 (c) $8.79
(b) $7.24 (d) $7.03

2(a) $79.71 (c) $338.96
(b) $126.19

3(a) $359.10 (c) $129.60

4(a) 19.25% (c) 39.44%

5. Mower $5/week
Car $1780

6. Determine the true interest rate on purchasing the mower of Question 5, if its cash price is $78.

7. Find the accumulated value of the following notes at the end of the period. Interest is compounded annually.

	Amount Invested	*Rate*	*Time (years)*
(*a*)	$300	7.5%	2
(*b*)	$800	$8\frac{1}{4}$%	1
(*c*)	$500	8%	3

8. Calculate the commission earnings based on the following information.

	Sales	*Rate of Commission*
(*a*)	$32 000	3.8%
(*b*)	$22 750	4.5%

9. Compute the total of the deductions for CPP, income tax, and UIC for the following workers.
   - (*a*) Herb A. has a personal exemption of $3522 and earns $126.50/week.
   - (*b*) Joyce B. has a personal exemption of $1878 and earns $118.25/week.

10. Complete the following Cumulative Earnings Record. Time in excess of 40 h is overtime and paid at $1\frac{1}{2}$ times the regular rate.

Cumulative Earnings Record

Name *I.N. Voyce*
Department *Warehouse*
Exemptions $4226
Hourly rate $2.85

Date	*Total Hours*	*Reg. Pay*	*O.T. Pay*	*Gross Pay*	*Deductions*				*Net Pay*
					UIC	*CPP*	*Inc. Tax*	*Total*	
Feb. 6	42								
13	45								
20	46								
27	40								
Total	173								

6. 27.6%
7(a) $346.69
8(b) $1023.75
9(b) $16.85
10. Total net pay $481.20

Summary of Units 20 to 30

	Do you understand . . . ?	*Can you . . . ?*
Unit 20 Introduction to Vectors		
20.1 Experimenting with Forces	force and reaction force vector quantity geometric vector scalar quantity	name a vector in 2 ways use a scale diagram to represent a vector
20.2 Working with Parallel Vectors	equal vectors opposite vectors parallel vectors resultant vector vector sum zero vector	add parallel vectors
20.3 Multiplying Vectors by Scalars	scalar multiple of a vector	express any vector as a scalar multiple of a given parallel vector
Unit 21 Adding Non-Parallel Vectors		
21.1 Resultant of Two Vectors	equilibrant Parallelogram Law of Vector Addition Triangle Law of Vector Addition	use the Parallelogram Law or the Triangle Law to determine the resultant of 2 vectors (by means of a scale diagram)
21.2 Vector Polygons	vector polygon	use the Polygon Law of Vector Addition to find the resultant of several vectors
Unit 22 Application of Vectors		
22.1 Navigation Problems	bearing and opposite bearing	draw accurate scale diagrams to solve navigation problems involving bearing
22.2 Physics and Shop Problems	resultant equilibrant equilibrium	use a scale diagram to determine the resultant and equilibrant of a system of forces

	Do you understand . . . ?	*Can you . . . ?*
Unit 23 **Insurance**		
23.1 Insuring Your Property	policy face premium Homeowner's Policy co-insurance clause indemnity	use tables to calculate Homeowners' insurance premiums or return of premiums calculate the loss paid under a co-insurance clause
23.2 Insuring Your Life	beneficiary level premium system term insurance whole life insurance limited-payment life insurance endowment insurance "par" and "non-par" policies dividend non-forfeiture benefits cash value paid-up insurance extended term insurance group insurance	use tables to calculate premium payments on each type of policy calculate premiums based on monthly, quarterly, or semi-annual payments
Unit 24 **Simple Interest**		
24.1 Calculating Interest	percent interest, principal, rate, time, amount per annum the simple interest formula	convert percents to decimals or fractions use the formula $I = Prt$ to calculate interest
24.2 Finding the Amount	compound interest annually, semi-annually, or quarterly compounded interest	use the formula $A = P + I$ to determine the amount of a loan complete an interest table to show the amount of a loan
24.3 Manipulating Interest Formulas		isolate P, r, or t in the formula $I = Prt$

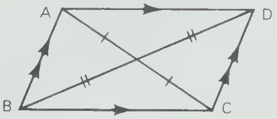
	Do you understand. . . ?	*Can you. . . ?*
Unit 25 **Owning an Automobile**		
25.1 Operating Expenses	depreciation average monthly cost of operating a car cost/100 km	use the table of depreciation rates to find the value of a used car calculate monthly cost and the cost/100 km of operating a car
25.2 Insuring an Automobile or Motorcycle	third-party liability planned PD, medical, collision, and comprehensive coverage deductible clause	use tables to calculate the annual premiums for automobile or motor cycle insurance
25.3 Purchasing an Automobile	initial cost interest charges replacement cost operating costs	calculate interest charges when purchasing a new automobile on "time" compare the cost of owning a car with the cost of leasing a car
Unit 26 **Direct Financing**		
26.1 Discounts on Cash Sales	credit buying direct financing terms of an invoice net amount retail discounts list price and net price trade discounts	determine the cash discount on a dated invoice under the terms of the invoice determine the net price when the list price is subjected to a discount find the single discount equivalent to a series of discounts calculate the average loan for an instalment purchase
26.2 Instalment Buying	buying "on time" true interest rate average loan carrying charge	use the formula $r = \frac{2NC}{P(n + 1)}$ to determine the true interest rate on an instalment purchase
26.3 Retail Buying and Credit	charge accounts service charge credit cards revolving credit account maximum debt limit monthly payment chart credit rating	calculate interest charges and payments required on a revolving credit account

	Do you understand . . . ?	*Can you . . . ?*
Unit 27 **Third-Party Financing**		
27.1 Borrowing From a Bank	promissory note discount, discount rate, proceeds, face, discount period, due date 3 days of grace personal instalment loan plan note demand note collateral credit rating	calculate the proceeds of a promissory note that is discounted find the actual interest rate on a discounted promissory note use a table to determine total interest charged on a demand note
27.2 Borrowing From a Credit Union or Loan Company	credit union shares loan committee collateral interest rebate shareholder dividend loan or finance company instalment loans	make a table showing the calculation of interest on a loan from a credit union calculate the true rate of interest charged by a finance company
Unit 28 **Saving and Investing Money**		
28.1 Deposit Accounts	savings and personal chequing accounts minimum monthly balance	calculate interest for a bank account or a credit union account
28.2 Investing in Short-Term Notes, Bonds, and Stocks	30-, 60-, or 90-day certificates trust certificates bonds stocks mutual fund	calculate the accumulated value of a certificate or a bond

	Do you understand . . . ?	*Can you . . . ?*
Unit 29 **Earning Money**		
29.1 Calculating Gross Earnings	the differences between payment by salary, by wage, and by commission overtime payments shift premiums lateness penalty incentive rate	convert an annual salary to a weekly or a monthly amount compute gross earnings using a time card calculate commission or incentive earnings
29.2 Calculating "Take-Home" Pay	income tax unemployment insurance (UIC) Canada Pension Plan (CPP)	use the tables to find deductions for income tax, UIC, and CPP calculate taxable income and "take-home" pay
Unit 30 **Payroll**		
30.1 The Cumulative Earnings Record	vacation pay	complete a Cumulative Earnings Record calculate vacation pay
30.2 The Payroll Summary	analysis of labor costs currency analysis for a payroll	complete a Payroll Summary calculate the amounts to be sent to the Federal Government for income tax, UIC, and CPP

Review Exercise, Units 20 to 30

1. In the diagram, locate and name segments that represent
   (*a*) equal vectors
   (*b*) opposite vectors
   (*c*) parallel vectors

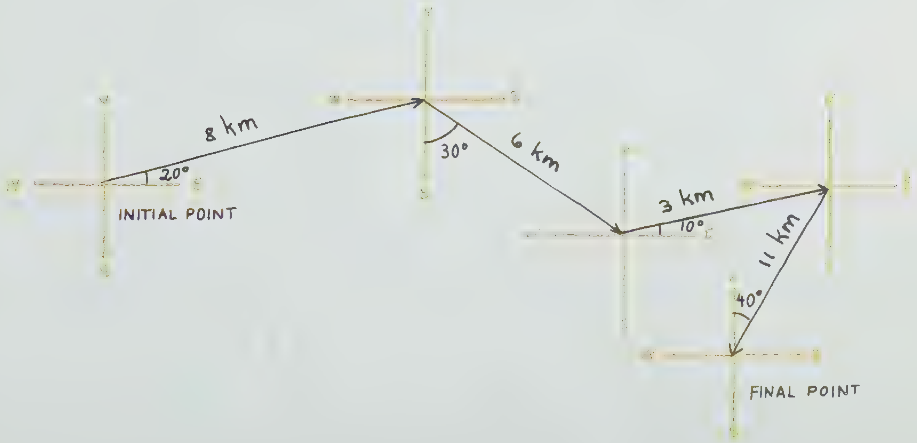


2. Use a scale diagram to find the resultant of each pair of forces acting at a point.

	Forces	*Angle between forces*
(*a*)	25 N, 40 N	35°
(*b*)	29 N, 15 N	72°
(*c*)	80 N, 120 N	119°

3. A ship on a bearing of 145° is travelling at 30 km/h and experiences a current of 8 km/h from the west and a wind of 15 km/h from the south. Determine the actual speed and direction of the ship.

4. On graph paper, make a scale diagram of the sketch, and determine
   (*a*) distance from initial point to final point,
   (*b*) bearing of initial point from final point,
   (*c*) bearing of final point from initial point.

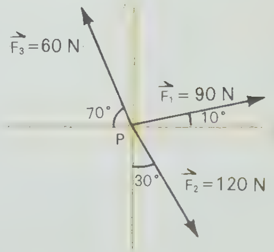


5. Make a scale diagram of the system of forces acting at point P, and use the Parallelogram Law to find the equilibrant of the system. (See margin diagram.)

2(a) 62 N at 13° to 40 N force
3. 27 km/h, bearing 111°
4(a) 12.2 km
5. 132 N, at 56° to 60 N force

Remember?
N
means
"newtons"
(unit of force)

5.



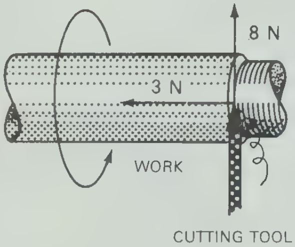
6. At 12 00 a ship heads out from harbor on a bearing of 190°, maintains a constant speed of 20 km/h, and makes course changes as follows.

Time	*New bearing*
13 00	130°
14 30	50°
15 45	155°

At 17 00, determine the distance of the ship from the harbor and the bearing of the harbor from the ship.

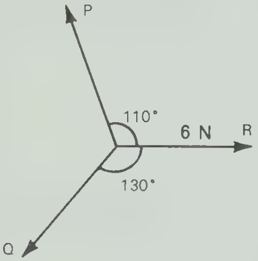
7. A cutting tool on a lathe exerts 2 forces on the work, as shown. Use a scale diagram to find the resultant force that the cutting tool exerts on the work.

7.



8. Forces P, Q, and R act on a point O, and are in equilibrium. Use a scale diagram to find the size of forces P and Q.

8.



9. Find the cost of a \$1500 loan at 7.5% for 7 months.

10. How long will it take \$1000 to amount to \$1019.73 at 8% interest?

11. Fenroy Finsterwald invests \$2000 in a business venture. If interest is compounded semi-annually at 9%/year, use a table to determine the amount of Fenroy's investment at the end of 2 years.

12. What interest rate must be paid for \$1200 to accumulate to \$1404 after 24 months?

13. Mr. C. D. Smashup purchases a \$200 000 PL and PD, \$50-deductible collision policy, with comprehensive and medical coverage, for his new car. If Mr. Smashup is 23 years old and drives his car to and from work in the city, what is his annual insurance premium?

14. Cindy Cyclemore (age 28) purchases a "350" motorbike for \$1000 to ride to and from work in the city. If Cindy buys a \$200 000 PL and PD, \$50-deductible collision insurance policy, with comprehensive coverage, and is allowed a 15% reduction in her total premium because of her driver-training certificate, calculate her total annual insurance premium.

6. 67 km, 313°
7. 8.5 N at 69° to the 3 N force
8. Magnitude of P is 5.3 N; magnitude of Q is 6.5 N.
9. \$65.63
10. 90 days
11. \$2385.04
12. 8.5%
13. \$529
14. \$127.33

15. Celia Citizen receives the invoice shown below for renovations to her house.

Home Repair Service *Date* June 4

Customer Celia Citizen

Item	*Materials*	*Labor*
Paint	$ 35.40	$ 30.00
Shingles	42.00	15.50
Eavestroughing	18.50	7.25
Patio stones	74.88	55.00

Terms: 5/10, n/30, 1%/ month interest charged on all overdue accounts

Determine the amount that will settle the account on each of the following dates.

(*a*) June 10
(*b*) July 2
(*c*) Aug. 4

16. Milford Mueller can purchase a dining room suite from the Acme Furniture Company for $475, less 10% and 15%, or he can obtain the same suite from the N. E. Wunder Company for $500, less 5% and 25%. Which company offers the better buy, and by what amount?

17. A camper trailer may be purchased for $650 cash, or may be bought for $100 down and $35 per month for 18 months. Calculate the true rate of interest being charged.

18. Mrs. U. B. Long has a revolving credit account with the U. Buymore department store. Her balance on May 1 was $46.25. During May and June she purchased items as follows.

Date	*Purchase*
May 5	$ 9.50
May 27	14.98
June 17	19.95

She also returned goods worth $11.49 on June 8. If the store charges 1.5% on the unpaid balance of the previous month, calculate Mrs. Long's balance on June 20.

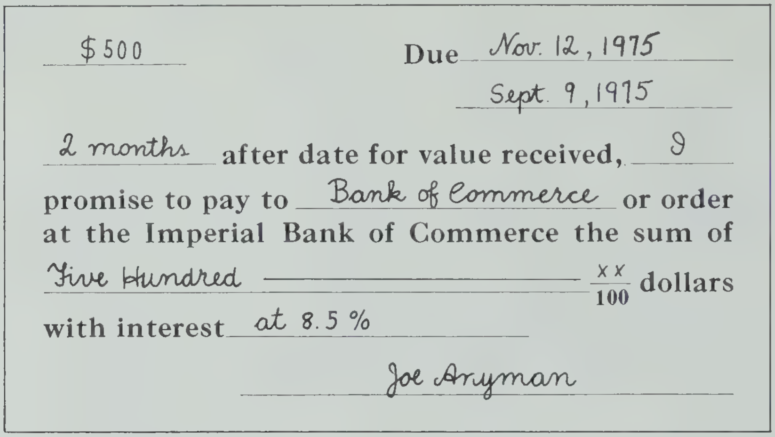
HOME
REPAIR
SERVICE

Recall $r = \frac{2NC}{P(n+1)}$

Use the chart, page 379!

15(a) $264.60 (c) $281.32
17. 18.4%
18. $74.88

19. Determine the proceeds of the promissory note shown below.



$500 Due Nov. 12, 1975

Sept. 9, 1975

2 months after date for value received, I promise to pay to Bank of Commerce or order at the Imperial Bank of Commerce the sum of Five Hundred ——— $\frac{xx}{100}$ dollars with interest at 8.5 %

Joe Aryman

Why not Nov. 9?

20. Simone G. Funkel borrows $600 on July 14 on a demand note at 9% and makes payments as follows: July 31, $200; Aug. 31, $150; Sept. 20, $150; Sept. 30, $100. Use a table to calculate Simone's total cost for the loan.

21. The E. Z. Credit Company offers $800 in cash in return for 12 equal monthly instalments based on principal and interest. If the total of interest and service charges is $123, determine
    (*a*) the amount of each monthly instalment, and
    (*b*) the true rate of interest charged.

22. Clarissa Clinkhammer borrows $1000 from her credit union on April 1, agreeing to repay the loan in 8 monthly instalments of $125 each as well as interest at the rate of 1%/month on the unpaid balance.
    (*a*) Use a table to help calculate Clarissa's total interest payments.
    (*b*) If a rebate of 15% is given on December 31, determine the actual interest rate Clarissa paid for the loan.

19. $492.55
20. $7.06
21(a) $76.92
(b) 28.4%
22(a) $45 (b) 10.2%

23. Mortimer Mobile is a travelling sales rep who drives 63 000 km in one year. Expenses during the year are as follows: gas, oil, lubrication, \$996; insurance, \$383; licence, \$35; repairs, \$290; depreciation \$625. Calculate
    (*a*) Mort's average monthly cost of operating the car, and
    (*b*) the cost/100 km to operate the car.

24. Arlene Anygirl buys a new car worth \$4200 by paying \$1000 down and paying off the balance in 24 monthly instalments of \$140 each. If Arlene had been able to invest the \$1000 down payment at 7.5%/year, determine her total interest cost.

25. Jennifer Homebody wishes to purchase a deluxe \$25 000 homeowner's insurance policy for her frame house.
    (*a*) Calculate Jennifer's premium for a 1-year policy.
    (*b*) If she purchased a 1-year policy and cancelled it 47 days later, what refund should she receive from the insurance company?
    (*c*) If Jennifer's property is worth \$27 000 and her policy contained an 80% co-insurance clause, what loss would she have to absorb if a fire caused \$12 000 damage?

Use the tables pages 448 and 449!

$$\left.\begin{matrix}\text{Loss}\\\text{paid}\end{matrix}\right\} = \frac{\text{Face}}{80\%\text{ of value}} \times \text{Loss}$$

26. If the policies run full term, what would be the difference in cost between
    (*a*) an endowment to age 60 policy for \$20 000 issued at age 35, with quarterly premium payments, and
    (*b*) a limited-payment to age 65 policy for \$25 000 issued at age 25, with monthly premium payments?

Table, page 450!

27. "No-Hit" Norman signed as a pitcher with a major league baseball team and received a bonus of \$15 000 for signing. Until he should decide how to use this sum, his manager advised him to purchase a 7.5% 60-day note. Find the value of the bonus 60 days later.

28. To finance local improvements many cities will now sell debentures "over-the-counter" to citizens who wish to invest in this way. Florence Foresight purchased a \$1000 debenture whose interest rate was $9\frac{1}{4}\%$. Calculate the value of the annual interest coupon on this debenture.

23(a) \$194.08
(b) \$3.70/100 km

24. \$315.63

25(a) \$87
(b) \$65.25

26. \$1230.00

27. \$15 184.93

28. \$92.50

29. John decides to save money to purchase a car. On January 1, 1975 he deposits $500 in a savings account that pays 6% interest, compounded annually. He continues to save by depositing $500 on January 1, 1976 and January 1, 1977. Find the value of his account at the end of 1977.

30. Which pays better, and by how much per week, a job paying $3.35/h for a 40 h week or another that pays $585/month?

31. Find the "take-home" pay for the following employees of Saunders Supply Ltd.
    (*a*) Jean is a shipper earning $105.50 weekly. Her personal exemptions is $1878 and miscellaneous deductions total $15/week.
    (*b*) Jack is a sales representative who earns $80/week plus 5% on sales in excess of $2000/week. His personal exemptions are $3874, and his sales are $3200 for the first week of April.
    (*c*) Jill earns $2.45/h on an assembly line and is paid overtime at "time-and-a-half" for more than 8 h in a day. Jill is single, pays union dues of $1.25/week, and for the week of March 20 works as follows: Monday, 9 h; Tuesday $8\frac{1}{2}$ h; Wednesday, $7\frac{1}{2}$ h; Thursday, 10 h; Friday $9\frac{1}{2}$ h.

32. Complete the following payroll register using an $1878 exemption for each employee.

Employee	*Hours*		*Rate*	*Gross Pay*	*UIC**	*CPP**	*Taxable Pay*	*Income* Tax*	*Union* Dues*	*Total Ded'ns*	*Net Pay*
	Reg.	*OT*									
J. D. Taylor	40	2	$3.10								
W. F. Morton	40	5	2.70								
W. C. Guyatt	40	8	2.60								
E. C. Jones	40	4	3.20								
N. E. Orr	40	1	3.50								
Totals											

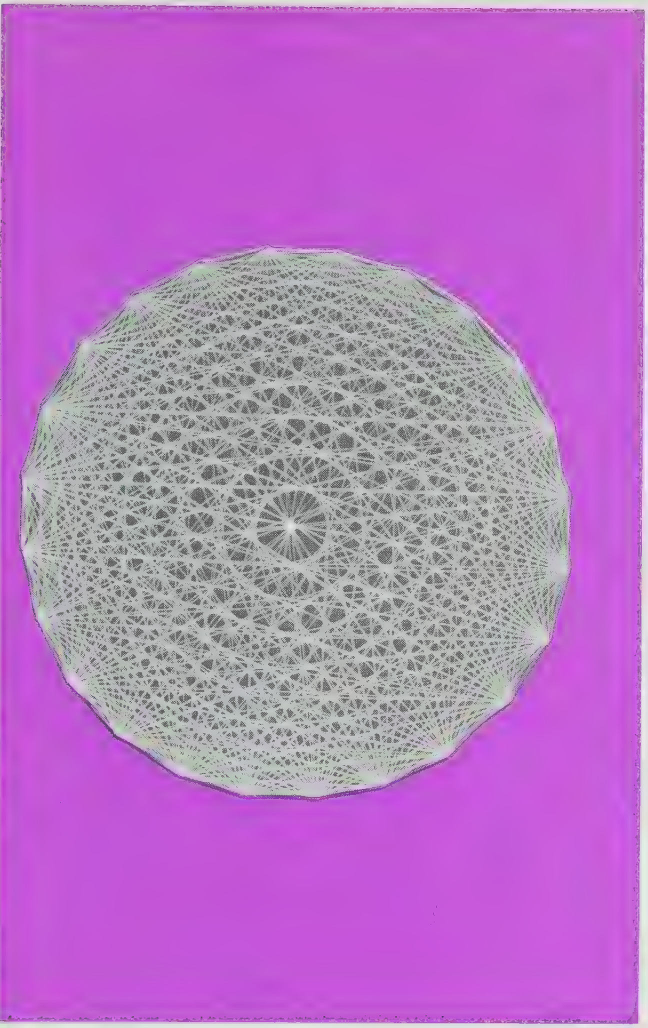
**Deductions*

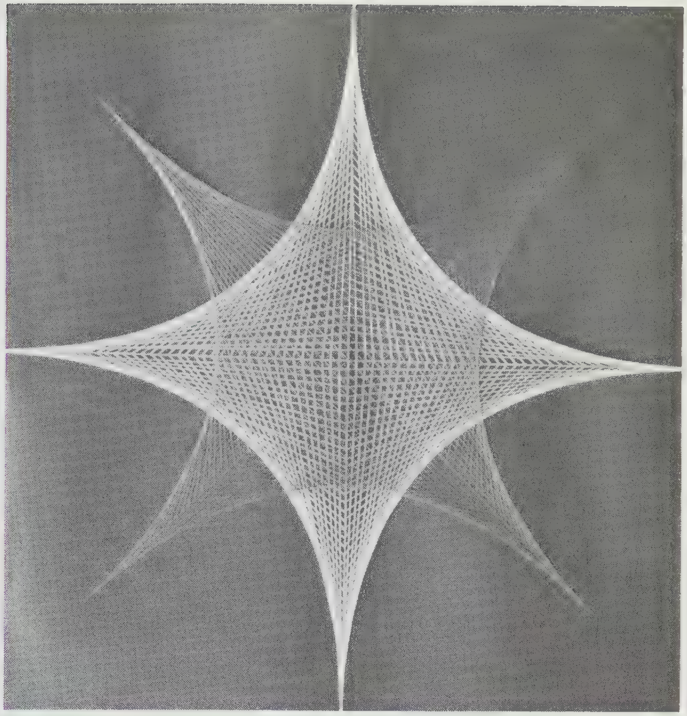
29. $1687.31

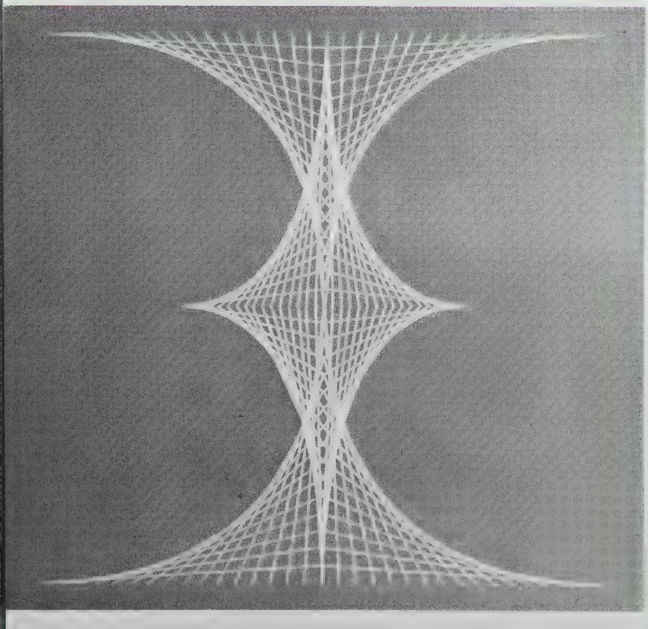
30. $585/month is $1/week better

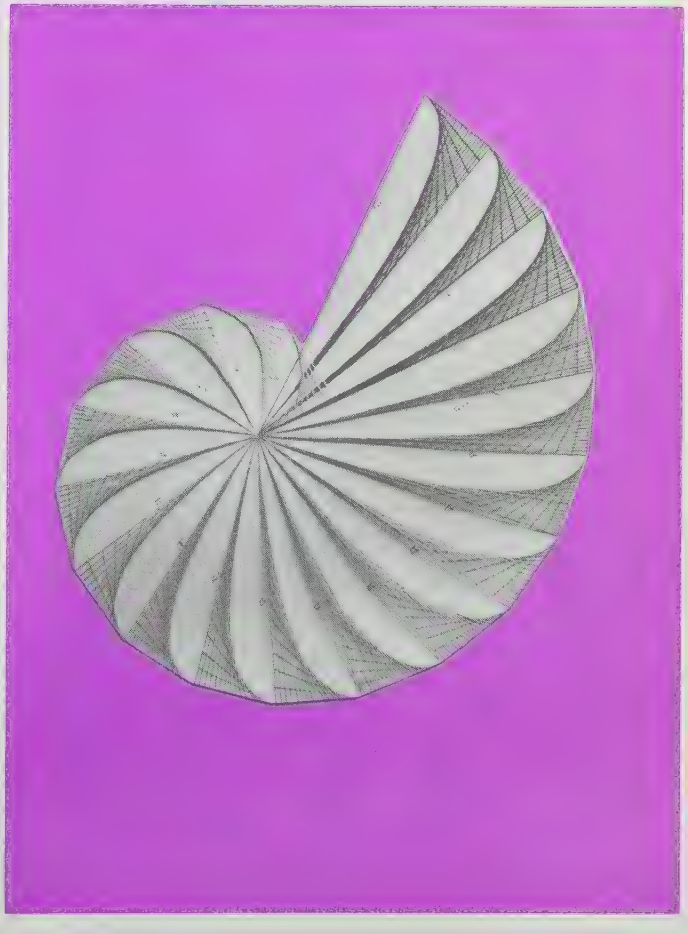
31(a) $77.26 (c) $98.26
(b) $127.16

32. Total net earnings $576.00









Square Root Table

n	$\sqrt{n}$	n	$\sqrt{n}$	n	$\sqrt{n}$	n	$\sqrt{n}$	n	$\sqrt{n}$
1	1.00	51	7.14	101	10.05	151	12.29	201	14.18
2	1.41	52	7.21	102	10.10	152	12.33	202	14.21
3	1.73	53	7.28	103	10.15	153	12.37	203	14.25
4	2.00	54	7.35	104	10.20	154	12.41	204	14.28
5	2.24	55	7.42	105	10.25	155	12.45	205	14.32
6	2.45	56	7.48	106	10.30	156	12.49	206	14.35
7	2.65	57	7.55	107	10.34	157	12.53	207	14.39
8	2.83	58	7.62	108	10.39	158	12.57	208	14.42
9	3.00	59	7.68	109	10.44	159	12.61	209	14.46
10	3.16	60	7.75	110	10.49	160	12.65	210	14.49
11	3.32	61	7.81	111	10.54	161	12.69	211	14.53
12	3.46	62	7.87	112	10.58	162	12.73	212	14.56
13	3.61	63	7.94	113	10.63	163	12.77	213	14.59
14	3.74	64	8.00	114	10.68	164	12.81	214	14.63
15	3.87	65	8.06	115	10.72	165	12.85	215	14.66
16	4.00	66	8.12	116	10.77	166	12.88	216	14.70
17	4.12	67	8.19	117	10.82	167	12.92	217	14.73
18	4.24	68	8.25	118	10.86	168	12.96	218	14.76
19	4.36	69	8.31	119	10.91	169	13.00	219	14.80
20	4.47	70	8.37	120	10.95	170	13.04	220	14.83
21	4.58	71	8.43	121	11.00	171	13.08	221	14.87
22	4.69	72	8.49	122	11.05	172	13.11	222	14.90
23	4.80	73	8.54	123	11.09	173	13.15	223	14.93
24	4.90	74	8.60	124	11.14	174	13.19	224	14.97
25	5.00	75	8.66	125	11.18	175	13.23	225	15.00
26	5.10	76	8.72	126	11.22	176	13.27	226	15.03
27	5.20	77	8.77	127	11.27	177	13.30	227	15.07
28	5.29	78	8.83	128	11.31	178	13.34	228	15.10
29	5.39	79	8.89	129	11.36	179	13.38	229	15.13
30	5.48	80	8.94	130	11.40	180	13.42	230	15.17
31	5.57	81	9.00	131	11.45	181	13.45	231	15.20
32	5.66	82	9.06	132	11.49	182	13.49	232	15.23
33	5.74	83	9.11	133	11.53	183	13.53	233	15.26
34	5.83	84	9.17	134	11.58	184	13.56	234	15.30
35	5.92	85	9.22	135	11.62	185	13.60	235	15.33
36	6.00	86	9.27	136	11.66	186	13.64	236	15.36
37	6.08	87	9.33	137	11.70	187	13.67	237	15.39
38	6.16	88	9.38	138	11.75	188	13.71	238	15.43
39	6.24	89	9.43	139	11.79	189	13.75	239	15.46
40	6.32	90	9.49	140	11.83	190	13.78	240	15.49
41	6.40	91	9.54	141	11.87	191	13.82	241	15.52
42	6.48	92	9.59	142	11.92	192	13.86	242	15.56
43	6.56	93	9.64	143	11.96	193	13.89	243	15.59
44	6.63	94	9.70	144	12.00	194	13.93	244	15.62
45	6.71	95	9.75	145	12.04	195	13.96	245	15.65
46	6.78	96	9.80	146	12.08	196	14.00	246	15.68
47	6.86	97	9.85	147	12.12	197	14.04	247	15.72
48	6.93	98	9.90	148	12.17	198	14.07	248	15.75
49	7.00	99	9.95	149	12.21	199	14.11	249	15.78
50	7.07	100	10.00	150	12.25	200	14.14	250	15.81

Square Root Table

n	$\sqrt{n}$	n	$\sqrt{n}$	n	$\sqrt{n}$	n	$\sqrt{n}$	n	$\sqrt{n}$
251	15.84	301	17.35	351	18.73	401	20.02	451	21.24
252	15.87	302	17.38	352	18.76	402	20.05	452	21.26
253	15.91	303	17.41	353	18.79	403	20.07	453	21.28
254	15.94	304	17.44	354	18.81	404	20.10	454	21.31
255	15.97	305	17.46	355	18.84	405	20.12	455	21.33
256	16.00	306	17.49	356	18.87	406	20.15	456	21.35
257	16.03	307	17.52	357	18.89	407	20.17	457	21.38
258	16.06	308	17.55	358	18.92	408	20.20	458	21.40
259	16.09	309	17.58	359	18.95	409	20.22	459	21.42
260	16.12	310	17.61	360	18.97	410	20.25	460	21.45
261	16.16	311	17.64	361	19.00	411	20.27	461	21.47
262	16.19	312	17.66	362	19.03	412	20.30	462	21.49
263	16.22	313	17.69	363	19.05	413	20.32	463	21.52
264	16.25	314	17.72	364	19.08	414	20.35	464	21.54
265	16.28	315	17.75	365	19.10	415	20.37	465	21.56
266	16.31	316	17.78	366	19.13	416	20.40	466	21.59
267	16.34	317	17.80	367	19.16	417	20.42	467	21.61
268	16.37	318	17.83	368	19.18	418	20.45	468	21.63
269	16.40	319	17.86	369	19.21	419	20.47	469	21.66
270	16.43	320	17.89	370	19.24	420	20.49	470	21.68
271	16.46	321	17.92	371	19.26	421	20.52	471	21.70
272	16.49	322	17.94	372	19.29	422	20.54	472	21.73
273	16.52	323	17.97	373	19.31	423	20.57	473	21.75
274	16.55	324	18.00	374	19.34	424	20.59	474	21.77
275	16.58	325	18.03	375	19.36	425	20.62	475	21.79
276	16.61	326	18.06	376	19.39	426	20.64	476	21.82
277	16.64	327	18.08	377	19.42	427	20.66	477	21.84
278	16.67	328	18.11	378	19.44	428	20.69	478	21.86
279	16.70	329	18.14	379	19.47	429	20.71	479	21.89
280	16.73	330	18.17	380	19.49	430	20.74	480	21.91
281	16.76	331	18.19	381	19.52	431	20.76	481	21.93
282	16.79	332	18.22	382	19.54	432	20.78	482	21.95
283	16.82	333	18.25	383	19.57	433	20.81	483	21.98
284	16.85	334	18.28	384	19.60	434	20.83	484	22.00
285	16.88	335	18.30	385	19.62	435	20.86	485	22.02
286	16.91	336	18.33	386	19.65	436	20.88	486	22.05
287	16.94	337	18.36	387	19.67	437	20.90	487	22.07
288	16.97	338	18.38	388	19.70	438	20.93	488	22.09
289	17.00	339	18.41	389	19.72	439	20.95	489	22.11
290	17.03	340	18.44	390	19.75	440	20.98	490	22.14
291	17.06	341	18.47	391	19.77	441	21.00	491	22.16
292	17.09	342	18.49	392	19.80	442	21.02	492	22.18
293	17.12	343	18.52	393	19.82	443	21.05	493	22.20
294	17.15	344	18.55	394	19.85	444	21.07	494	22.23
295	17.18	345	18.57	395	19.87	445	21.10	495	22.25
296	17.20	346	18.60	396	19.90	446	21.12	496	22.27
297	17.23	347	18.63	397	19.92	447	21.14	497	22.29
298	17.26	348	18.65	398	19.95	448	21.17	498	22.32
299	17.29	349	18.68	399	19.97	449	21.19	499	22.34
300	17.32	350	18.71	400	20.00	450	21.21	500	22.36

Table of Trigonometric Ratios

θ	sin θ	cos θ	tan θ
0	0.000	1.000	0.000
1	0.017	1.000	0.017
2	0.035	0.999	0.035
3	0.052	0.999	0.052
4	0.070	0.998	0.070
5	0.087	0.996	0.087
6	0.105	0.995	0.105
7	0.122	0.993	0.123
8	0.139	0.990	0.141
9	0.156	0.988	0.158
10	0.174	0.985	0.176
11	0.191	0.982	0.194
12	0.208	0.978	0.213
13	0.225	0.974	0.231
14	0.242	0.970	0.249
15	0.259	0.966	0.268
16	0.276	0.961	0.287
17	0.292	0.956	0.306
18	0.309	0.951	0.325
19	0.326	0.946	0.344
20	0.342	0.940	0.364
21	0.358	0.934	0.384
22	0.375	0.927	0.404
23	0.391	0.921	0.424
24	0.407	0.914	0.445
25	0.423	0.906	0.466
26	0.438	0.899	0.488
27	0.454	0.891	0.510
28	0.469	0.883	0.532
29	0.485	0.875	0.554
30	0.500	0.866	0.577
31	0.515	0.857	0.601
32	0.530	0.848	0.625
33	0.545	0.839	0.649
34	0.559	0.829	0.675
35	0.574	0.819	0.700
36	0.588	0.809	0.727
37	0.602	0.799	0.754
38	0.616	0.788	0.781
39	0.629	0.777	0.810
40	0.643	0.766	0.839
41	0.656	0.755	0.869
42	0.669	0.743	0.900
43	0.682	0.731	0.933
44	0.695	0.719	0.966
45	0.707	0.707	1.000
46	0.719	0.695	1.036
47	0.731	0.682	1.072
48	0.743	0.669	1.111
49	0.755	0.656	1.150
50	0.766	0.643	1.192
51	0.777	0.629	1.235
52	0.788	0.616	1.280
53	0.799	0.602	1.327
54	0.809	0.588	1.376
55	0.819	0.574	1.428
56	0.829	0.559	1.483
57	0.839	0.545	1.540
58	0.848	0.530	1.600
59	0.857	0.515	1.664
60	0.866	0.500	1.732
61	0.875	0.485	1.804
62	0.883	0.469	1.881
63	0.891	0.454	1.963
64	0.899	0.438	2.050
65	0.906	0.423	2.145
66	0.914	0.407	2.246
67	0.921	0.391	2.356
68	0.927	0.375	2.475
69	0.934	0.358	2.605
70	0.940	0.342	2.747
71	0.946	0.326	2.904
72	0.951	0.309	3.078
73	0.956	0.292	3.271
74	0.961	0.276	3.487
75	0.966	0.259	3.732
76	0.970	0.242	4.011
77	0.974	0.225	4.331
78	0.978	0.208	4.705
79	0.982	0.191	5.145
80	0.985	0.174	5.671
81	0.988	0.156	6.314
82	0.990	0.139	7.115
83	0.993	0.122	8.144
84	0.995	0.105	9.514
85	0.996	0.087	11.430
86	0.998	0.070	14.301
87	0.999	0.052	19.081
88	0.999	0.035	28.636
89	1.000	0.017	57.290
90	1.000	0.000	

AUTOMOBILE INSURANCE

Table 1 Typical annual premiums for a $200 000 inclusive Public Liability and Property Damage Insurance policy.

AGE OF DRIVER(S)	URBAN AREA			RURAL AREA		
	AUTOMOBILE USED FOR			AUTOMOBILE USED FOR		
	BUSINESS	COMMUTING	PLEASURE	BUSINESS	COMMUTING	PLEASURE
Over 25-	$129	$104	$ 81	$108	$ 82	$ 67
One over 25; one under 25	$250	$200	$159	$210	$161	$132
Under 25	$310	$283	$245	$262	$214	$195

Table 2 Typical annual premiums for a $50-deductible Collision Insurance policy.

AGE OF DRIVER(S)	URBAN AREA						RURAL AREA					
	AUTOMOBILE USED FOR						AUTOMOBILE USED FOR					
	BUSINESS		COMMUTING		PLEASURE		BUSINESS		COMMUTING		PLEASURE	
	NEW	OLD	NEW	OLD	NEW	OLD	NEW	OLD	NEW	OLD	NEW	OLD
Over 25	$101	$ 51	$ 75	$ 38	$ 56	$ 24	$ 90	$ 45	$ 69	$ 35	$ 55	$ 23
Under 25	$220	$ 88	$220	$ 88	$220	$ 88	$212	$106	$212	$106	$212	$106

Table 3 Typical annual premiums for a Comprehensive Insurance policy.

AGE OF DRIVER	URBAN AREA		RURAL AREA	
	NEW	OLD	NEW	OLD
Over 25	$ 17	$ 9	$ 19	$ 10
Under 25	$ 23	$ 12	$ 26	$ 13

MOTORCYCLE INSURANCE

Table 4 PL and PD Insurance ($200 000 inclusive) These rates are based on "pleasure" rates for automobiles and then adjusted as indicated. Premium = Basic rate × Adjustment factor

(*a*) Basic rate

AGE	URBAN	RURAL
Over 25	$ 81	$ 67
Under 25	$245	$195

(*b*) Adjustment factor

* BIKE SIZE	FACTOR
0 -100	40%
101 -200	60%
201 -500	100%
above 500	125%

*
Bikes are rated by engine size. "500" means 500 cm^3.

Table 5 Collision Insurance ($50-Deductible)

Premium = Basic rate × Adjustment factor

(*a*) Basic rate

AGE	RATE
Over 25	$ 56
Under 25	$118

(*b*) Adjustment factor

LIST PRICE	FACTOR
$0- 250	40%
251- 500	60%
501- 1000	80%
1001- 2000	100%
above 2000	125%

Table 6 Comprehensive Insurance

LIST PRICE	PREMIUM
$0- 250	$12
251- 500	$18
501- 1000	$24
1001- 2000	$30
above 2000	$36

COMPANY PREMIUM RETENTION FOR CANCELLED POLICIES

TIME POLICY IN FORCE UP TO	PERCENT OF PREMIUM KEPT BY INSURANCE COMPANY	TIME POLICY IN FORCE UP TO	PERCENT OF PREMIUM KEPT BY INSURANCE COMPANY	TIME POLICY IN FORCE UP TO	PERCENT OF PREMIUM KEPT BY INSURANCE COMPANY
15 days	13%	135 days	47%	255 days	76%
1 month	19%	5 months	50%	9 months	80%
45 days	22%	165 days	51%	285 days	83%
2 months	25%	6 months	52%	10 months	86%
75 days	31%	195 days	55%	315 days	90%
3 months	35%	7 months	60%	11 months	93%
105 days	39%	225 days	65%	345 days	96%
4 months	43%	8 months	73%	12 months	100%

HOMEOWNERS' PREMIUM RATES (FULL COVERAGE)

	BRICK		FRAME	
AMOUNT OF INSURANCE	STANDARD	DELUXE	STANDARD	DELUXE
$ 5 000	$37	$49	$41	$53
6 000	37	50	42	54
7 000	39	51	44	56
8 000	40	52	44	57
9 000	41	54	45	58
10 000	42	55	46	59
11 000	43	57	48	62
12 000	44	58	49	63
13 000	44	59	50	64
14 000	45	60	50	65
15 000	46	61	51	66
16 000	47	62	52	68
17 000	48	64	53	69
18 000	49	64	54	70
19 000	49	64	55	70
20 000	50	65	55	70
21 000	51	68	57	73
22 000	52	70	58	76
23 000	55	72	61	78
24 000	58	77	64	84
25 000	61	80	68	87
26 000	64	85	72	92
27 000	69	90	77	98
28 000	72	96	81	104
29 000	76	101	84	109
30 000	80	105	89	114
31 000	84	110	93	120
32 000	88	116	97	125
33 000	91	121	102	131
34 000	96	126	106	137
35 000	99	131	110	143
36 000	104	137	116	149
37 000	108	143	120	155
38 000	111	147	124	160
39 000	116	152	129	165
40 000	119	157	132	171
41 000	122	162	136	175
42 000	125	165	139	180
43 000	129	171	144	185
44 000	132	175	147	190
45 000	136	179	151	194
46 000	139	184	155	200
47 000	143	189	159	205
48 000	147	194	164	211
49 000	150	198	167	215
50 000	153	203	171	220
Each additional $1000	3.00	4.00	3.40	4.40

LIFE INSURANCE: ANNUAL PREMIUMS
(PER $1000 WORTH OF INSURANCE)

AGE AT ISSUE	TERM		WHOLE LIFE (TO AGE 85)	LIMITED-PAYMENT LIFE (TO AGE 65)	ENDOWMENT	
	10 YEARS	15 YEARS			TO AGE 60	TO AGE 65
15	$3.58	$4.08	$11.14	$11.82	$14.35	$14.98
16	3.64	4.10	11.48	12.21	14.85	15.86
17	3.70	4.13	11.82	12.60	15.34	16.74
18	3.76	4.15	12.16	12.99	15.84	17.62
19	3.83	4.17	12.50	13.38	16.34	18.50
20	3.90	4.20	12.84	13.77	16.84	19.37
21	3.96	4.22	13.25	14.26	17.48	20.17
22	4.02	4.24	13.66	14.75	18.13	20.97
23	4.09	4.26	14.07	15.24	18.78	21.77
24	4.16	4.28	14.48	15.73	19.45	22.57
25	4.23	4.31	14.87	16.23	20.07	23.38
26	4.29	4.42	15.39	16.90	20.92	24.48
27	4.36	4.53	15.91	17.58	21.77	25.58
28	4.43	4.64	16.44	18.26	22.62	26.68
29	4.50	4.75	16.97	18.94	23.48	27.79
30	4.57	4.86	17.48	19.61	24.34	28.90
31	4.74	5.10	18.15	20.54	25.52	30.49
32	4.91	5.34	18.83	21.47	26.70	32.08
33	5.09	5.58	19.51	22.40	27.88	33.67
34	5.27	5.82	20.20	23.32	29.05	35.26
35	5.46	6.05	20.87	24.24	30.22	36.85
36	5.78	6.44	21.71	25.51	31.88	39.21
37	6.10	6.83	22.56	26.78	33.55	41.57
38	6.42	7.22	23.41	28.05	35.23	43.93
39	6.74	7.61	24.26	29.32	36.90	46.29
40	7.10	8.00	25.10	30.61	38.55	48.65
41	7.58	8.60	26.20	32.50	41.00	52.49
42	8.06	9.20	27.30	34.39	43.45	56.33
43	8.54	9.80	28.40	36.28	45.90	60.17
44	9.02	10.40	29.50	38.17	48.34	64.00
45	9.53	10.99	30.60	40.05	50.82	67.83
46	10.28	12.00	32.02	43.07	54.74	75.25
47	11.03	13.01	33.44	46.09	58.66	82.67
48	11.78	14.02	34.86	49.11	62.58	90.09
49	12.53	15.03	36.28	52.13	66.50	97.50
50	13.32	16.04	37.72	55.15	70.41	104.91
51	14.75	17.70	39.58	60.86	77.97	
52	16.18	19.36	41.44	66.57	85.53	
53	17.61	21.02	43.30	72.28	93.09	
54	19.04	22.68	45.16	77.99	100.64	
55	20.47	24.34	47.02	83.68	108.19	
56	22.81	25.90	49.60			
57	25.15	27.46	52.18			
58	27.49	29.02	54.76			
59	29.83	30.59	57.35			
60	32.17	32.17	59.94			

CANADA PENSION PLAN CONTRIBUTIONS AND UNEMPLOYMENT INSURANCE PREMIUMS

COTISATIONS AU RÉGIME DE PENSIONS DU CANADA ET PRIMES D'ASSURANCE-CHÔMAGE

WEEKLY PAY PERIOD — *PÉRIODE HEBDOMADAIRE DE PAIE*

$93.19 — $518.17

Remuneration *Rémunération* From-*de*	To-*à*	C.P.P. *R.P.C.*	U.I. Premium *Prime d'a.-c.*	Remuneration *Rémunération* From-*de*	To-*à*	C.P.P. *R.P.C.*	U.I. Premium *Prime d'a.-c.*	Remuneration *Rémunération* From-*de*	To-*à*	C.P.P. *R.P.C.*	U.I. Premium *Prime d'a.-c.*	Remuneration *Rémunération* From-*de*	To-*à*	C.P.P. *R.P.C.*	U.I. Premium *Prime d'a.-c.*
$ 93.19 -	93.73	1.44	1.31	$ 133.19 -	133.73	2.16	1.87	$ 173.19 -	173.73	2.88	2.43	$ 213.19 -	218.17	3.64	2.59
93.74 -	94.29	1.45	1.32	133.74 -	134.29	2.17	1.88	173.74 -	174.29	2.89	2.44	218.18 -	223.18	3.73	2.59
94.30 -	94.84	1.46	1.32	134.30 -	134.84	2.18	1.88	174.30 -	174.84	2.90	2.44	223.19 -	228.17	3.82	2.59
94.85 -	95.40	1.47	1.33	134.85 -	135.40	2.19	1.89	174.85 -	175.40	2.91	2.45	228.18 -	233.18	3.91	2.59
95.41 -	95.95	1.48	1.34	135.41 -	135.95	2.20	1.90	175.41 -	175.95	2.92	2.46	233.19 -	238.17	4.00	2.59
95.96 -	96.51	1.49	1.35	135.96 -	136.51	2.21	1.91	175.96 -	176.51	2.93	2.47	238.18 -	243.18	4.09	2.59
96.52 -	97.07	1.50	1.36	136.52 -	137.07	2.22	1.92	176.52 -	177.07	2.94	2.48	243.19 -	248.17	4.18	2.59
97.08 -	97.62	1.51	1.36	137.08 -	137.62	2.23	1.92	177.08 -	177.62	2.95	2.48	248.18 -	253.18	4.27	2.59
97.63 -	98.18	1.52	1.37	137.63 -	138.18	2.24	1.93	177.63 -	178.18	2.96	2.49	253.19 -	258.17	4.36	2.59
98.19 -	98.73	1.53	1.38	138.19 -	138.73	2.25	1.94	178.19 -	178.73	2.97	2.50	258.18 -	263.18	4.45	2.59
98.74 -	99.29	1.54	1.39	138.74 -	139.29	2.26	1.95	178.74 -	179.29	2.98	2.51	263.19 -	268.17	4.54	2.59
99.30 -	99.84	1.55	1.39	139.30 -	139.84	2.27	1.95	179.30 -	179.84	2.99	2.51	268.18 -	273.18	4.63	2.59
99.85 -	100.40	1.56	1.40	139.85 -	140.40	2.28	1.96	179.85 -	180.40	3.00	2.52	273.19 -	278.17	4.72	2.59
100.41 -	100.95	1.57	1.41	140.41 -	140.95	2.29	1.97	180.41 -	180.95	3.01	2.53	278.18 -	283.18	4.81	2.59
100.96 -	101.51	1.58	1.42	140.96 -	141.51	2.30	1.98	180.96 -	181.51	3.02	2.54	283.19 -	288.17	4.90	2.59
101.52 -	102.07	1.59	1.43	141.52 -	142.07	2.31	1.99	181.52 -	182.07	3.03	2.55	288.18 -	293.18	4.99	2.59
102.08 -	102.62	1.60	1.43	142.08 -	142.62	2.32	1.99	182.08 -	182.62	3.04	2.55	293.19 -	298.17	5.08	2.59
102.63 -	103.18	1.61	1.44	142.63 -	143.18	2.33	2.00	182.63 -	183.18	3.05	2.56	298.18 -	303.18	5.17	2.59
103.19 -	103.73	1.62	1.45	143.19 -	143.73	2.34	2.01	183.19 -	183.73	3.06	2.57	303.19 -	308.17	5.26	2.59
103.74 -	104.29	1.63	1.46	143.74 -	144.29	2.35	2.02	183.74 -	184.29	3.07	2.58	308.18 -	313.18	5.35	2.59
104.30 -	104.84	1.64	1.46	144.30 -	144.84	2.36	2.02	184.30 -	184.84	3.08	2.58	313.19 -	318.17	5.44	2.59
104.85 -	105.40	1.65	1.47	144.85 -	145.40	2.37	2.03	184.85 -	185.40	3.09	2.59	318.18 -	323.18	5.53	2.59
105.41 -	105.95	1.66	1.48	145.41 -	145.95	2.38	2.04	185.41 -	185.95	3.10	2.59	323.19 -	328.17	5.62	2.59
105.96 -	106.51	1.67	1.49	145.96 -	146.51	2.39	2.05	185.96 -	186.51	3.11	2.59	328.18 -	333.18	5.71	2.59
106.52 -	107.07	1.68	1.50	146.52 -	147.07	2.40	2.06	186.52 -	187.07	3.12	2.59	333.19 -	338.17	5.80	2.59
107.08 -	107.62	1.69	1.50	147.08 -	147.62	2.41	2.06	187.08 -	187.62	3.13	2.59	338.18 -	343.18	5.89	2.59
107.63 -	108.18	1.70	1.51	147.63 -	148.18	2.42	2.07	187.63 -	188.18	3.14	2.59	343.19 -	348.17	5.98	2.59
108.19 -	108.73	1.71	1.52	148.19 -	148.73	2.43	2.08	188.19 -	188.73	3.15	2.59	348.18 -	353.18	6.07	2.59
108.74 -	109.29	1.72	1.53	148.74 -	149.29	2.44	2.09	188.74 -	189.29	3.16	2.59	353.19 -	358.17	6.16	2.59
109.30 -	109.84	1.73	1.53	149.30 -	149.84	2.45	2.09	189.30 -	189.84	3.17	2.59	358.18 -	363.18	6.25	2.59
109.85 -	110.40	1.74	1.54	149.85 -	150.40	2.46	2.10	189.85 -	190.40	3.18	2.59	363.19 -	368.17	6.34	2.59
110.41 -	110.95	1.75	1.55	150.41 -	150.95	2.47	2.11	190.41 -	190.95	3.19	2.59	368.18 -	373.18	6.43	2.59
110.96 -	111.51	1.76	1.56	150.96 -	151.51	2.48	2.12	190.96 -	191.51	3.20	2.59	373.19 -	378.17	6.52	2.59
111.52 -	112.07	1.77	1.57	151.52 -	152.07	2.49	2.13	191.52 -	192.07	3.21	2.59	378.18 -	383.18	6.61	2.59
112.08 -	112.62	1.78	1.57	152.08 -	152.62	2.50	2.13	192.08 -	192.62	3.22	2.59	383.19 -	388.17	6.70	2.59
112.63 -	113.18	1.79	1.58	152.63 -	153.18	2.51	2.14	192.63 -	193.18	3.23	2.59	388.18 -	393.18	6.79	2.59
113.19 -	113.73	1.80	1.59	153.19 -	153.73	2.52	2.15	193.19 -	193.73	3.24	2.59	393.19 -	398.17	6.88	2.59
113.74 -	114.29	1.81	1.60	153.74 -	154.29	2.53	2.16	193.74 -	194.29	3.25	2.59	398.18 -	403.18	6.97	2.59
114.30 -	114.84	1.82	1.60	154.30 -	154.84	2.54	2.16	194.30 -	194.84	3.26	2.59	403.19 -	408.17	7.06	2.59
114.85 -	115.40	1.83	1.61	154.85 -	155.40	2.55	2.17	194.85 -	195.40	3.27	2.59	408.18 -	413.18	7.15	2.59
115.41 -	115.95	1.84	1.62	155.41 -	155.95	2.56	2.18	195.41 -	195.95	3.28	2.59	413.19 -	418.17	7.24	2.59
115.96 -	116.51	1.85	1.63	155.96 -	156.51	2.57	2.19	195.96 -	196.51	3.29	2.59	418.18 -	423.18	7.33	2.59
116.52 -	117.07	1.86	1.64	156.52 -	157.07	2.58	2.20	196.52 -	197.07	3.30	2.59	423.19 -	428.17	7.42	2.59
117.08 -	117.62	1.87	1.64	157.08 -	157.62	2.59	2.20	197.08 -	197.62	3.31	2.59	428.18 -	433.18	7.51	2.59
117.63 -	118.18	1.88	1.65	157.63 -	158.18	2.60	2.21	197.63 -	198.18	3.32	2.59	433.19 -	438.17	7.60	2.59
118.19 -	118.73	1.89	1.66	158.19 -	158.73	2.61	2.22	198.19 -	198.73	3.33	2.59	438.18 -	443.18	7.69	2.59
118.74 -	119.29	1.90	1.67	158.74 -	159.29	2.62	2.23	198.74 -	199.29	3.34	2.59	443.19 -	448.17	7.78	2.59
119.30 -	119.84	1.91	1.67	159.30 -	159.84	2.63	2.23	199.30 -	199.84	3.35	2.59	448.18 -	453.18	7.87	2.59
119.85 -	120.40	1.92	1.68	159.85 -	160.40	2.64	2.24	199.85 -	200.40	3.36	2.59	453.19 -	458.17	7.96	2.59
120.41 -	120.95	1.93	1.69	160.41 -	160.95	2.65	2.25	200.41 -	200.95	3.37	2.59	458.18 -	463.18	8.05	2.59
120.96 -	121.51	1.94	1.70	160.96 -	161.51	2.66	2.26	200.96 -	201.51	3.38	2.59	463.19 -	468.17	8.14	2.59
121.52 -	122.07	1.95	1.71	161.52 -	162.07	2.67	2.27	201.52 -	202.07	3.39	2.59	468.18 -	473.18	8.23	2.59
122.08 -	122.62	1.96	1.71	162.08 -	162.62	2.68	2.27	202.08 -	202.62	3.40	2.59	473.19 -	478.17	8.32	2.59
122.63 -	123.18	1.97	1.72	162.63 -	163.18	2.69	2.28	202.63 -	203.18	3.41	2.59	478.18 -	483.18	8.41	2.59
123.19 -	123.73	1.98	1.73	163.19 -	163.73	2.70	2.29	203.19 -	203.73	3.42	2.59	483.19 -	488.17	8.50	2.59
123.74 -	124.29	1.99	1.74	163.74 -	164.29	2.71	2.30	203.74 -	204.29	3.43	2.59	488.18 -	493.18	8.59	2.59
124.30 -	124.84	2.00	1.74	164.30 -	164.84	2.72	2.30	204.30 -	204.84	3.44	2.59	493.19 -	498.17	8.68	2.59
124.85 -	125.40	2.01	1.75	164.85 -	165.40	2.73	2.31	204.85 -	205.40	3.45	2.59	498.18 -	503.18	8.77	2.59
125.41 -	125.95	2.02	1.76	165.41 -	165.95	2.74	2.32	205.41 -	205.95	3.46	2.59	503.19 -	508.17	8.86	2.59
125.96 -	126.51	2.03	1.77	165.96 -	166.51	2.75	2.33	205.96 -	206.51	3.47	2.59	508.18 -	513.18	8.95	2.59
126.52 -	127.07	2.04	1.78	166.52 -	167.07	2.76	2.34	206.52 -	207.07	3.48	2.59	513.19 -	518.17	9.04	2.59
127.08 -	127.62	2.05	1.78	167.08 -	167.62	2.77	2.34	207.08 -	207.62	3.49	2.59				
127.63 -	128.18	2.06	1.79	167.63 -	168.18	2.78	2.35	207.63 -	208.18	3.50	2.59				
128.19 -	128.73	2.07	1.80	168.19 -	168.73	2.79	2.36	208.19 -	208.73	3.51	2.59				
128.74 -	129.29	2.08	1.81	168.74 -	169.29	2.80	2.37	208.74 -	209.29	3.52	2.59				
129.30 -	129.84	2.09	1.81	169.30 -	169.84	2.81	2.37	209.30 -	209.84	3.53	2.59				
129.85 -	130.40	2.10	1.82	169.85 -	170.40	2.82	2.38	209.85 -	210.40	3.54	2.59				
130.41 -	130.95	2.11	1.83	170.41 -	170.95	2.83	2.39	210.41 -	210.95	3.55	2.59				
130.96 -	131.51	2.12	1.84	170.96 -	171.51	2.84	2.40	210.96 -	211.51	3.56	2.59				
131.52 -	132.07	2.13	1.85	171.52 -	172.07	2.85	2.41	211.52 -	212.07	3.57	2.59				
132.08 -	132.62	2.14	1.85	172.08 -	172.62	2.86	2.41	212.08 -	212.62	3.58	2.59				
132.63 -	133.18	2.15	1.86	172.63 -	173.18	2.87	2.42	212.63 -	213.[illegible]	3.59	2.59				

"For remuneration in excess of $518.17:

(a) refer to page 8 under "Employee's Contribution—Calculation Method";

(b) the Unemployment Insurance premium is $2.59."

"Si la rémunération dépasse $518.17:

a) se reporter à la rubrique «Cotisation de l'employé—Méthode par le calcul», à la page 8;

b) la prime d'assurance-chômage est $2.59."

TABLE 232

WEEKLY TAX DEDUCTIONS
Basis—52 Pay Periods per Year

DÉDUCTIONS D'IMPÔT PAR SEMAINE
Base—52 périodes de paie par année

WEEKLY PAY Use appropriate bracket — PAIE PAR SEMAINE Utilisez le palier approprié	IF THE EMPLOYEE'S "NET CLAIM CODE" ON FORM TD1 IS — SI LE «CODE DE RÉCLAMATION NETTE» DE L'EMPLOYÉ SELON LA FORMULE TD1 EST DE													See note on page 14. Voir remarque p. 14.
	1	2	3	4	5	6	7	8	9	10	11	12	13	
	DEDUCT FROM EACH PAY—DÉDUISEZ SUR CHAQUE PAIE													Column A Colonne A
92.00 - 93.99	7.55	6.50	5.40	4.35	3.30	2.25	1.20	.95	.70	.45	.25	.15		
94.00 - 95.99	8.05	7.00	5.90	4.80	3.75	2.70	1.65	1.05	.80	.60	.35	.20	.05	.05
96.00 - 97.99	8.55	7.50	6.40	5.30	4.25	3.20	2.15	1.15	.90	.70	.45	.25	.10	.10
98.00 - 99.99	9.10	8.05	6.90	5.80	4.75	3.70	2.65	1.60	1.05	.80	.55	.30	.15	.15
100.00 - 101.99	9.60	8.55	7.45	6.35	5.25	4.20	3.15	2.10	1.15	.90	.65	.45	.20	.20
102.00 - 103.99	10.10	9.05	7.95	6.85	5.75	4.70	3.65	2.60	1.55	1.00	.80	.55	.25	.25
104.00 - 105.99	10.65	9.60	8.50	7.40	6.30	5.20	4.15	3.10	2.05	1.15	.90	.65	.35	.30
106.00 - 107.99	11.15	10.10	9.00	7.90	6.80	5.70	4.65	3.60	2.55	1.50	1.00	.75	.45	.30
108.00 - 109.99	11.65	10.65	9.55	8.45	7.35	6.20	5.15	4.10	3.05	2.00	1.10	.85	.55	.30
110.00 - 111.99	12.20	11.15	10.05	8.95	7.85	6.75	5.65	4.60	3.55	2.50	1.45	1.00	.65	.35
112.00 - 113.99	12.75	11.65	10.60	9.45	8.35	7.25	6.15	5.05	4.05	3.00	1.95	1.10	.75	.35
114.00 - 115.99	13.30	12.20	11.10	10.00	8.90	7.80	6.70	5.55	4.50	3.45	2.40	1.35	.90	.45
116.00 - 117.99	13.85	12.75	11.60	10.50	9.40	8.30	7.20	6.10	5.00	3.95	2.90	1.85	1.00	.85
118.00 - 119.99	14.40	13.30	12.15	11.05	9.95	8.85	7.75	6.60	5.50	4.45	3.40	2.35	1.10	1.10
120.00 - 121.99	14.95	13.85	12.70	11.55	10.45	9.35	8.25	7.15	6.05	4.95	3.90	2.85	1.40	1.40
122.00 - 123.99	15.50	14.40	13.25	12.10	11.00	9.90	8.75	7.65	6.55	5.45	4.40	3.30	1.90	1.40
124.00 - 125.99	16.05	14.95	13.80	12.65	11.50	10.40	9.30	8.20	7.10	6.00	4.90	3.80	2.40	1.40
126.00 - 127.99	16.60	15.50	14.35	13.20	12.00	10.90	9.80	8.70	7.60	6.50	5.40	4.30	2.90	1.40
128.00 - 129.99	17.15	16.05	14.90	13.75	12.55	11.45	10.35	9.25	8.10	7.00	5.90	4.80	3.40	1.40
130.00 - 131.99	17.70	16.60	15.45	14.30	13.10	11.95	10.85	9.75	8.65	7.55	6.45	5.30	3.90	1.40
132.00 - 133.99	18.25	17.15	16.00	14.85	13.65	12.50	11.40	10.25	9.15	8.05	6.95	5.80	4.35	1.45
134.00 - 135.99	18.80	17.70	16.55	15.40	14.20	13.05	11.90	10.80	9.70	8.60	7.50	6.35	4.85	1.50
136.00 - 137.99	19.35	18.25	17.10	15.90	14.75	13.60	12.45	11.30	10.20	9.10	8.00	6.85	5.35	1.50
138.00 - 139.99	19.90	18.80	17.65	16.45	15.30	14.15	13.00	11.85	10.75	9.65	8.55	7.40	5.90	1.50
140.00 - 141.99	20.45	19.35	18.20	17.00	15.85	14.70	13.55	12.40	11.25	10.15	9.05	7.90	6.40	1.50
142.00 - 143.99	21.00	19.90	18.75	17.55	16.40	15.25	14.10	12.95	11.75	10.65	9.55	8.45	6.90	1.55
144.00 - 145.99	21.55	20.45	19.30	18.10	16.95	15.80	14.65	13.45	12.30	11.20	10.10	8.95	7.45	1.55
146.00 - 147.99	22.10	21.00	19.80	18.65	17.50	16.35	15.20	14.00	12.85	11.70	10.60	9.45	7.95	1.55
148.00 - 149.99	22.65	21.55	20.35	19.20	18.05	16.90	15.75	14.55	13.40	12.25	11.15	10.00	8.50	1.55
150.00 - 151.99	23.20	22.10	20.90	19.75	18.60	17.45	16.30	15.10	13.95	12.80	11.65	10.50	9.00	1.55
152.00 - 153.99	23.75	22.65	21.45	20.30	19.15	18.00	16.85	15.65	14.50	13.35	12.20	11.05	9.55	1.55
154.00 - 155.99	24.30	23.20	22.00	20.85	19.70	18.55	17.40	16.20	15.05	13.90	12.75	11.55	10.05	1.55
156.00 - 157.99	24.90	23.70	22.55	21.40	20.25	19.10	17.95	16.75	15.60	14.45	13.30	12.10	10.60	1.55
158.00 - 159.99	25.50	24.30	23.10	21.95	20.80	19.65	18.50	17.30	16.15	15.00	13.85	12.65	11.10	1.55
160.00 - 161.99	26.10	24.90	23.65	22.50	21.35	20.20	19.00	17.85	16.70	15.55	14.40	13.20	11.60	1.60
162.00 - 163.99	26.70	25.50	24.25	23.05	21.90	20.75	19.55	18.40	17.25	16.10	14.95	13.75	12.15	1.60
164.00 - 165.99	27.30	26.10	24.85	23.60	22.45	21.30	20.10	18.95	17.80	16.65	15.50	14.30	12.70	1.60
166.00 - 167.99	27.90	26.70	25.45	24.15	23.00	21.85	20.65	19.50	18.35	17.20	16.00	14.85	13.25	1.60
168.00 - 169.99	28.50	27.30	26.05	24.75	23.55	22.40	21.20	20.05	18.90	17.75	16.55	15.40	13.80	1.60
170.00 - 174.99	29.55	28.35	27.10	25.80	24.55	23.35	22.20	21.00	19.85	18.70	17.55	16.35	14.75	1.60
175.00 - 179.99	31.05	29.85	28.60	27.30	26.05	24.80	23.55	22.40	21.20	20.05	18.90	17.70	16.10	1.60
180.00 - 184.99	32.55	31.35	30.10	28.80	27.55	26.30	25.00	23.75	22.60	21.45	20.25	19.10	17.50	1.60
185.00 - 189.99	34.05	32.85	31.60	30.35	29.05	27.80	26.50	25.25	23.95	22.80	21.65	20.45	18.85	1.60
190.00 - 194.99	35.55	34.35	33.10	31.85	30.55	29.30	28.00	26.75	25.45	24.20	23.00	21.80	20.25	1.60
195.00 - 199.99	37.05	35.85	34.60	33.35	32.05	30.80	29.50	28.25	26.95	25.70	24.45	23.20	21.60	1.60
200.00 - 204.99	38.65	37.35	36.10	34.85	33.55	32.30	31.00	29.75	28.45	27.20	25.95	24.60	23.00	1.60
205.00 - 209.99	40.30	38.95	37.60	36.35	35.05	33.80	32.50	31.25	29.95	28.70	27.45	26.10	24.40	1.70
210.00 - 214.99	41.90	40.60	39.20	37.85	36.55	35.30	34.00	32.75	31.45	30.20	28.95	27.60	25.90	1.70
215.00 - 219.99	43.55	42.25	40.85	39.45	38.10	36.80	35.50	34.25	32.95	31.70	30.45	29.10	27.40	1.70
220.00 - 224.99	45.15	43.85	42.50	41.10	39.70	38.35	37.00	35.75	34.50	33.20	31.95	30.65	28.90	1.75
225.00 - 229.99	46.80	45.50	44.10	42.75	41.35	39.95	38.60	37.25	36.00	34.70	33.45	32.15	30.40	1.75
230.00 - 234.99	48.45	47.10	45.75	44.35	43.00	41.60	40.20	38.85	37.50	36.20	34.95	33.65	31.90	1.75
235.00 - 239.99	50.05	48.75	47.35	46.00	44.60	43.25	41.85	40.45	39.10	37.70	36.45	35.15	33.40	1.75
240.00 - 244.99	51.70	50.40	49.00	47.65	46.25	44.85	43.50	42.10	40.70	39.35	37.95	36.65	34.90	1.75
245.00 - 249.99	53.40	52.00	50.65	49.25	47.90	46.50	45.10	43.75	42.35	40.95	39.60	38.15	36.40	1.75

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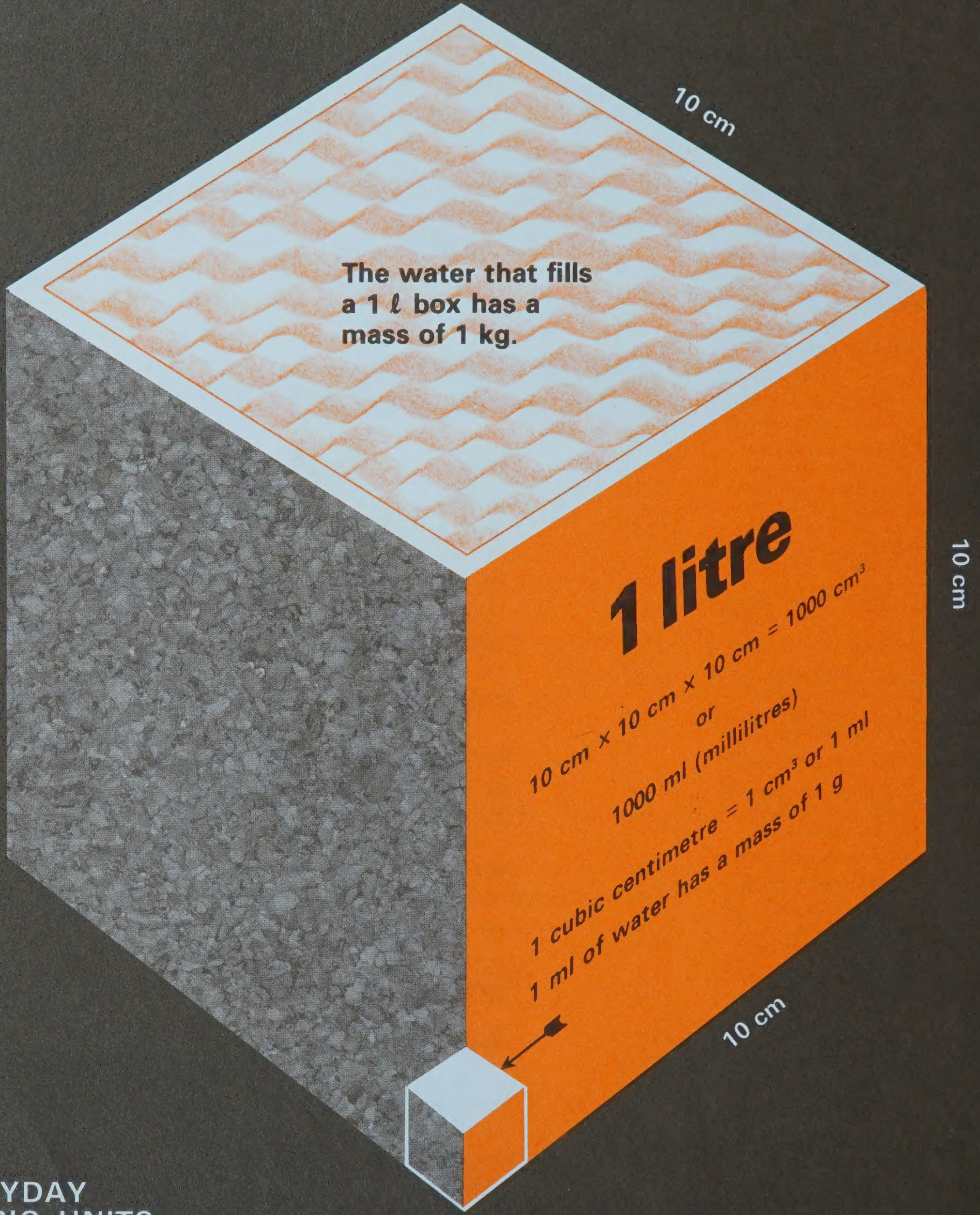
10 cm
The water that fills a 1 ℓ box has a mass of 1 kg.
10 cm
1 litre
10 cm × 10 cm × 10 cm = 1000 cm³
or
1000 ml (millilitres)
1 cubic centimetre = 1 cm³ or 1 ml
1 ml of water has a mass of 1 g
10 cm

EVERYDAY METRIC UNITS

Length		*Area*		*Volume*		*Mass*	
millimetre	mm	square centimetre	cm^2	cubic centimetre	cm^3 (or ml)	gram	g
centimetre	cm	square metre	m^2	litre (1000 cm^3)	litre or ℓ	kilogram	kg
metre	m	hectare (10 000 m^2)	ha	kilolitre	kl	tonne	t
kilometre	km	square kilometre	km^2				

SI Base Units

Quantity	*Unit*	*Symbol*
length	metre	m
mass	kilogram	kg
time	second	s
temperature	kelvin*	K
current	ampere	A
luminous intensity	candela	cd
amount of substance	mole	mol

*degrees Celsius (°C) for everyday use

SI Prefixes

Prefix	*Multiplication Factor*		*Symbol*
tera	1 000 000 000 000	$= 10^{12}$	T
giga	1 000 000 000	$= 10^{9}$	G
mega	1 000 000	$= 10^{6}$	M
kilo	1 000	$= 10^{3}$	k
hecto	100	$= 10^{2}$	h
deca	10	$= 10^{1}$	da
deci	0.1	$= 10^{-1}$	d
centi	0.01	$= 10^{-2}$	c
milli	0.001	$= 10^{-3}$	m
micro	0.000 001	$= 10^{-6}$	µ
nano	0.000 000 001	$= 10^{-9}$	n
pico	0.000 000 000 001	$= 10^{-12}$	p

SI Derived Units

Quantity	*Unit*	*Symbol*	*Formula*
area	square metre	m^2	
volume	cubic metre	m^3	
density	kilogram per cubic metre	kg/m^3	
frequency	hertz	Hz	1/s
velocity, linear	metre per second	m/s	
acceleration, linear	metre per second per second	m/s^2	
force	newton	N	$kg \cdot m/s^2$
energy	joule	J	$N \cdot m$
power	watt	W	J/s
pressure	pascal	Pa	N/m^2
electric charge	coulomb	C	$A \cdot s$
potential difference	volt	V	W/A
electric resistance	ohm	Ω	V/A
luminous flux	lumen	lm	$cd \cdot sr$
illumination	lux	lx	lm/m^2

SI Conventions

- No period after the symbol, except to end a sentence.
- Same symbol used for singular and plural.
- Most symbols are small letters. Capitals are used when the unit is named after a famous person, as N for the newton.
- Units of area are written cm^2, m^2, and volume cm^3, m^3.
- Use a space instead of a comma for numbers of more than four digits, as 10 000 m or 0.000 75 mm.

gage

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metric métrique

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ISBN 0 7715 3593 7